N72-22884

N. A. Rynin

INTERPLANETARY FLIGHT AND COMMUNICATION

Volume II, No. 6

Superaviation and Superartillery

TRANSLATED FROM RUSSIAN

Published for the National Aeronautics and Space Administration
and the National Science Foundation, Washington, D.C.
by the Israel Program for Scientific Translations

N.A. Rynin

INTERPLANETARY FLIGHT AND COMMUNICATION

(Mezhplanetnye soobshcheniya)

Volume II, No. 6

SUPERAVIATION AND SUPERARTILLERY

(Superaviatsiya i superartilleriya)

Leningrad 1929

Translated from Russian

Israel Program for Scientific Translations
Jerusalem 1971

TT 70-50116
NASA TT F-645

Published Pursuant to an Agreement with
THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
and
THE NATIONAL SCIENCE FOUNDATION, WASHINGTON, D. C.

IPST Cat.No.5817

Translated by Dipl.-Ing. T. Pelz, M. Sc.

Printed in Jerusalem by Keter Press

Available from the
U.S. DEPARTMENT OF COMMERCE
National Technical Information Service
Springfield, Va. 22151

Table of Contents

PREFACE

This volume is the fifth in a series of books published separately by the author under the general title "Interplanetary Flight and Communication."

The following four books have already appeared:

1. Dreams, Legends, and Early Fantasies.
2. Spacecraft in Science Fiction.
3. Rockets.
4. Theory of Rocket Propulsion.

The sixth volume, "Radiant Energy. Science Fiction and Scientific Projects," is being printed and will appear in the near future.

The remaining volumes are:

7. K. E. Tsiolkovskii. Life, Writings and Rockets.
8. Theory of Space Flight.
9. Astronavigation. Theory, Annals, and Bibliography.

Chronicles and a bibliography have been prepared for printing, but for financial reasons it is still unknown when they will be published.

Any comment on the volumes already published should be addressed to the author, Nikolai Alekseevich Rynin, 37, Kolomenskaya ul., ap. 25, Leningrad.

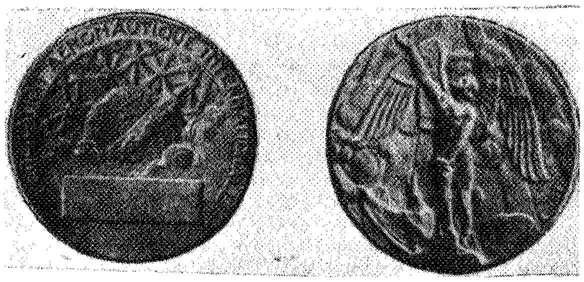


FIGURE 1. Flight among the stars *

5

INTRODUCTION

The problem of rapid transportation has long attracted attention, and a large number of scientists and engineers have worked and are working on it. This problem has now been solved for persons and freight through aviation, and for means of destruction, through artillery. However, the present state of technology obviously limits human flight in altitude, range, and speed. The "present state of technology" is understood as the use of propeller-driven aircraft with wings. Such a limit also exists for modern artillery using ordinary guns, even of large size, with internal charges and firing projectiles carrying explosives. New ways have to be found both in aviation and in artillery to overcome the above-mentioned limitations. New problems will then have to be solved under the old condition, common to both, "to carry a given load over a maximum distance as fast as possible."

Speeds of approximately 160 m/sec (575 km/hr) were [at the time of writing] attained in aviation, while the maximum [muzzle] velocities of artillery guns are about 1,600 m/sec. However, human minds cannot rest at these achievements, and attempt to exceed these limits.

Three schools exist in aviation, in connection with these attempts:

1. The **normal** school, which bases itself on present achievements of aeronautical engineering, and proceeds in the direction of increasing the size of airplanes and the power of engines.

2. The **evolutionary** school which anticipates success in increasing the speed of transportation through flight in the stratosphere, i.e., at altitudes of 12–25 km, where the air is very thin and where the drag is thus very small.

The adherents of this school retain the engine driving the propeller in the airplane but claim that flight in the stratosphere requires special airscrews and that the engine must be designed for the propeller, i.e., that the usual procedure in aviation should be reversed. The engine must be supercharged. Flight speeds of 1,000–2,000 km/hr might be possible under these conditions.

* Figure 1 shows a medal by the sculptor Blin. One side represents Victory with wings, standing on a very high mountain and stretching out a hand toward the sky in order to greet the heroes of the air. The other side represents the lines of communications around our planet amongst the stars.

3. The **revolutionary** school asserts that rapid transportation requires leaving the atmosphere and flying outside it, since the air restricts the flight speed. This eliminates the propeller and introduces rocket engines whose operation is not only independent of the atmosphere, but is even improved by its absence.

In this respect the revolutionary school is close to those scientists and engineers who work on problems of space flight and interplanetary communication, where flight speeds of up to 12 km/sec are being considered.

Attempts to increase the muzzle velocity and range of guns caused the formation of analogous schools in artillery, namely:

1. The **normal** school, anticipating success with larger guns.
2. The **evolutionary** school, which plans guns firing rocket missiles.
3. The **revolutionary** school, which considers its task to be the design of rocket missiles not fired by guns.

The complex of problems forming the subject of research by adherents of the above-mentioned schools may be defined by the terms **superaviation** and **superartillery**; it is, however, more correct to assign these terms only to those problems which are being considered by the evolutionary and the revolutionary schools, and apply the word "super" to the flight of airplanes and missiles above the troposphere, i. e., in and above the stratosphere.

An attempt is made in this book to discuss superaviation and superartillery, and to show that they are closely linked in regard to the methods of solving the problems encountered.

In particular, we shall consider all investigations dealing with high-altitude flights within the terrestrial atmosphere, and the firing of projectiles from guns to large distances.* In addition, the book also includes four monographs whose contents are closely related to superaviation, the first of which also deals with superartillery.

These are:

1. Data on the properties of the atmosphere up to high altitudes.
2. The living conditions for human beings at high altitudes.
3. The influence of large accelerations on human beings.
4. The operation of instruments measuring the acceleration during flight.

* Problems concerning rocket missiles were already considered in the volume "Rockets."

7

FIGURE 2. Bird Simurg

Chapter I.

SUPERAVIATION

Epigraph

Eternally, never dying the bird Simurg spreads its gray wings above the peaks of Elbrus.

With one eye it looks at the past and with the other at the future. Peering forward, it powerfully flaps its wings, and then everything on earth seethes in a mighty stream, strong winds rise, the sea boils, and new life is born in tempests and storms.

That eye, however, which looks toward the nebulous smoke of the past, is needed by the bird Simurg in order to flap its wings again and again, stronger and stronger to move the violet spring floods of life toward the beckoning new coast. The gray wings do not cease to move.

(Caucasian legend).

1. FLIGHT RECORDS

The progress of aviation greatly advanced the [first] World War and has not stopped since then. Many new types of aircraft have appeared whose performance considerably exceeds that of the planes used in World War I. The achievements of aeronautical engineering have found expression during the last years in so-called flight records. The principal flight records refer to the altitude, speed, range, and duration of continuous flight without landing [endurance].

Figure 3 shows the changes in these records between 1908 and 1929.

Although these curves rise steeply up to 1929, it should be remembered how difficult it was to achieve the latest records. It may be assumed that at the present state of technology it will hardly be possible to exceed these records considerably, particularly in altitude and flight speed. The diagram shows that the latest altitude record is 12,739 m, the latest speed record, 514.308 km/hr,* the longest distance covered without landing or refueling during flight, 7,666.6 km,* and the maximum endurance 65 hrs and 28 min. Engineer Bréguet, in his report to the French Academy of Sciences on 9 May 1927, stated that it is theoretically possible to cover 25,000 km in an airplane without landing.

Table 1 gives data on existing land- and seaplanes (Nos. 1–52). Figures 4–12 show some of them. These planes have not attained flight records but are remarkable for their size, power, and load-carrying capacity. Their engine power varies between 940 and 6,000 hp, their load-carrying capacity between 1,100 and 16,000 kg, and their gross weight in flight between 5,400 and 51,000 kg at speeds of up to 250 km/hr. We present data on these airplanes in order to show that despite their large size and corresponding names (Superwal, Supermarine, Supergoliath, etc.) they do not belong to the superaviation machines which will be considered later.

* These records have been slightly improved while this book was in print. The speed record is now 575.6 km/hr, while the maximum range is approximately 7840 km.

(12)

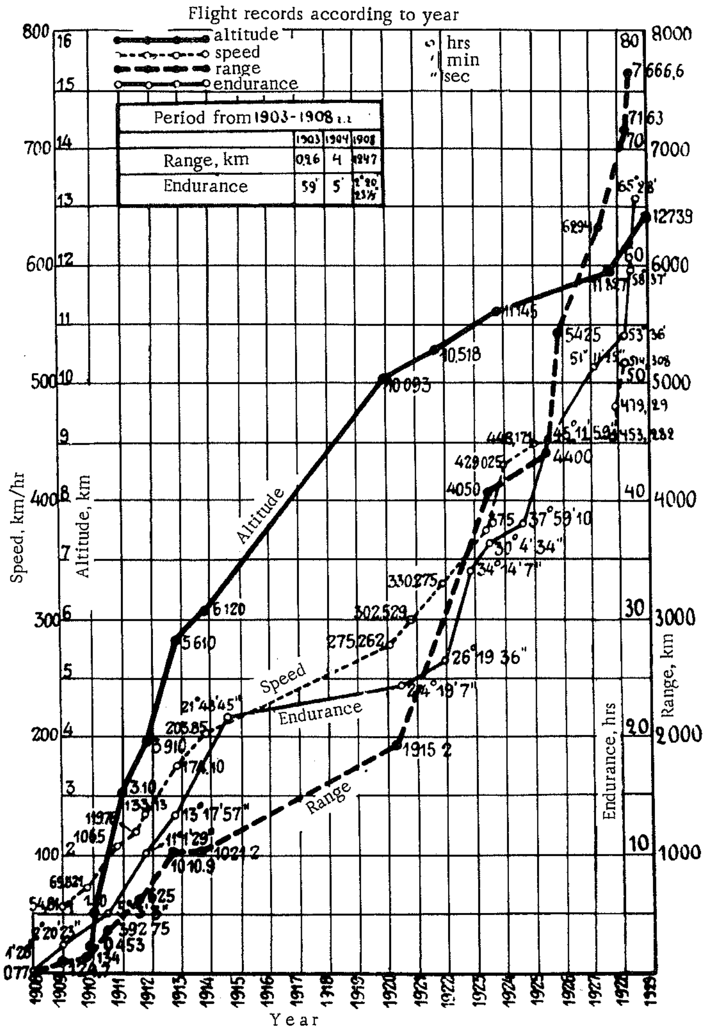


FIGURE 3. Flight records according to years

(13)

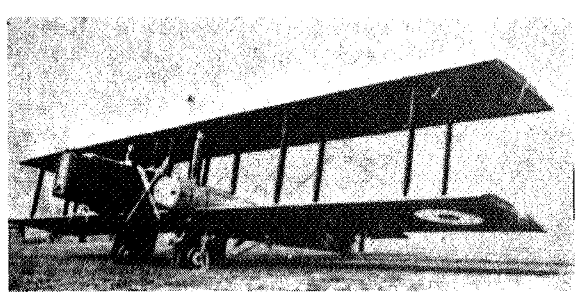


FIGURE 4. Farman Supergoliath

8
9
TABLE 1. Large existing and planned airplanes

Serial number	Designation	Type	Engines			Dimensions			
			number	power of each, hp	total power, hp	span, m	length, m	height, m	wing area, m^2
						A. Existing			
1	Supermarine Southampton	flying boat	2	470	940	22.81	14.8	5.7	133
2	N.S.Navy (P.N.$_9$)	"	2	500	1,000	22.3	15.0	5.0	125
3	Macchi (Italy)	"	2	500	1,000	21.75	14.03	—	110
4	Savoia-Marchetti	"	2	500	1,000	27.32	—	—	105
5	Caproni "73-ter"	wheel landing gear	2	500	1,000	25.0	15.1	5.6	143
6	Blériot No.127	"	2	500	1,000	23.2	14.5	3.4	88
7	Cams 54 G.R.	flying boat	2	500	1,000	20.4	14.84	—	113
8	Latham 47 G.R.	"	2	500	1,000	25.5	16.0	—	130
9	Savoia 55	"	2	500	1,000	26.34	—	—	96.0
10	Cams S 53	"	2	500	1,000	20.4	14.82	5.52	115
11	Farman 180	wheel landing gear	2	500	1,000	26.0	18.0	5.0	175
12	Blériot 195	"	4	250	1,000	23.2	—	—	90
13	Fokker T.4	seaplane	2	540	1,080	25.8	17.6	6.0	96
14	Besson (M.B.36)	flying boat	3	380	1,140	25.0	18.75	4.70	130
15	Strana Sovetov	wheel landing gear	2	600	1,200	28.7	18.01	6.09	120
16	Junkers G.31	"	3	400	1,200	30.3	16.2	6.0	94.5
17	Cant 16-ter	flying boat	3	400	1,200	23.2	14.94	5.72	139
18	Loland-Pillard	"	3	420	1,260	30.0	18.0	4.5	140
19	Lioret-Olivier 15	"	3	420	1,260	28.1	16.95	—	136
20	Supermarine-Solent	"	3	425	1,275	22.8	15.3	5.8	146.6
21	Fokker F.10	wheel landing gear	3	425	1,275	24.17	15.25	3.86	78
22	Dornier Superwal	flying boat	2	650	1,300	28.5	24.6	5.2	143
23	Rohrbach Rocco	"	2	650	1,300	26.0	19.3	6.65	94
24	Rohrbach Rocco V[6]	"	2	650	1,300	26.7	19.0	6.5	94
25	Short Cromarty	"	2	650	1,300	34.1	18.25	7.0	172.5
26	Short Singapore	"	2	670	1,340	—	—	—	—
27	Short Calcutta	"	3	450	1,350	28.35	19.75	6.78	170
28	DeHavilland Hercules (66)	wheel landing gear	3	450	1,350	26.1	18.2	6.3	143
29	Berny-Konings	flying boat	3	450	1,350	37.5	26.74	7.6	178
30	Junkers G.31 A	wheel landing gear	3	450	1,350	30.3	16.5	6.0	94.6
31	Brunelli	"	2	700	1,400	27.0	14.1	—	94.57
32	Rohrbach Robbe II	flying boat	2	700	1,400	21.5	15.2	5.9	55.0
33	Keystone-Patrican	wheel landing gear	3	525	1,575	27.45	19.21	4.0	94.0
34	Latham H.R.5	flying boat	4	400	1,600	32.5	21.0	—	255
35	Boeing P.B.I.	"	2	800	1,600	26.35	16.0	5.32	—
36	Blackburn Iris	"	3	650	1,950	—	—	—	—

Weight, kg			Load, kg		speed, km/hr	ceiling, m	range, km	Remarks
empty	payload	gross	per m^2	per hp				
Airplanes								
4,000	2,400	6,400	48	6.9	174	4,300	1,100	
4,000	4,000	8,000	71.2	8.8	—	—	—	
3,700	1,850	5,550	45.5	5.0	190	—	1,000	
3,900	2,200	6,100	—	—	210	—	—	for 10 men
3,400	2,000	5,400	37.76	—	180	5,000	—	10 men + 2 crew + 300 kg freight for $5^1/_2$ hr.
3,365	1,101	4,466	50.8	4.46	221	8,100	—	
4,400	4,000	8,400	74.0	8.4	210	2,300	4,300	
—	4,000	—	—	—	180	—	4,300	
4,650	3,500	8,150	88.5	8.15	197	—	—	
4,070	2,210	6,280	54.6	6.28	220	5,000	1,000	
4,000	4,000	8,000	45.5	8.00	195	4,000	1,000	
3,600	4,900	8,500	—	—	—	—	—	
4,150	2,400	6,550	68.2	7.3	200	4,400	—	
4,870	2,865	7,735	59.5	6.8	185	—	—	
4,290	2,170	6,460	53.83	5.38	207	4,760	—	
—	—	7,700	—	—	185	—	—	
4,500	2,500	7,000	50.3	5.8	195	—	—	
5,100	3,200	8,200	58.5	6.5	200	5,000	1,500	for 13 men
3,950	2,700	6,650	48.8	5.27	150	—	—	
4,469	2,931	7,400	50.5	5.85	178.5	3,300	1,530	
3,397	2,250	5,647	72.6	4.44	233	5,450	965	
6,550	5,250	11,800	82.5	9.1	192	2,300	1,430	
5,990	3,610	9,600	102	7.38	220	3,150	1,300	for 16 men
5,375	—	7,000	95.8	6.9	210	2,600	—	
4,810	3,510	8,320	48.15	6.4	153	—	—	
—	—	—	—	—	—	—	—	
5,730	3,455	9,185	54	5.83	193	1,190	—	
4,300	2,370	6,670	47	4.9	200	—	—	
7,500	5,400	12,900	72	7.5	221	—	3,200	according to prospectus
4,970	2,730	7,700	81.5	5.7	200	4,300	1,000	
3,915	1,800	5,715	—	—	184	5,000	—	20 passengers
3,600	2,080	5,680	103	4.1	224	5,900	4,000	6 hr; 20 men
3,853	2,943	6,796	73.87	4.31	250	4,900	—	6 hr
7,800	—	10,950	43.0	6.85	160	—	—	
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	

10

11

Serial number	Designation	Type	Engines			Dimensions		
			number	power of each, hp	total power, hp	span, m	length, m	height, m
37	Saunders Valkyrie	flying boat	3	650	1,950	29.6	20.1	—
38	Beardmore Inflexible	wheel landing gear	3	650	1,950	48.0	23.0	6.45
39	Dornier Superwal	flying boat	4	500	2,000	28.6	24.6	5.9
40	Farman Supergoliath	wheel landing gear	4	500	2,000	35.0	19.7	6.5
41	Caproni 79	"	4	500	2,000	—	—	—
42	Lot XXIII (Bréguet)	seaplane	2	1,000	2,000	—	—	—
43	Pennoët	flying boat	5	420	2,100	39.25	27.0	6.5
44	Rohrbach Inflexible	"	3	700	2,100	—	—	—
45	Fokker F.32	wheel landing gear	4	525	2,100	—	—	—
46	Rohrbach Romar	flying boat	3	720	2,160	36.91	22.55	8.47
47	Barling Bomber	wheel landing gear	6	420	2,400	36.6	19.8	8.55
48	Fairey No.4 Atalanta	flying boat	4	600	2,400	26.55	16.0	5.32
49	Junkers J.38	wheel landing gear	2 2	850 400	2,500	45.0	23.0	5.3
50	Fairey Atalanta I	flying boat	4	650	2,600	42.4	20.1	—
51	Blackburn	"	6	500	3,000	—	—	—
52	Dornier X	"	12	500	6,000	48.0	40.05	6.45
							B. Planned	
53	Fokker	flying boat	4	700	2,800	60	—	—
54	Dornier	"	—	—	3,000	70	—	—
55	Schiffer	"	6	600	3,600	24	26	5.8
56	Lorenz (Spain)	seaplane wheel landing gear	6	750	4,500	—	—	—
57	Levin Chagniard Kartewell	"	7	800	5,600	60	—	—
58	Parseval	flying boat	6	1,000	6,000	76	—	—
59	Kussner	wheel landing gear	4	—	—	—	—	—
60	Kirst	"	6	—	6,300	—	—	—
61	Kussner	"	6	1,250	7,500	—	—	—
62	Richard	flying boat	9	1,000	9,000	—	—	—
63	Rumpler No.1	"	10	1,000	10,000	94	39.3	9.2
64	Rumpler No.2	"	10	1,000	10,000	88	48.7	13.3
65	Grùlich	"	10	1,000	10,000	115	60	—
66	Junkers J.1,000 Amphibian	flying wing	4	3,000	12,000	72	—	—
67	Klampt	flying boat	5	3,000	15,000	140	46	17
68	Bréguet	flying wing	—	—	—	—	—	—
69	Moës	flying boat	—	—	—	—	—	—
70	Brénner	"	—	—	—	—	—	—
71	Blériot	"	4	—	—	—	—	—
72	French twin-deck	wheel landing gear	4	405	1,620	—	—	—
73	Sikorsky	"	—	—	—	—	—	—
74	Caprön	"	6	1,000	6,000	—	28	10

TABLE 1 (Continued)

	Weight, kg			Load, kg					Remarks
wing area, m^2	empty	payload	gross	per m^2	per hp	speed, km/hr	ceiling, m	range, km	
183	8,140	3,960	12,100	66.2	6.05	201	4,570	—	
183	—	—	17,000	—	—	—	—	—	
143.8	—	—	15,000	—	—	220	—	—	20 passengers
266	7,150	4,500	11,650	43.8	5.82	190	6,000	—	Fig.4
—	—	—	—	—	—	—	—	—	
—	—	1,500+ fuel	—	—	—	180	—	—	for 8 hr.
280	12,500	5,600	18,100	65	8.6	160	—	—	Fig.5
—	—	—	20,000	—	—	—	—	—	
—	—	—	—	—	—	—	—	—	
170	6,900	4,600	14,500	89.85	6.39	223	—	4,000	
371	8,750	9,400	18,150	48.5	7.5	150	3.000	1,800	Fig.6
122	5,100	5,700	10,800	68.5	6.75	201	—	—	Fig.7
290	11,000	7,000	18,000	69.0	8.3	—	—	—	35 passengers, Figs. 8–9
269.4	9,714	3,286	13,000	44.6	5	162	4,300	—	
—	—	—	30,000	—	—	—	—	1,600	50 passengers
467.7	35,000	16,000	51,000	110	8.6	250	—	—	60 passengers + 10 crew Figs. 10–12
Airplanes									
—	—	—	—	—	—	—	—	—	60 passengers (old project)
—	—	—	—	—	—	—	—	—	old project
100	7,070	12,930	20,000	200	5.5	400		—	Fig.25
—	11,000	9,000	20,000	—	—	—	—	—	50 passengers
—	—	—	43,000	—	—	—	—	—	50 passengers; Fig.19
833	20,000	3,000	50,000	60	8.0	347	15,000	—	
145	—	—	—	—	—	—	—	—	Diesel engine, Fig.26
—	—	—	55,000	—	—	—	—	—	theoretical limit
—	—	—	40,000	—	—	—	—	—	Diesel engine, Fig.26
—	4,200	48,000	90,000	—	—	—	—	6,500	42,000 kg fuel; 4,000 kg payload, 2,000 kg crew
1,000	58,000	57,000	115,000	115	11.5	275	—	—	100 passengers, Fig.20
—	50,000	65,000	115,000	—	—	300	—	—	135 passengers+ +35 crew
—	—	—	—	—	—	160	—	2,350	70 passengers, Fig.17
—	—	—	50,000	—	—	200	—	—	flying wing, Figs. 14–15
2,750	95,000	100,000	195,000	70.5	12.9	300	6,500	—	Fig.18
—	—	—	—	—	—	—	—	—	flying wing, Fig.13
—	—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	—	
—	—	—	24,000	—	—	250	—	—	
—	—	—	8,000	—	—	200	—	1,260	35 passengers
—	—	—	100,000	—	—	1,250	40,000	—	200 passengers
—	15,000	2,000	35,000	—	—	250	—	—	built in 1929

13

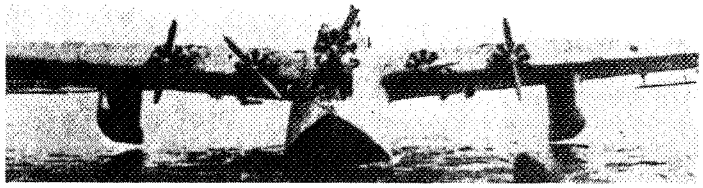


FIGURE 5. Penhoët flying boat

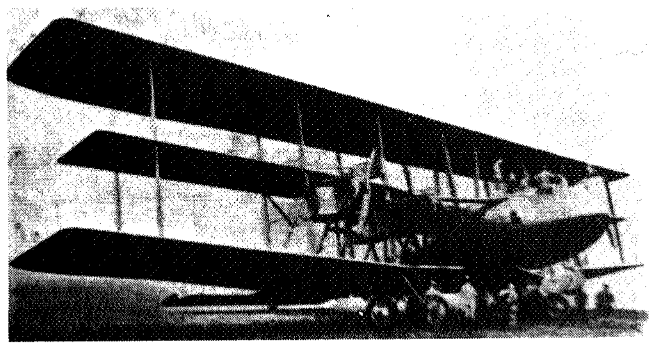


FIGURE 6. Barling bomber

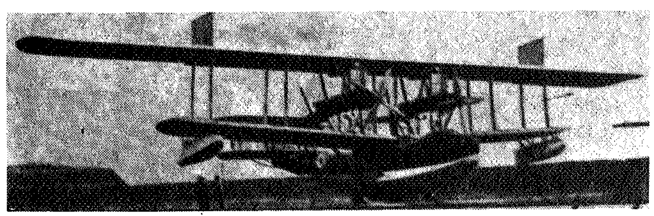


FIGURE 7. Fairey No.4 Atalanta flying boat

14

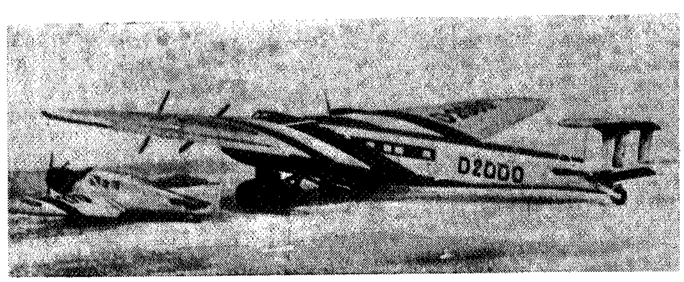


FIGURE 8. Junkers J.38

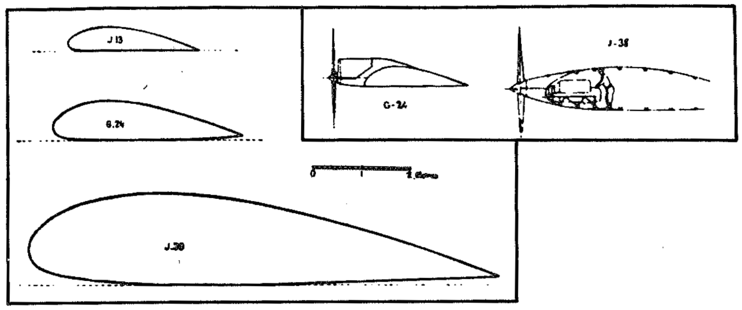


FIGURE 9. Airfoils used in Junkers planes

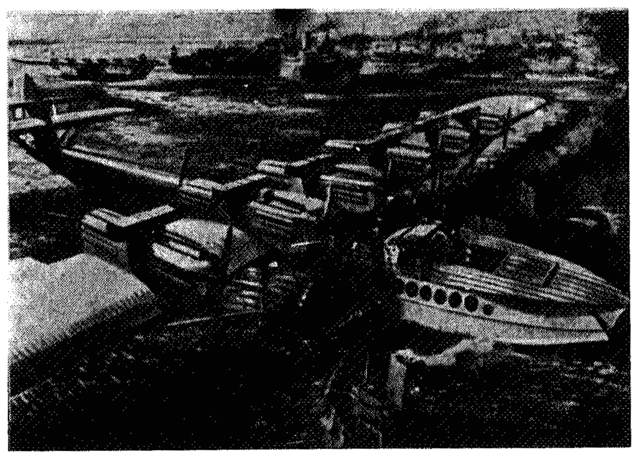


FIGURE 10. Dornier X flying boat according to initial information

15

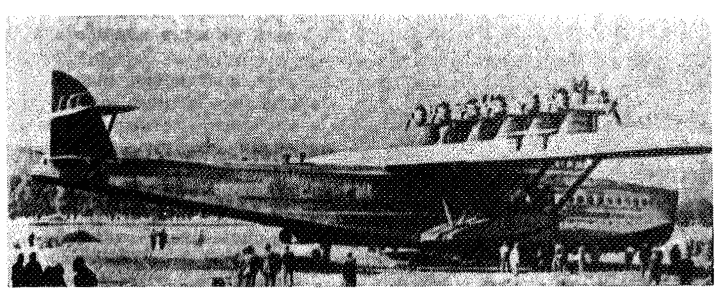


FIGURE 11. Dornier X flying boat. Overall view

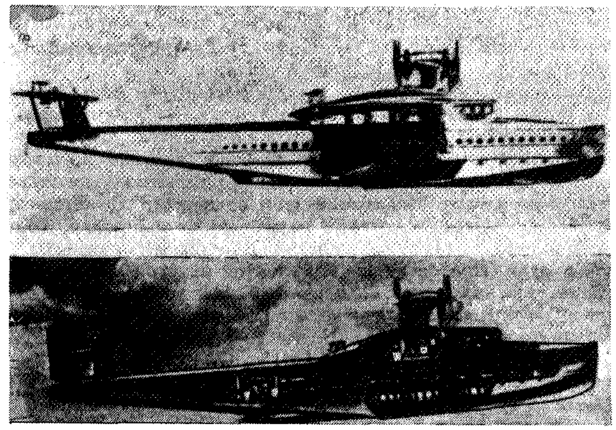


FIGURE 12. Dornier X flying boat in flight (top), section (bottom)

The Italian engineer Cerrado Gustozza has given the characteristics of future high-power airplanes (Table 2).

TABLE 2

Power [hp]	Wing dimensions				Weight, t			Load, kg	
	span, m	width, m	thickness, m	area, m^2	empty	payload	gross	per m^2	per hp
2,000	27	5.4	0.90	145	7	7	14	95	6.5
3,000	34	6.4	1.06	222	10–11	9–10	20	95	6.7
4,500	40	8.0	1.33	320	15–16	14–15	30	90	6.5
6,000	46	9.2	1.53	423	20–21	19–20	40	95	6.5
8,000	52	10.5	1.75	546	25–27	23–25	50	90	6.3
10,000	60	12.5	2.0	720	30–32	33–35	65	90	6.5
16,000	75	15	2.56	1,125	45–50	50–55	100	90	6.3

16 2. METHODS OF INCREASING THE PERFORMANCE OF AIRPLANES

The performance of airplanes must be improved in order to establish new records. This purpose may be achieved in the following ways:

a) by reducing the drag of the airplane;
b) by reducing the structural weight of the airplane;
c) by increasing the lift/drag ratio of the airplane;
d) by increasing the engine power of the airplane;
e) by selecting the optimum propeller.

We shall now consider all these methods in the manner previously employed in aviation.

a) **Reduction of drag.** To reduce the drag designers strive to eliminate from the aircraft structure everything superfluous and all protuberances in order to obtain a shape streamlined to the maximum. A simple large flying wing would be ideal in this respect. It would accommodate the engines, fuel, passengers, crew, and freight; only the propellers, rudder, and elevator protrude during flight. The undercarriage is extended for landing; fixed landing gear with either wheels or floats is provided. Such planes have not yet been built but have been designed by the French engineer Bréguet (Figure 13, No. 68 in Table 1) and the German engineer Junkers (Figure 14 and 15, No. 66 in Table 1).

b) **Reduction of structural weight of airplane.** The structural weight of an airplane can be reduced through efficient design, operating conditions, and the use of light and strong materials like Duralumin, silicone, Aeron, etc. However, certain limits are set to this weight reduction by design and operational considerations.

c) **The lift/drag ratio can be increased** by finding the optimum type of wing. Aerodynamics apparently has already indicated the optimum airfoil section. In this respect we mentioned the work of Driggs, Gullev, and Dryden in the U. S. A. who tested airplane wings in wind-tunnels at speeds of approximately 300 m/sec (velocity of sound). These tests showed that the lift coefficient decreases, while the drag coefficient increases when the flight speed approaches the velocity of sound. The propeller efficiency also decreases when the blade-tip speed approaches the velocity of sound.

17

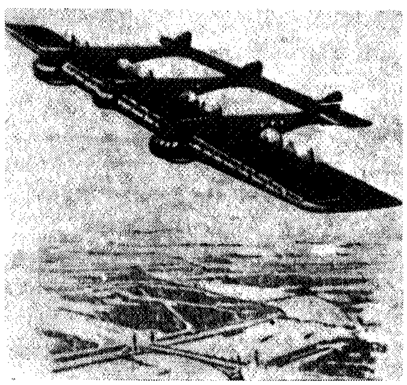


FIGURE 13. Bréguet flying wing (project)

The drag at flight speeds equal to or exceeding the velocity of sound [transonic and supersonic speeds, respectively] was investigated theoretically by D. Ryabushinskii in his paper "Sur la résistance de l'air aux grandes vitesses" (On the Resistance of Air at High Velocities), published in the Sixth Bulletin of the Aerodynamical Institute at Kutchino, Paris, 1920. However, designers have not yet encountered any limitations on absolute load-carrying capacity, and airplanes of ever increasing size are being projected.

(16)

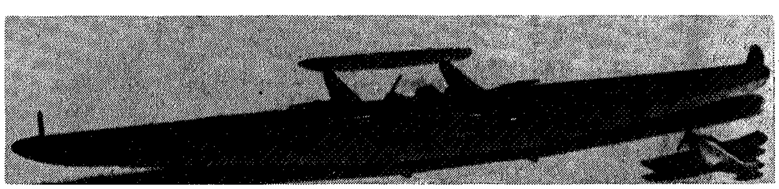


FIGURE 14. Overall view of Junkers flying wing (project)

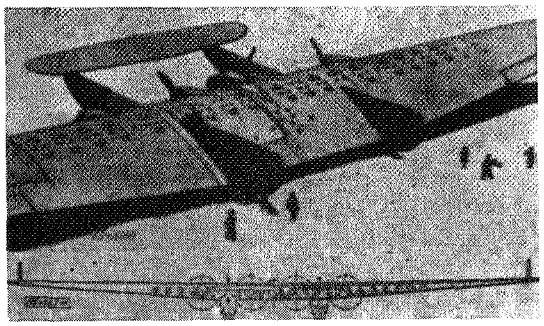


FIGURE 15. Detail of Junkers flying wing (project)

d) **Increasing the engine power.**

The engine power can be increased in various ways:

a) by equipping the airplane with one or several powerful engines;
b) by increasing the cylinder capacity (superdimensioning);
c) by increasing the compression ratio of the fuel mixture;
d) by using high-altitude carburetors;
e) by using turbochargers;
f) by selecting the optimum propeller.

a) Equipping the airplane with engines of maximum power/weight ratio is the simplest solution.

This can be done in three ways: 1) By maintaining the power of the engine constant but reducing its weight, so that a more powerful engine may be obtained at a given weight. Figure 16 shows how Isotta-Fraschini engines were improved between 1908 and 1926: in 1908 a 60 hp engine had a power/weight ratio of 1 hp/2130 g, whereas in 1926 a 500 hp engine had a power/weight ratio of 1 hp/800 g. In other words, 1 kg engine weight provided only 0.46 hp in 1908 at a total power of 60 hp, but 1.25 hp in 1926 at a total
18 engine power of 500 hp. Most recent data for 1929 indicate that a 500 hp engine has a power/weight ratio of 1 hp/640 g (1.57 kg/hp), while the "Asso 1,000" engine develops 1,200 hp at a weight of 805 kg (670 g/hp). 2) The number of engines can be increased while the power/weight ratio of each engine remains as before. This increases the total power of the airplane. 3) Both methods may be employed jointly. In fact, recent airplane designs provide for powerful engines. Data on such planes are given in Table 1. Their power varies between 2,800 and 15,000 hp. Nearly all are designed as flying boats, since this facilitates takeoff and landing. Figures 13–15, 17–20, 25, and 26 show these airplanes. The most representative of them is the Rumpler flying boat (Figure 20, Nos. 63 and 64 in Table 1). Its inventor calls it an airplane with infinite wings. The reason for this is that lengthening the wings and increasing the number of fuselages and engines driving the propellers makes it possible to build airplanes of any desired load-carrying capacity.

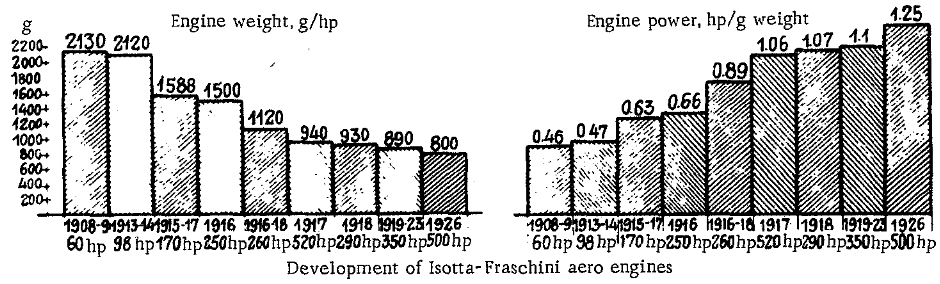


FIGURE 16. Evolution of Isotta-Fraschini engines

20 We shall now give parts of the content of Rumpler's report on large airplanes, and on his airplane in particular.

A large size is required for transoceanic flight. It is seen from Figure 21 that such a plane becomes excessively large if the weight is mainly concentrated in one fuselage. An n-fold increase in the linear dimensions alters the particulars of the airplane in the following manner if the load per unit wing area is assumed to remain constant:

(18)

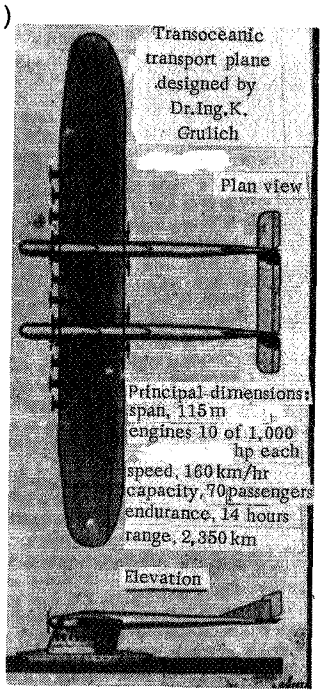


FIGURE 17. Grulich flying boat (project)

The area of the supporting surfaces increases as n^2. ; the weight G_{sp} of the wing spars increases as n^3. In other words, the weight of the wing spars increases as the gross weight to the power of 1.5, and not proportionally to it (Lanchester's formula).

The weight of those wing parts which do not take up loads, such as the wires, liners, etc., increases only proportionally to G_n, i.e., as n^2. It is further assumed that the weight G_1 of those wing parts which take up loads accounts for 80%, and that of the remainder (G_0), for 20% of the total weight G_{F_0} of the wings.

In other words,

$$G_{F_0} = G_1 + G_0 = 0.8\ G_{F_0} + 0.2\ G_{F_0}.$$

An n-fold increase in the linear dimensions of the airplane thus causes the weight of the wings to become

$$G_{F_n} = G_{F_0}\ (0.8n^3 + 0.2n^2).$$

Let the gross weight of the airplane be G_n, as before, and assume that its weight without wings and payload is $0.5 G_n$. We then have:

$$G_n = 0.5\ G_n + G_{F_n} + G_{sn},$$

where G_{zn} is the payload. Hence,

$$G_{zn} = 0.5\ G_n - G_{Fn} = 0.5 G_0 n^2 - G_{F_0}\ (0.8 n^3 + 0.2 n^2).$$

where G_0 is the gross weight of the airplane before its linear dimensions were increased n-fold.

(19)

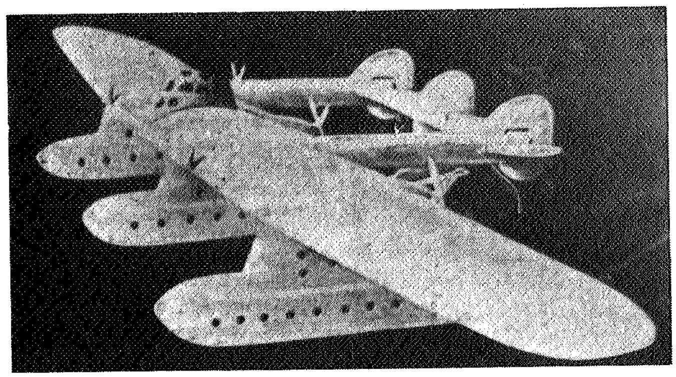


FIGURE 18. Klamt flying boat (project)

FIGURE 19. Model of Chagniard-Kartewell airplane (project)

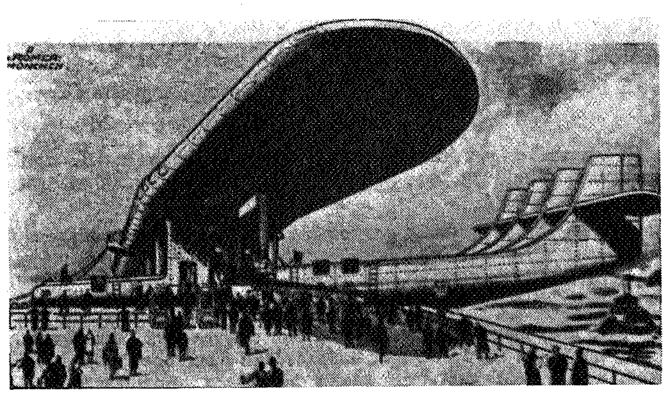


FIGURE 20. Overall view of Rumpler flying boat (project)

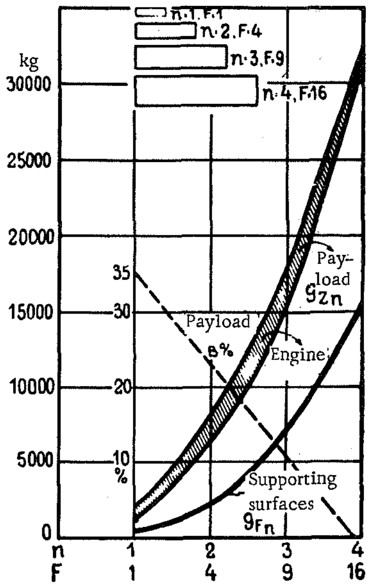


FIGURE 21. Diagram according to Rumpler

Figure 21 corresponds to the case where $G_0 = 2{,}000$ kg ($n=1$), with

$$G_{F_0} = 0{,}15\ G_0 = 300\ \text{kg},$$

and $G_{Z0} = 0.35\ G_0$

An n-fold increase in the linear dimensions of the airplane thus leads to $G_n = 2{,}000\,n^2$ kg.

The weight of the wings will then be

$$G_{F_n} = 300\ (0.8n^3 + 0.2n^2).$$

and the payload,

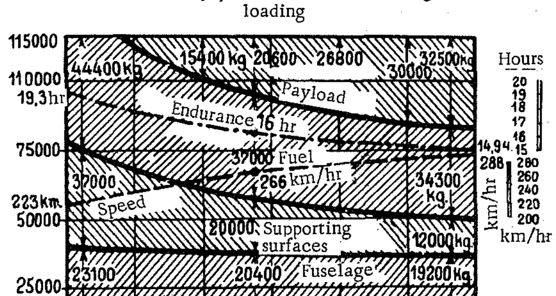
$$G_{z_n} = 0.5 \cdot 2000n^2 - 300\ (0.8n^3 + 0.2n^2).$$

The curves plotted on the basis of this calculation show that the payload decreases rapidly when the size of the airplane is increased, and becomes zero when the gross weight of the airplane is approximately 30,000 kg. This variation of the payload, expressed in % [of the gross weight], is represented in Figure 21 by the broken line.

21 Thus, increasing the size of the airplane when the load is concentrated at one place is not advisable.

The variation of the payload in Rumpler's planned airplane is more favorable since it is distributed over several floats. This is seen from Figure 22 which shows the variation of the payload as function of the wing loading for an airplane having a gross weight of 115,000 kg.

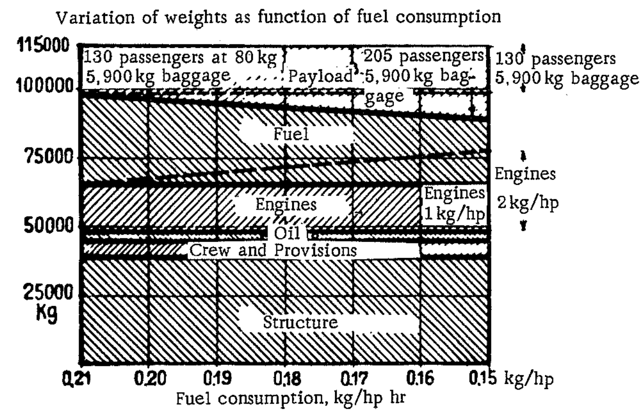
Variation of payload as function of wing loading



Gross weight, 115,000 kg, power. 10,000 hp; range, 4,300 km

FIGURE 22. Diagram according to Rumpler

The payload can, however, be increased further if the possibilities of reducing the fuel consumption are utilized as shown in Figure 23.



Gross weight, 115,000 kg; power, 10,000 hp; endurance, 16 hrs

FIGURE 23. Diagram according to Rumpler

In general, Rumpler computed the weight distribution for his plane as indicated in Figure 24, which also shows, for comparison, the weight
22 distribution for an ordinary plane.* The idea of distributing the payload over the wings is not new, having been first proposed by I. Sikorsky for his "Russkii Vityaz'" airplane.

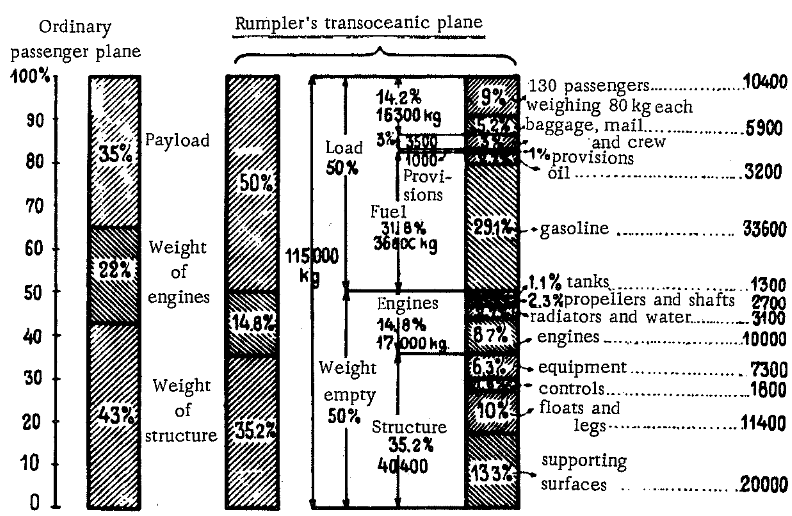


FIGURE 24. Diagram according to Rumpler

* Das Trans-Ozean Flugzeug (The Transoceanic Airplane). Lecture given by F. Rumpler. Reports and Transactions of the [German] Scientific Society for Aeronautics, 1926, No. 14, p. 37.

Schiffer's project is also of interest (Figure 25 and No. 55 in Table 1). In this design the ailerons are replaced by louvers passing through the wings; opening or closing the louvers changes the pressure on the main wings and thus alters the inclination about the longitudinal axis. In addition, louvers on the stabilizer partly act as elevators. The designers of such airplanes consider it possible to fly with them from Europe to America in one day.

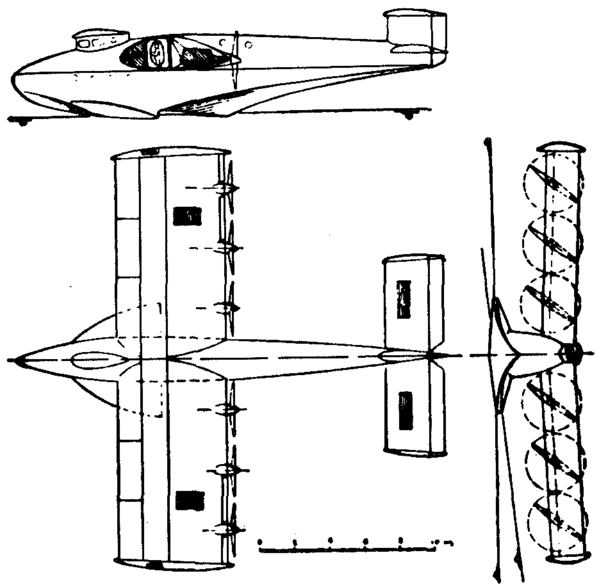


FIGURE 25. Schiffer's flying boat (project)

23 Engineer H. Borck in his paper "Höhenflugzeuge und ihre Kritik auf Grund der Leistungsanalyse"* (High-altitude Planes and their Evaluation on the the Basis of Performance Analyses)* presents an interesting method of evaluating different airplanes with respect to their capability of flying in general, and at large altitudes in particular. He found an analogy between the motion of air during the flight of an airplane and the same motion during operation of a propeller. In particular, he concluded that the Rumpler, Klampt, and Dornier airplanes would not be able to take off, whereas the Rohrbach Rofix airplane, equipped with a 450 hp engine, could attain an altitude of 13,300 m.

Borck developed his ideas in the paper "Der Leistungsbedarf von Höhenflugzeugen" (The Power Requirements of High-altitude Airplanes) (Illustrierte Flugwoche, 1927, p. 244)**, where he presented a number of diagrams characterizing the performance of the Junkers G 24 and the Rohrbach Rofix airplanes at different altitudes. Table III gives a comparison between the results obtained by him for the ceilings and the corresponding speeds of the heavier Junkers and the lighter Rohrbach plane.

* "Illustrierte Flugwoche," 1927, p.52.

** A critical review of Borck's paper was published by V.Vedrov in "Tekhnika Vozdushnogo Flota," 1928, p.51.

TABLE 3.

	Total power, hp	Power delivered to propellers, hp	Ceiling, m	Speed at ceiling, km/hr
Junkers G 24	750	591	6,350	200
Rohrbach-Rofix	450	348	24,000	540

Typical representatives of large airplanes of normal design are those built and planned by Prof. Junkers. In 1918 he was granted a patent for a freely supporting wing which was used in his well-known airplane No. 13. In recent years he has designed a series of all-metal planes with ever larger wings built according to this principle. The J-38 plane (Figure 8) is at present being built, and the J-1,000 plane (Figures 14 and 15) is planned.

Table 4 gives the main particulars of five different Junkers planes.

TABLE 4

Designation	F–13 L	W–33 L	G–24 L	G–31 L	J–38
Wing span, m	17.75	17.7	29.9	30.3	44
Wing area, m^2	43	43	100	94.6	290
Load per m^2 of wing area	58	58	65	84.6	69
Load per hp	8.5	8.5	7.35	5.3	8.3
Gross weight, kg	1,361	1,347	4,285	5,425	–
Crew, kg	75	75	150	150	375
Payload at range of 500 km	571	1,000	1,670	2,350	–
Payload at range of 1,000 km	421	810	1,170	1,645	–

24 Figure 9 shows at the same scale the airfoil sections of the J-13, G-24, and J-38 airplanes. The last-mentioned plane has a 1.80 m high wing extending beyond the engine which is installed in its interior (cf. drawing on right). Figures 14 and 15 show the planned airplane, designed by Prof. Junkers as flying wing. Its size becomes apparent from a comparison with the F. 13 shown beside it, whose wing span is 15.75 m. The J-38 is to be propelled by oil engines.

The idea of large transatlantic airplanes found expression in the projects of the German engineer Kussner who presented two designs (Figure 26) with four and six engines, respectively (Nos. 59 and 61 in Table 1). The designs were based on the use of thick wings with cabins and several engines.

Kussner proposed flight speeds of 300 to 400 km/hr for commercial planes carrying payloads of 2–3 t across the Atlantic. The gross weight of such an airplane may attain 40 t. Figure 26 shows schematically the four-engine plane on top and the six-engine one at the bottom. Diesel engines developing 1,250 hp each are to be installed. Wheeled landing gear is envisaged as causing the least drag. The ceiling is 4,000 m. The wings are almost delta-shaped; the wing loading may attain 145 kg/m^2 on long flights (up to 3,600 km) and 125 kg/m^2 at ranges of up to 2,000 km. The

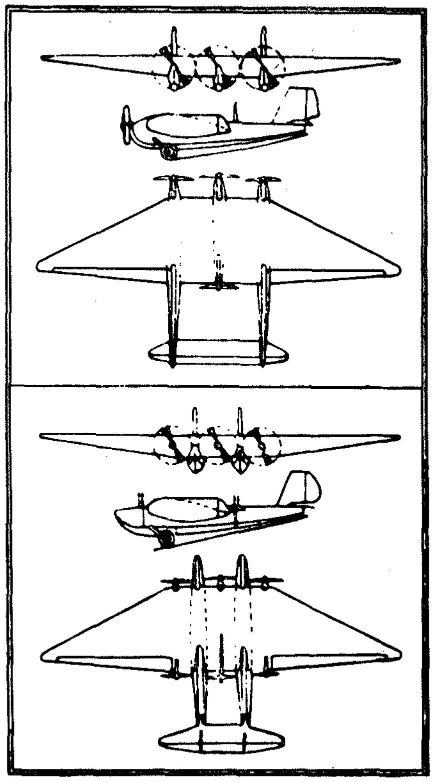


FIGURE 26. Planes proposed by Kussner

takeoff distance is to be approximately 850 m with wooden propellers having a pitch of 1.1 m and 1,350 m with metal propellers having a pitch of 1.35 m. Rocket-assisted takeoff is also possible. Opel found that such rockets develop a thrust of 200 kg per kg propellant. Air for combustion is supplied to the engines by 185 hp-Brown-Boveri turbochargers, delivering 5,630 kg air per hr.

b) The use of **superdimensioned engines** involves an increase in the engine weight, which obviously does not improve its performance. The results will probably be similar to those obtainable in the case discussed under a). This also involves the use of high-altitude carburetors discussed below.

c) **The compression ratio** can be increased up to a certain limit; this method is already used in present-day engines. Operation of an engine using precompressed air, where the pressure ahead of the carburetor is higher than the gas outlet pressure, differs considerably from operation of the same engine aspirating air directly from the atmosphere. Two main phenomena occur in this case. The excess inlet pressure causes the engine to operate like a compressed-air motor; reduction of the resistance in the cylinder, due to the improved
25 inflow, permits better charging with fresh mixture. This is very important; however, this also induces a large load on the engine, which is permissible only if the engine has been designed accordingly. Should this not be the case, it is nevertheless an advantage that the engine will develop more power at high altitudes if it aspirates compressed air, than on the ground if it aspirates atmospheric air there. It is difficult to compute the additional power thus gained, since the effects of the residual (burnt) charge, heating of the cylinder walls, and vaporization of the fuel are not known. Experiments have still to be carried out on this subject.

In his paper, published in the "Jahrbuch der W. G. L.,* 1927, p. 116, Kamm gives the increase in engine power (in % of that on the ground) at a compression ratio of 1:6. His results are presented in Figure 27, allowance being made for the decrease in the oxygen content of the air up to an altitude of 11 km. Above 11 km the effect of supercharging decreases (compression at a temperature difference of 70°C, polytropic efficiency of 0.5). The diagram shows that up to an altitude of 10 km the engine power exceeds that on the ground.

* Wissenschaftliche Gesellschaft für Luftfahrt.

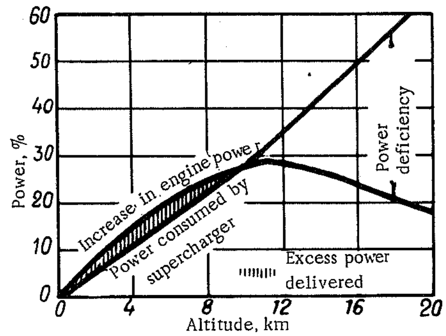


FIGURE 27

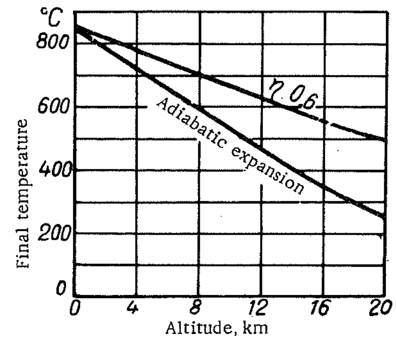


FIGURE 28

One source of energy is not utilized in this case. This is the pressure energy of the exhaust gases, which increases when the external pressure decreases.

The turbochargers used at present are still far from perfect, due to the high temperatures developed in them. However, a favorable effect of increasing altitude is that the temperature of the expanded gas decreases with the external pressure. Figure 28 shows the final temperature in the case of adiabatic expansion in an ideal Laval nozzle and in a turbine having an efficiency of 60%. An initial gas temperature of 850°C and 1 atm have been assumed. It is seen that the final gas temperature varies inversely as the altitude and the efficiency.

Utilization of the exhaust gases is increased and the thermal load on the turbine is reduced if the parts through which the gas passes are shaped like a divergent nozzle. The resistance to the flow is then eliminated, and the entire pressure energy is converted into kinetic energy. The turbine will then operate as a pure impulse turbine. Tests show how close we are to this goal.

d) **High-altitude carburetors** permit, if desired by the pilot, the amount
26 of air aspirated from the outside into the carburetor to be varied by moving a handle. Such carburetors are fitted to so-called **high-altitude** engines. An example of these is the BMW engine whose power may be varied between 185 and 230 hp. It develops only 185 hp on the ground and at altitudes up to 4,000 m. The mixture is initially supplied from the carburetor to the cylinders while the throttle is only partly open. The throttle is opened fully only at an altitude of 4,000 m.

The German flyer Diemer attained an altitude of 9.6 km in June 1919 with a DFW (T. 37) biplane equipped with a similar 185 hp engine (BMW IIIa) without supercharger.

Another example of the installation of a high-altitude carburetor is shown in Figure 29, which illustrates the new 450 hp BMW engine. It has 12 cylinders in a V arrangement, the high-altitude carburetor being located between the two rows. This carburetor consists of two float chambers, three suction pipes, and five throttles (Figure 30). Three throttles ((10), (13), (14)) are used at low altitudes and are adjusted by means of handle (17).

Throttles (11) and (12) have holes on their circumferences, through which air can always enter from the outside, these two throttles are adjusted by means of handle (19). The carburetor can maintain the engine power constant up to an altitude of 2.7 km. About 630 hp can be developed by this engine on the ground if all throttles are fully open.

e) **A turbocharger** [or supercharger] is a device in an engine which
supplies the carburetor with additional air required at the given altitude
for complete combustion of the fuel mixture. The use of turbochargers
permits a considerable increase in the airplane ceiling. The principle
of a turbocharger is as follows. The exhaust gas from the engine
27 impinges on the blades of a turbine whose shaft carries a blower; the
latter aspirates air from the atmosphere and delivers the required amount
to the carburetor and thence to the engine. Below we present schematically
four types of turbochargers. These are the French Rateau, the U. S. General
Electric (designed by Moss), the German Lorenz, and the U. S. Roots.

26

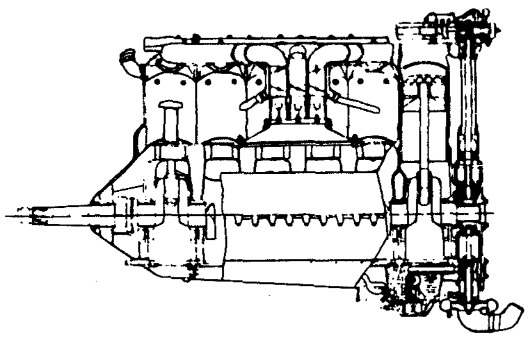


FIGURE 29. BMW high-altitude engine developing 450 hp

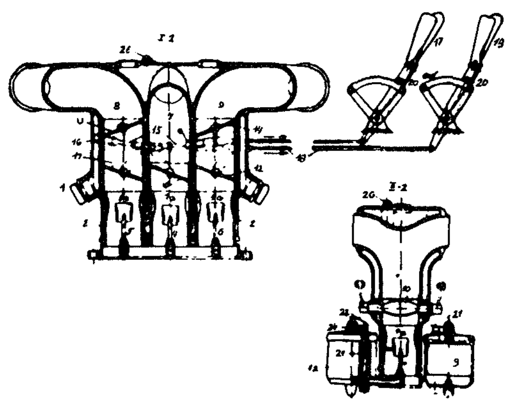


FIGURE 30. High-altitude carburetor

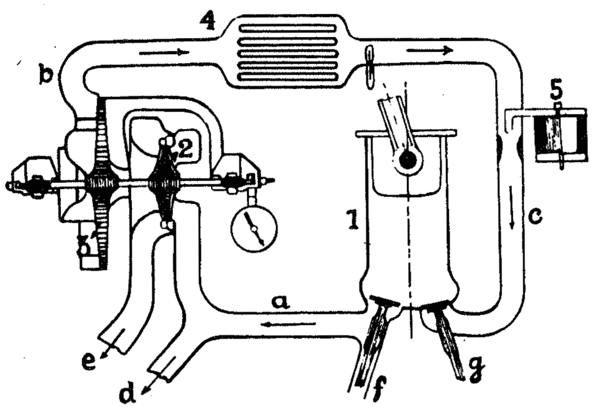


FIGURE 31. Rateau turbocharger

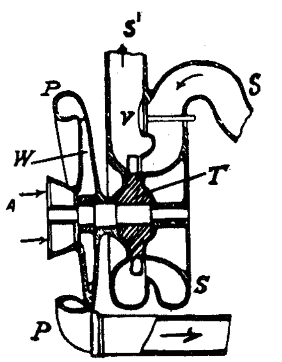


FIGURE 32. Moss (GEC) turbocharger

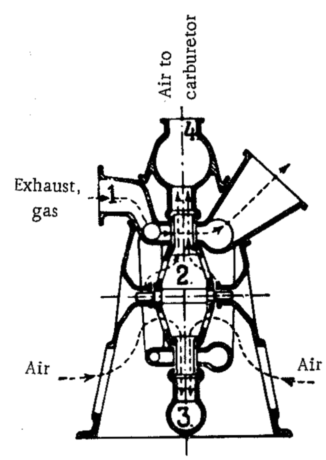


FIGURE 33. Lorenz turbocharger

Figure 31 shows schematically the Rateau turbocharger.*

The gas is exhausted from engine cylinder (1) through valve (f) and pipe (a) to turbine (2) where it impinges on the blades and thus drives it and centrifugal air blower (3) carried on the same shaft. The blower forces air through pipe (b) and cooler (4) past carburetor (5) where gasoline is injected into it. The fuel mixture thus formed enters the cylinder via valve (g). The gas is discharged from the turbine into the
28 atmosphere through pipe (e). At low altitudes the gas is not led from the cylinder to the turbine but is discharged directly to the atmosphere through pipe (d). The turbine speed is up to 30,000 rpm.

Figure 32 shows schematically the Moss (GEC) turbocharger, which is similar to the Rateau turbocharger. The exhaust gas flows through

* We also mention the Schwade, Brown-Boveri and Cie, A.E.G., Bristol-Jupiter, Cozette, Powerplus, Scherbondy, Pratt and Whitney, and Armstrong-Siddeley turbochargers.

pipe (S) to turbine (T). The excess gas is discharged directly to the atmosphere through valve (V). Blower (W) is carried on the turbine shaft. The blower aspirates air from the atmosphere through intake (A) and delivers it to the engine through pipe (P). The turbine speed is 30,000 rpm. The complete turbocharger with instruments weighs 80 kg.

Figure 33 shows the Lorenz turbocharger. The exhaust gas flows through pipe (1) and impinges on the hollow turbine blades whose interiors communicate with the widened central part of turbine disk (2) which has openings through which air is aspirated from the atmosphere, delivered by centrifugal force through the hollow blades into annular casing (3) surrounding the turbine blades, and thence supplied to the engine through pipe (4).

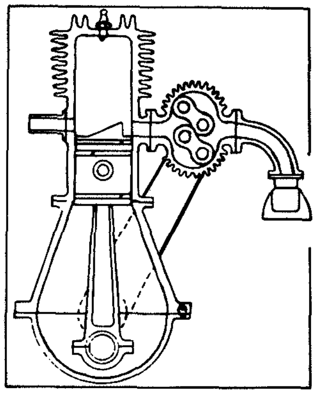


FIGURE 34. Roots blower

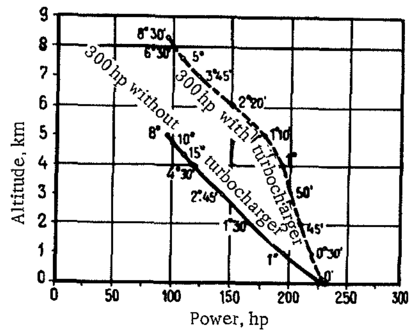


FIGURE 35

The Roots blower (USA) consists of a dismountable drum-like aluminum housing in which two rotors, each shaped like a figure 8, turn uniformly at the same speed. The rotors have cycloidal profiles and are arranged at a phase difference of 90° (Figure 34). The rotors are hollow and are also made of aluminum alloy; their wall thickness is 5 mm, they are 280 mm long, and their maximum diameter is 245 mm (for 400 hp). One rotor is driven from the crankshaft and transmits its motion to the other rotor via gears. The efficiency of such a blower varies between 75 and 83%; it increases when the rotational speed or the pressure difference is reduced. This blower requires 4 hp during idling on the ground while 40 hp are required in flight. It weighs 68 kg. The air temperature is raised by 80°C during compression, as compared with 110°C in the Rateau blower.

Figure 35 gives two curves obtained during flight tests of an airplane weighing 1,700 kg and having a wing area of 40 m^2. The engine develops 300 hp on the ground. The wing loading is thus 42.5 kg/m^2, while the power/weight ratio is 1 hp/5.7 kg. The numbers against the points on the curves indicate the angle of attack of the wings. The full line refers to engine operation without turbocharging, while the broken curve refers to

operation with turbocharging. It is seen that the airplane ceiling is 5,000 m
29 without turbocharging, but 8,300 m with turbocharging.

Figure 36 shows the flight speed as a function of the angle of attack at different altitudes with (curve ***ACD***) and without (curve ***AB***) turbocharging. Thus, the engine makes possible a speed of 200 km/hr on the ground at approximately zero angle of attack (point ***A***). The speed developed at an altitude of 5,000 m is 163 km/hr at an angle of attack of 8° without turbocharging (point ***B***) and 220.5 km/hr with turbocharging at an angle of attack of 1° (point ***C***). The speed is approximately 190 km/hr at the ceiling with turbocharging (point ***D***).

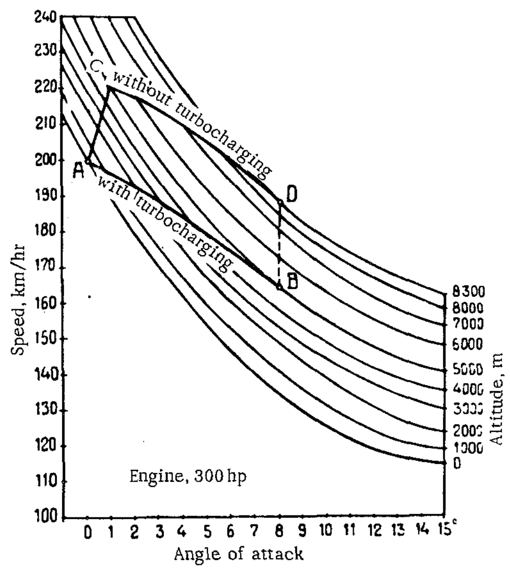


FIGURE 36

Figure 37 gives for the same airplane the following data for flight with and without turbocharging at various angles of attack: horizontal flight speed, km/hr; rate of climb, m/sec; climbing time, min.

It is seen from the diagrams that turbocharging considerably increases the ceiling and flight speed.

Figure 38 shows the takeoff diagram for the French Goudrou-Lezère fighter plane equipped with a 420 hp Jupiter engine.*

The air-force academy in Prague has supplied information on flight tests of a single-engined Avia-BH 33 fighter plane, successively equipped with Gnome-Rhône Jupiter series VI and VII engines developing 450 hp. The last-mentioned engine was turbocharged. Figure 39 gives the results of these tests.

* Note concerning the history of superchargers: The Cadillac plant (U.S.A.) in 1913 used a blower to supply mixture from the carburetor to the exhaust [sic] valves of a car engine. The German designer Junkers in 1918 used a piston compressor to charge a two-stroke internal-combustion engine. See also M.Levin's paper "Nadduv motorov" (Supercharging of Engines) in "Tekhnika Vozdushnogo Flota," 1928, p.529.

30 Interesting tests were carried out with a Rateau turbocharger and a mark 11 W. E. Farman engine developing 500 hp, installed in a Bréguet 12 A 2 airplane weighing 3,050 kg in flight, including 1,000 kg lead ballast. These tests were performed in France in the summer of 1929, the airplane attaining an altitude of 9,500 m. The engine speed was 2,320 rpm at this altitude. The pilot started using oxygen at an altitude of 4,500 m.

Table 5 gives characteristic data on the engine during flight without supercharging and with supercharging at compression ratios of 1.52/1 and 2/1 respectively.

TABLE 5

Altitude, m	Engine power			Remarks
	without supercharging	with supercharging		
		compression ratio 1.52/1	compression ratio 2/1	
1,000	500	415	500	Power varied linearly between values given
1,500		Beginning of supercharging at 440		
2,000				
3,000				
3,600		500		
4,000				
5,000				
5,800	250	390	540	
6,000				
7,000				
7,500	200	340	470	
Weight per hp at altitude of 5,800 m	1.8 kg	1.35 kg	0.9 kg	

Figure 40 is the barogram of Donati's flight in Italy on 21 December 1928. Figure 41 is the barogram of Street and Stevens' takeoff at Dayton (U. S. A.) in 1929.

Development of turbochargers is, however, slow. In addition to the above-mentioned designs there also exist the Wittich and Zoeller blowers, and many other types. We have grouped separately those designs where the turbine forms a single set with the engine and the blower. Another example is the Pratt and Whitney radial engine with centrifugal blower. The latter is arranged in front of the cylinders and is driven from the engine shaft. The cylinders may be arranged in a U in order to improve the blower operation. Figure 42 shows how the blower, engine, and exhaust turbine may be arranged in a high-altitude engine (this is not an actual design).

The difficulties encountered with water-cooled engines should be mentioned. The boiling point of water decreases with the pressure, and many engine-cooling systems do not operate satisfactorily at an altitude of 8 km (Figure 43). It would be advantageous to replace water by some
31 other coolant, e. g., some light hydrocarbon, if this would offer some advantage as regards specific heat, latent heat of vaporization, weight, and

safety against fire. Otherwise there is no alternative than to locate the entire cooling system in a pressurized compartment. This, however, introduces complications. Another solution is to use air-cooled engines.

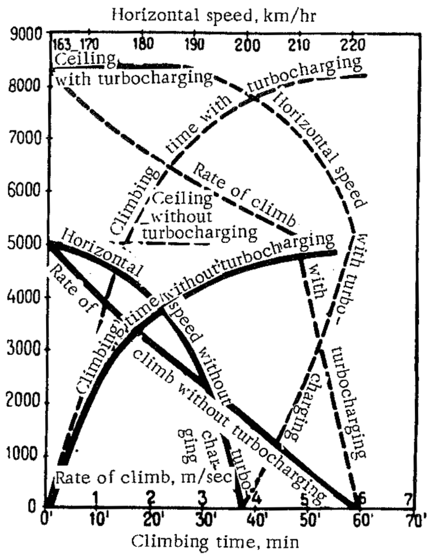


FIGURE 37

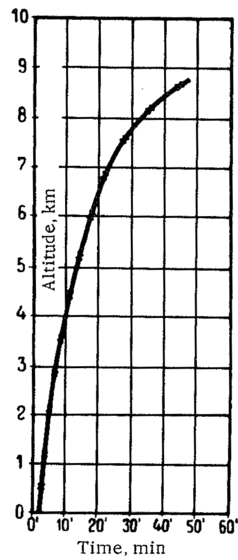


FIGURE 38

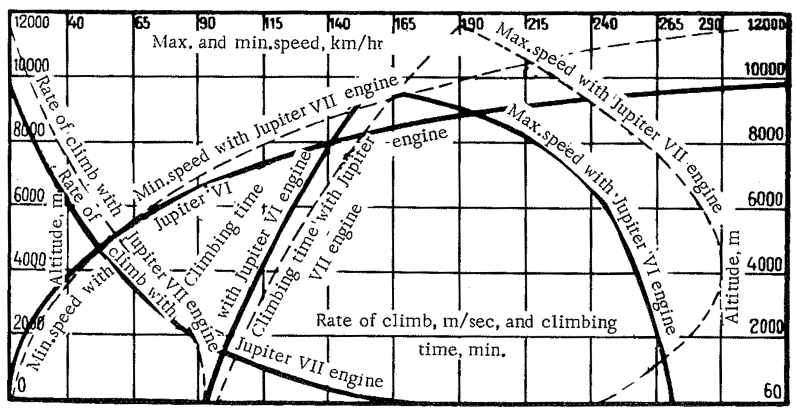
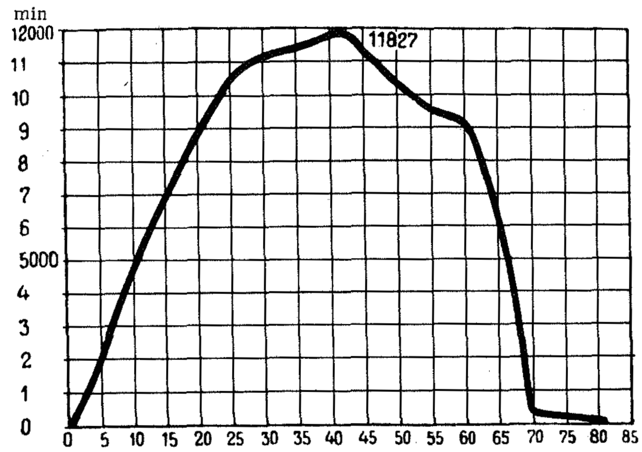


FIGURE 39

There is some sense in suggesting the use of a steam turbine for high-altitude flight. All reciprocatory motion and shocks can then be eliminated. This improves the operation of the engine and of the entire airplane. It has also been suggested that this ensures constant power at all altitudes, since a closed-cycle process is not affected by the external pressure.

32



Barogram of Donati's record flight (Italy, 21 Dec. 1928)

FIGURE 40

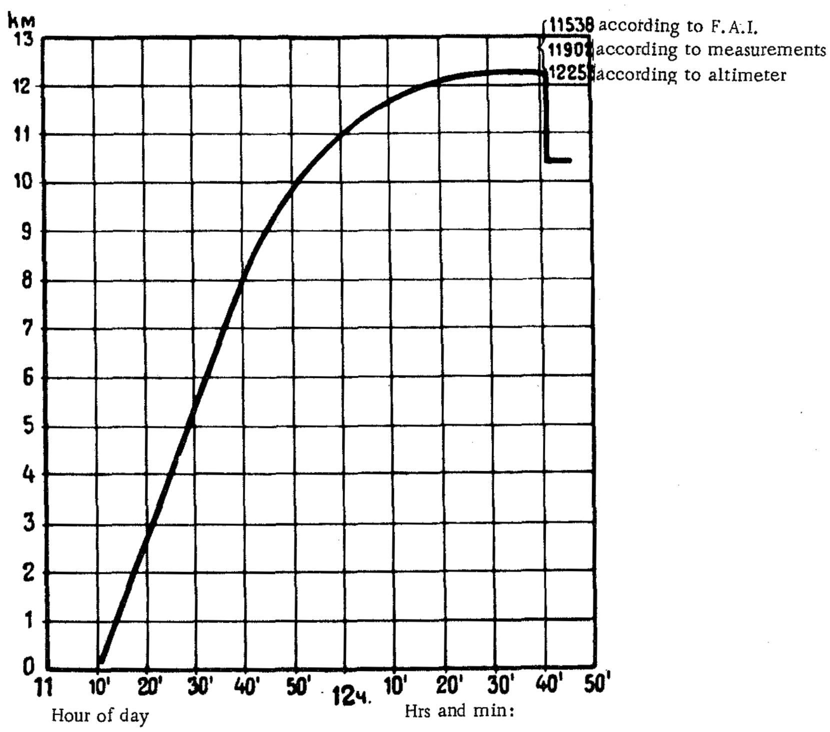


FIGURE 41

33 However, a disadvantage is the considerable weight of the installation which includes the steam generator, the steam compressor, the radiator and the condenser. It is also wrong to assume that the power of the installation does not depend on the [external] pressure. Heat transfer

in the boiler, air preheater, and condenser depends strongly on the air density, whose reduction necessitates increasing the areas of all surfaces across which heat is transmitted. The weight of the installation thus increases with the altitude. The size of the boiler can be reduced by increasing the air pressure in the furnace; this, however, requires the use of a compressor or blower which again increases the weight of the installation. The condenser surface will be almost five times larger than the [radiator?] surface of a gasoline engine developing the same power. This also increases the drag and thus reduces the flight speed.

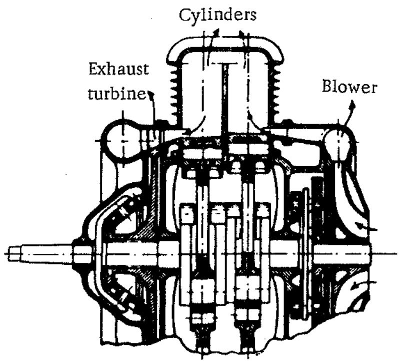


FIGURE 42

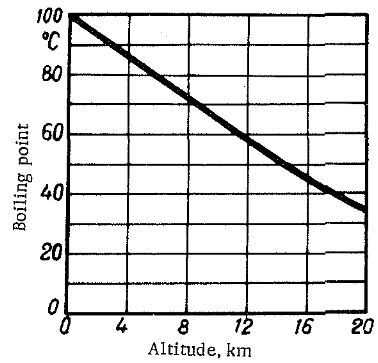


FIGURE 43

R. Wagner (Hamburg) carried out a series of investigations on the weight and efficiency of steam turbines for airplanes. He concluded that the following particulars are applicable to them: weight, 2 kg/hp; fuel consumption, 0.25kg/hp · hr; thermal efficiency, 25%. However, under these conditions such a light 1,000 hp installation must utilize the heat to the same extent as an optimum stationary installation, which is rather improbable. Taking into account the data for modern gasoline engines, it is hardly possible to assume that a turbine will weigh less than 2.5 kg/hp and consume less fuel than 0.3 kg/hp · hr. We shall use these data to compare power requirements of high-altitude airplanes of the same dimensions, equipped with a turbine and an internal-combustion engine respectively.

Figure 44 shows the results of the calculations. The airplanes considered have a wing span of 30 m, a gross weight of 10 t, an empty weight of 3.5 t, a payload (including the crew) of 1 t, and a propeller efficiency of 70%. The plane equipped with the turbine has a L/D ratio (11/1 instead of 12/1), due to the increased drag caused by the condenser. The diagram shows that the plane equipped with the internal-combustion engine has the higher speed and larger range. Planes equipped with steam turbines are therefore not suitable for flight in the stratosphere. However, further development of engines and turbines may change this.

The French engineer N. Cazanne believes that high-altitude flight requires radial air-cooled two-stroke engines. This would enable the

34 engine to be considerably simplified and its efficiency to be raised, since the rarefaction of the air facilitates the exhaust of the burnt gas. The engine may be either inside or outside an airtight compartment. The latter design is obviously preferable since only the magneto then has to be located inside the airtight compartment.

Much time is required for an airplane to climb to a large altitude on its own, despite the large power reserve available. Cazanne therefore suggested using an auxiliary towing plane equipped with an engine developing approximately 400 hp, whereas the towed plane would only have a 60 hp engine. Such a plane would develop a speed of 230 km/hr with a payload of 500 kg at low altitudes, but would have a speed of 460 km/hr at an altitude of 14,700 m.

In 1927 very interesting reports were submitted to the German Scientific Society for Aeronautics by Kamm and Schrenk on the limits and possibilities of using supercharged high-altitude engines, in which they determined the speeds and ranges of airplanes at altitudes up to 30 km. These reports were published in the "Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt E. V. (W. G. L.)" (Yearbook of the Scientific Society for Aeronautics), 1927, pp. 116 and 129.

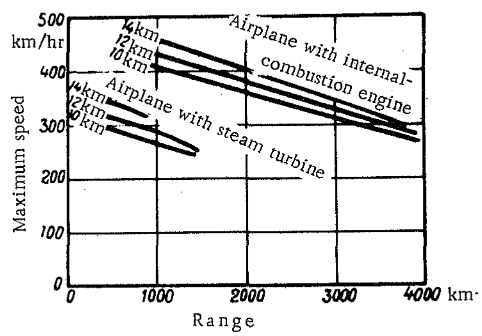


FIGURE 44

We first present briefly the contents of Kamm's report.

The power of an ordinary gasoline engine decreases with increasing altitude, although more rapidly at altitudes below 5 km than higher up. Tests in the U.S.A. have shown that this decrease ceases at an altitude of approximately 30 km, whereas Kamm believes that this occurs at an altitude of 15—17 km.

The use of high-altitude engines makes it possible to reduce this drop in power. Such an engine is supercharged by a blower or compressor at large altitudes. This improves the engine performance, but operation of a blower involves an additional power consumption which increases with altitude.

A favorable effect is caused by the reduced counterpressure to the exhaust of the gas at large altitudes, whereas the reduction in the oxygen content of the air with increasing altitude has an unfavorable effect.

Kamm also presented diagrams showing that the use of blowers driven mechanically by the engine causes the required useful power of the engine to be the same up to an altitude of 10 km as at ground level, whereas it drops to 42% of the latter value at an altitude of 20 km, and to 10% at an altitude of 30 km. The engine weight at altitudes up to 20 km has the permissible value of 2.5 kg/hp.

The weight of the supercharger depends on its design and the altitude. A mechanically driven blower weighs more than a turbocharger at altitudes up to 13 km. At larger altitudes, however, the weight of a turbocharger exceeds that of a mechanically driven blower (per kg of air delivered per sec). Turbochargers should therefore be used at altitudes up to 13 km in preference to mechanically driven blowers, whereas they become less advantageous than the latter at larger altitudes.

Cooling conditions become worse with increasing altitude in the case of air-cooled engines, but improve up to an altitude of 5 km in the case of water-cooled engines. Above this altitude conditions also become worse in the latter case, but to a smaller degree than in the former case.

Figure 45 is a diagram, given by Kamm, of the engine performance as function of the altitude.

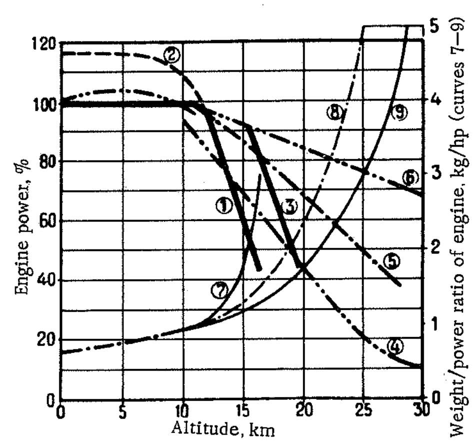


FIGURE 45

This diagram contains the following curves: (1) – power delivered to propeller with turbocharger having an efficiency of 54% above an altitude of 13 km; (2) – ditto, with overloaded engine; (3) – as (1), at turbocharger efficiency of 64%; (4) – with mechanically driven blower; (5) – with combination of turbocharger and mechanically driven blower; (6) – total shaft hp in case (5). It is seen that the engine has sufficient power up to an altitude of 10 km, above which the power begins to decrease rapidly. This involves an increase in the weight/power ratio of the engine, illustrated in the last three curves; (7) – with turbocharger; (8) – with mechanically driven blower; (9) – with combination of turbocharger and mechanically driven blower.

Full power is available up to an altitude of 13 km, 70% up to an altitude of 20 km, and 30% up to an altitude of 30 km if the combination of a turbocharger in the denser layers and a mechanically driven blower in the more rarefied ones is used. The weight/power ratio will in this case be as follows: 1 kg/hp at an altitude of 13 km, 1.7 kg/hp at an altitude of 20 km, and 2.8 kg/hp at an altitude of 25 km.

These results are valid for present-day blowers and turbines. Raising their efficiencies will increase the possibilities of superaviation, i. e., flight at large altitudes and high speeds.

Contents of Schrenk's paper. Ing. Schrenk presented a number of interesting diagrams illustrating the operation of engines at large altitudes, their corresponding weight/power ratios, the flight speeds, and the ranges.

Figure 46 shows some of these curves. Curve (1) represents the power N, required per kg of the engine weight G_e without turbocharging; curve (2) represents the N/G_e with turbocharging but without air being aspirated by the engine directly; curve (3) corresponds to the case when air is supplied to the engine both directly and via the turbocharger. Curve (4) represents the relative fuel consumption b/b_0, where b_0 is the fuel consumption at ground level. The curves form the envelopes of series of curves for different engines developing the maximum power. The optimum
36 results are in the case considered obtained at an altitude of 15 km. The limiting curve $N/\overline{G}_e$ refers to an ideal engine of constant weight, without propeller. Such an engine is later also assumed to be installed in an airplane.

Schrenk also presented curves of the power, speed (up to 460 km/hr), range (up to 4,500 km), fuel consumption (up to 3,500 kg), and altitude (up to 20 km). The airplane considered by him weighs 10 t, of which the structure accounts for 3.5 t, the payload for 1 t, and the fuel and propulsion plant for 5.5 t.

In conclusion he suggested that flight at high speeds should take place at altitudes of 12–16 km, variable-pitch propellers being used. Two to four propellers rotating at 600–800 rpm are proposed, each propeller requiring a power of 500–700 hp. The expense involved in high-altitude flight makes it advisable for mail carrying. Transportation of passengers would necessitate the provision of an airtight cabin supplied with oxygen, which would increase the weight of the airplane.

Schrenk expressed optimism about the development of superaviation, and considered it quite possible that this will take place in the near future.

The results of Ing. Schrenk's computations (cf. his contribution in the 1927 Yearbook of the WGL at the discussion following Kamm's lecture "Grenzleistungen im Höhenflug" (Performance Limits in High-altitude Flight), Jahrbuch der WEL, 1928, p. 60) are given in Figure 47. The computations were carried out for an airplane weighing 10 t, having a wing span of 30 m, and a L/D ratio of $^{12}/_{1}$. Minimizing the weight of the engine in such an airplane gives a power which varies with the altitude. The payload, including the crew, is 1 t. It is seen from the diagram that favorable relationships are obtained for altitudes between 12 and 16 km.

f) **High-altitude propellers.** High-altitude propellers may be divided into three groups:

1. Propellers of conventional design.
2. Propellers of conventional design, driven from the engine via gears.
3. Variable-pitch propellers.

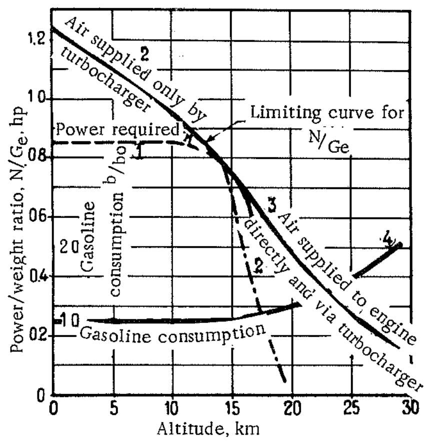


FIGURE 46

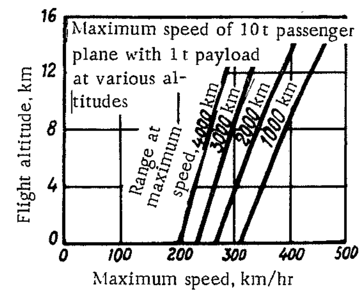


FIGURE 47

37 An interesting study was published by H. Borck in the Z. F. M.* 1927, p. 83. He considered the choice of propellers for flight altitudes between 12 and 15 km, where the specific weight of the air varies between 0.3 and 0.2 kg/m^3. At these altitudes he wants to achieve a speed which is twice that at ground level for the same range, and considers that the most important problem in this case is the selection of a suitable propeller.

This selection is linked to the choice of the engine, which may be of one of the following four types:

1. An engine of conventional design, developing full power on the ground and equipped with a turbocharger.
2. A partially superdimensioned engine with turbocharger.
3. A fully superdimensioned engine.
4. A steam engine.

Engines of types 1–3 may be either gasoline engines with electric ignition or Diesel engines. In carburetor engines the compression ratio of the mixture of gasoline vapors and air limits the torque. Diesel engines designed for overloads and dimensioned accordingly can within certain limits develop a given power at varying rpm and torques at different altitudes.

Engines may be divided into two groups as regards selection of the propeller:

a) **Engines** developing constant powers at different altitudes at equal torques and rpm.

b) **Engines** developing constant powers at different altitudes at varying torques and rpm.

In his paper Borck investigated which of the above-mentioned three groups of propellers may be used in combination with each type of engine.

As example he considered the case when a propeller rotating at 1,500 rpm takes up a power of 600 hp at a torque of 286 kg · m, the specific weight of the air being 0.3 kg/m^3. He assumed that the power required

* Zeitschrift für Flugtechnik und Motor-Luftschiffahrt.

is proportional to the air density and to the cube of the propeller rpm. He also postulated that at ground level, where the specific weight of the air is 1.2 kg/m^3, a torque of 455 kg·m should be developed at 943 rpm. In this case it is possible to use a conventional propeller without gears. However, this condition can be satisfied only by a geared Diesel engine or a steam engine.

The propeller rpm must, however, be increased proportionally if the flight speed is to be 1.5 or 1.7 times higher. It is then necessary either to take off without using the full power of the engine, if the airplane is not overloaded, or else to use a geared, variable-pitch propeller.

Schrenk came to the following general conclusions:

1. Flight at high altitudes (12–15 km) is possible with the aid of specially designed propellers having the same efficiency as at low altitudes. The propeller diameter and rpm will then be suitable for existing engines.

2. High-efficiency propellers can be designed for flight altitudes of 20–30 km, where the specific weight of air is 0.3–to 0.1 kg/m^3. The inverse problem has to be solved in this case: whereas for an ordinary airplane flying at the usual altitudes the propeller is chosen so as to fit a given engine, for flight altitudes of 20–30 km **it is necessary first to design the propeller, and then fit the airplane and the engine to it.**

38 The limits of propeller operation.*

The mathematical expression for the law of similarity, referred to propellers, is

$$N \propto \rho\, n^3 D^5 ,$$

where N is the power delivered to the propeller, ρ is the density of the air, n is the propeller rpm, D is the propeller diameter. For propellers with uniform pitch, for which this law is valid, we have

$$\frac{v}{nD} = \text{const.}$$

Consider flight at a constant lift coefficient, when

$$v \propto \rho^{-1/2},$$

and

$$n \propto \frac{1}{\rho^{1/2} D}.$$

Introducing this expression into the similarity formula, we obtain the diameter of a propeller of uniform pitch:

$$D \propto \sqrt[4]{\rho N^2},$$

i.e., the diameters of similar propellers of uniform pitch decrease with increasing altitude at equal powers developed. The propeller rpm is inversely proportional to the square root of the air density; the blade-tip speed increases accordingly.

* According to Schrenk, "Probleme des Höhenflugs" (Problems of High-altitude Flight), Z.F.M., 1928, p.471.

This increase in the blade-tip speed also characterizes a high-altitude propeller. The efficiency of the outermost propeller elements drops rapidly when the blade-tip speed approaches the velocity of sound. However, we do not know the exact law according to which this occurs. The blade-tip speed of racing planes is on the average $^2/_3$ of the velocity of sound; the latter, however, varies with the altitude approximately as the square root of the absolute temperature. The composition of the air also has an influence. The velocity of sound is approximately 340 m/sec at sea level and decreases to 295 m/sec at an altitude of 11 km, beyond which it remains almost constant at the heights attainable by airplanes.

The propeller advance ratio, which represents the ratio of the flight speed to the tangential propeller velocity must be increased if the tangential propeller velocity is to remain constant with increasing flight speed, which varies directly with the altitude. If [at a given flight speed] we desire to maintain constant the ratio of the tangential propeller velocity to the velocity of sound, we must increase the advance ratio since the velocity of sound decreases with increasing altitude. The blade-tip speed is the resultant of the tangential and translational velocities of the propeller, and increases even faster than the propeller pitch. Large-pitch propellers are therefore required for high-altitude flight.

Figure 48 gives data for AVA (Göttingen) propellers of 20 cm diameter. The ratio of the propeller pitch to the diameter increases up to 3. The diagram also shows the efficiency, which is very high in the case
39 considered.*

The exact relationship is obtained from Eifel's logarithmic diagram. Using it for the data given we find that the optimum propellers will have wide blades when the pitch is increased. The family of propellers considered will be optimum when the ratio of the propeller pitch to the diameter is between 1.2 and 1.5. Even wider blades and smaller diameters may be selected when this ratio is larger, in order to maintain a high efficiency.

High-altitude propellers thus must have wide blades, although this still requires further tests. The logarithmic diagrams enable us to investigate the relative influence of the propeller diameter and rpm at a given power if the blade-tip speed must be a fixed fraction of the velocity of sound at any altitude. Figure 50 refers to an airplane equipped with an engine developing an effective power of 600 hp. The blade-tip speed is taken as $^3/_4$ of the velocity of sound corresponding to the altitude. Doubling the diameter (and the pitch) requires that the propeller rpm almost be halved to maintain a low blade-tip speed. This shows how important it is to carry out experiments to determine the influence of the blade-tip speed on the efficiency, and thus find the maximum permissible tangential propeller velocity.

The power must be distributed over several propellers if the diameter of a single propeller becomes excessively large in relation to the size of the airplane.

* The load coefficient $\varphi = \frac{2{,}5}{\varphi F v^2}$ decreases when the pitch is increased; the efficiency $\eta_a = \frac{2}{1+\sqrt{1+\varphi}}$ thus gradually approaches unity. The influence of profile drag decreases when the main part of the blade makes an angle exceeding 40–45° with the plane of the propeller.

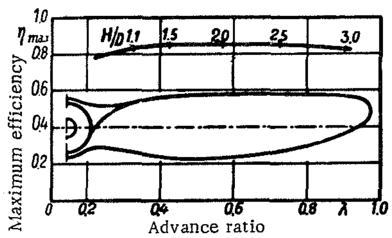


FIGURE 48

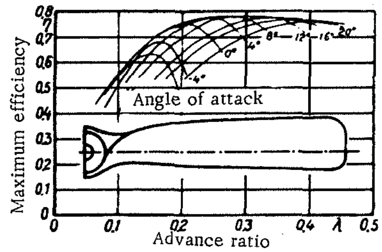


FIGURE 49

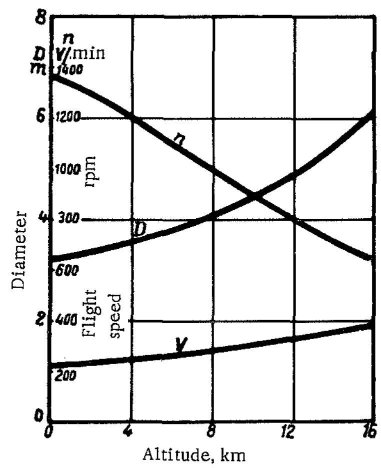


FIGURE 50

The above-mentioned different relationships, valid for flights at low and at high altitudes, can be satisfied only by means of variable-pitch propellers.

Figure 49 gives the efficiency of a variable-pitch propeller (No. 96), according to [NACA] report No. 151. It is seen that the propeller should operate mainly at small advance ratios.

40 3. CRITICAL REVIEW OF PROPELLER-DRIVEN AIRPLANES

Both the existing and the planned airplanes described have two main shortcomings due to which it is impossible to increase their flight speeds and ceilings considerably. These are:

1. A large power is required at high speeds **during flight at low altitudes**
where the air is quite dense, so that the propeller still provides sufficient
thrust and the wings adequate lift. The engine weight then accounts for
almost the total available lift if the airplane is of limited size. Rohrbach
therefore planned his above-mentioned airplane with "infinitely" long
wings. These, however, lead to the other shortcomings which are compounded
by the generally low efficiency of the propulsion plant, although it is higher
than that of many other engine types. Figures 51–56 represent the energy
balances of different types of engines. These diagrams show that a) only
27% of the energy contained in the fuel is transmitted to the shaft of an aero
41 engine, while only 20% is utilized by the propeller (Figure 51), b) only 12%
of the energy contained in the fuel is transmitted to the shaft of a steam
engine while only 9.5% is obtained at the shaft of an electric motor in the
case of electric transmission (Figure 52), and c) only 5% of the energy
contained in the fuel is available at the wheel rim of a steam locomotive
(Figure 53).

(40) Energy balance of aero engine driving propeller

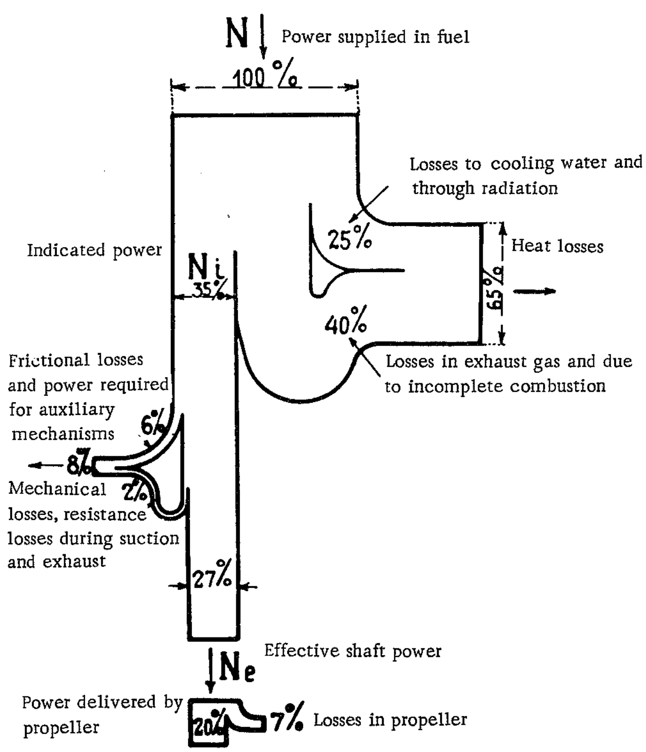


FIGURE 51

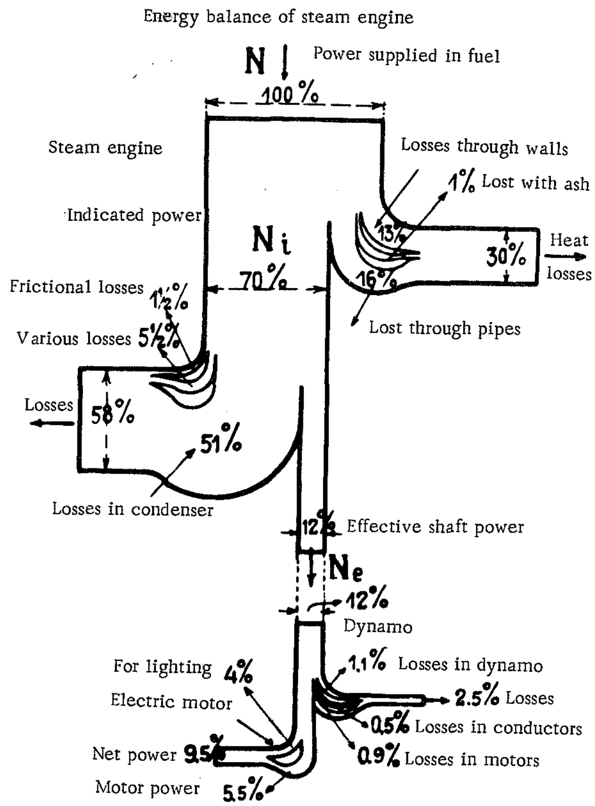


FIGURE 52

The energy balance of a steam turbine is more favorable (Figure 54). A net output equal to 24% of the energy contained in the fuel has been obtained in the U.S.A. This value usually does not exceed 18%.

Figure 55 gives the energy balance of a 30 hp truck, as found by Riedler. The losses in the engine amount to 76%, while those in the transmission account for 4.2% of the energy contained in the fuel. The work done in overcoming the resistance to motion of the truck and trailer accounts for another 12.1%. The excess power of 7.7% is used for acceleration and overcoming grades.

Figure 56 shows the energy balance of a steam tug as a function of its speed. The abscissas represent the speed in km/hr, while the ordinates indicate the energy distribution.*

* Figures 55 and 56 were reproduced from "Mekhanika Transporta" (Mechanics of Transport) by Prof. G. Dubelir, Collection No. 100, LIIPS.

Energy balance of steam locomotive

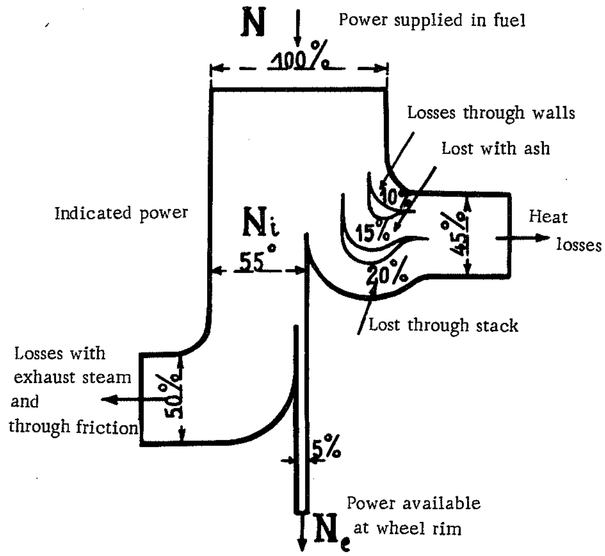


FIGURE 53

Energy balance of steam turbine

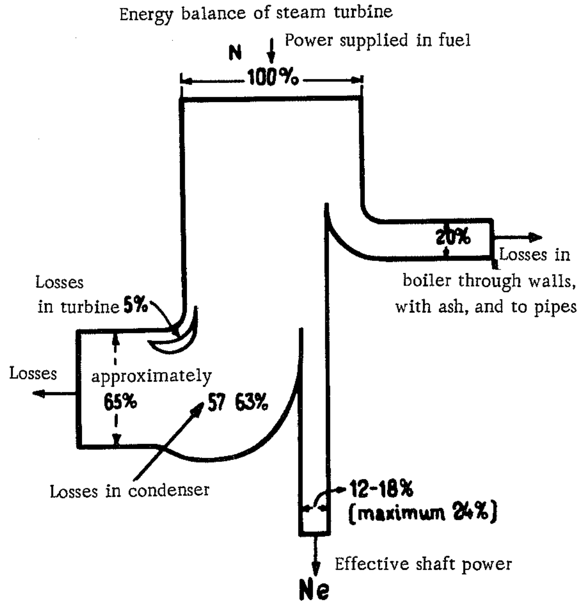


FIGURE 54

43 2. The power of the engine decreases **during flight at high altitudes** where the air is rarefied and drag is small, while the thrust of the propeller and the lift of the wings are low if their design is conventional, even when the angles of attack are increased.

We shall now consider high-altitude flight under these conditions. Such flights are economically justified, since the low density of the upper layers of the atmosphere permits high speeds. (The speed is inversely proportional to the square root of the air density if the lift coefficient is constant.)

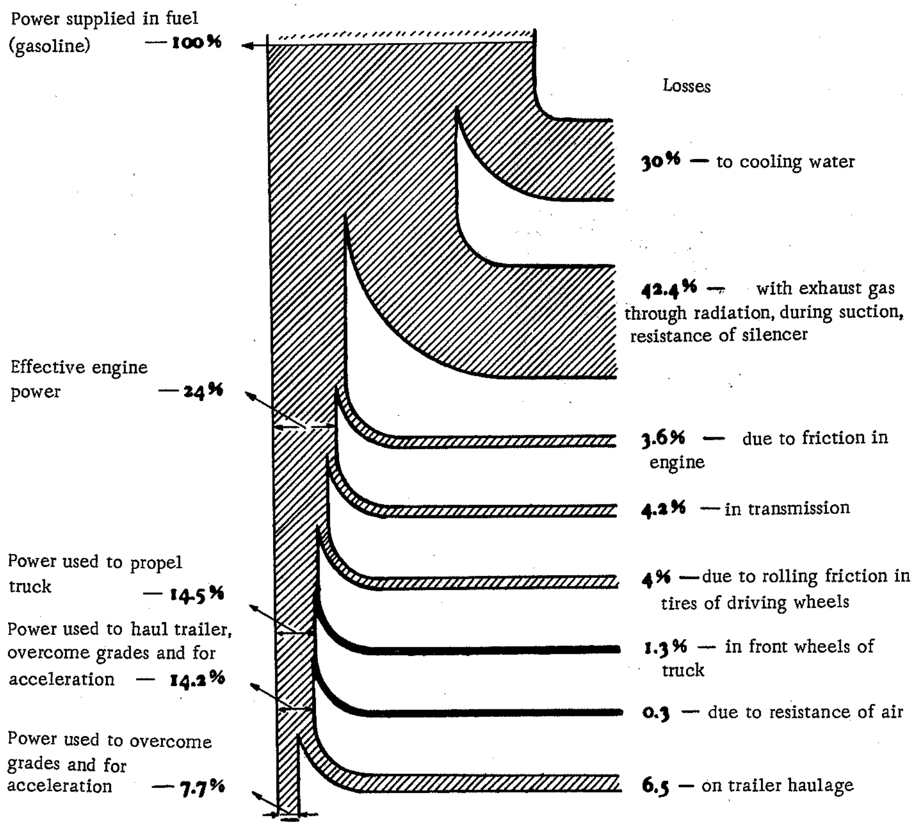


FIGURE 55. Energy balance of 30 hp truck, according to Riedler's experiments

However, this reasoning, although interesting in itself, does not lead to correct conclusions regarding variation of the speed with altitude, if the laws of mechanical similarity are applied. We shall arrive at more correct relationships if we assume a constant power requirement and then consider the maximum possible speeds and ranges at the various altitudes.

Neglecting induced drag, we may first assume that the speed is inversely proportional to the cubic root of the air density. It follows from this that the lift coefficient increases with altitude. Lastly, in contrast to flight

44 at maximum speed at ground level, the L/D ratio increases at first rapidly and then more slowly until it attains its optimum value at constant power and fuel consumption. The maximum speed and range will then be obtained simultaneously.

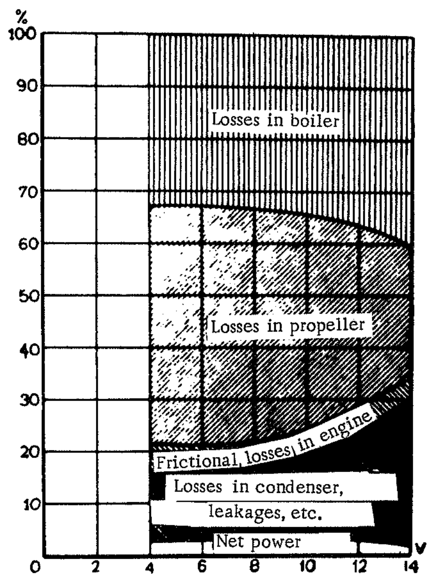


FIGURE 56. Energy balance of steam tug

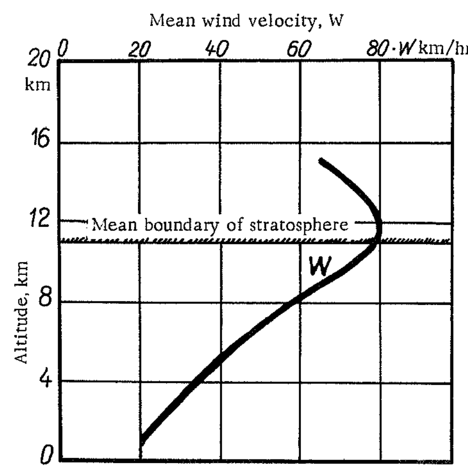


FIGURE 57

These relationships were investigated by Schrenk (cf. Yearbook of W. G. L.; 1927, discussion following 1928 lecture, and discussion following Lorenz's lecture). He did not consider one and the same airplane, which during flight at low altitudes would be overloaded by the superchargers and the high-altitude propulsion plant, but an ideal airplane having a propulsion plant designed for the given altitude and range. Only under these conditions shall we be able to assert that the maximum power is utilized at any altitude. It follows from this that it is difficult to increase the speed when the range is given.

Thus, the speed is increased only by 40—45% at an altitude of 12 km. On the other hand, the range increases considerably at a given speed; for instance, it is tripled at 300 km/hr.

The reason for this, is that the L/D ratio is optimum at the maximum flight speed; this indicates a significant advantage of high-altitude flight: the coincidence of high speed with low fuel consumption. A 10 t airplane consumes 1 kg of fuel per km. High-altitude flight offers another, equally important advantage. Transoceanic planes fly near the surface through storms, fog and ice clouds, with a watery grave a few meters underneath. On the other hand, flight in the stratosphere above an altitude of approximately 11 km (Figure 57), unaffected by terrestrial

temperature variations and in undisturbed layers of air, is free from these inconveniences. A perpetually clear sky extends above the flyer, and permits orientation by the stars during day and night. Gusts of wind do not trouble the passengers, and the pilot can control the plane at his
45 ease and hold course and altitude. Before a forced landing, with the engine stopped, the plane can glide inside a circle of 100–200 km radius where it is always possible to find a landing ground or a ship by radio.

Some drawbacks of high-altitude flight should also be mentioned. Strong winds (70–80 km/hr) blow at the lower boundary of the stratosphere (Figure 57). The wind velocity then decreases with increasing altitude. However, our knowledge of the winds at these altitudes is still imperfect, and we have no information on the optimum height for aerial communications. The payloads of the airplanes will be extremely limited due to the large power required and the need for high-altitude propulsion plants. Equipment protecting the passengers against the effects of low pressure will also be needed.

High-altitude flights will increase our knowledge of the meteorology and aerology of the upper layers of the atmosphere, in particular of the winds in the stratosphere.

In the following section we shall expound the latest views of various engineers on how to effect high-altitude flights at high speeds.

4. ELEMENTS OF THE THEORY OF SUPERAVIATION

a) General remarks

Superaviation is defined as the flight of airplanes at high altitudes and high speeds. The most important characteristic of high altitudes is the rarefaction of the air, due to which the drag of the airplane is small, enabling it to develop a high speed. Various methods have been proposed by different scientists in order to obtain such high speeds; these methods may be divided into two main groups.

a) **The first group** includes those proposed by Costantzi,* Garuffa,** Bambino,† Roeder,†† Pomiglio, and Borck (cf. above) who attempted to to prove:

1) it is possible to design airplanes flying at speeds of 1,000–2,000 km/hr without employing new methods of providing lift and thrust;

2) all efforts should be directed toward the study of high-altitude engines. This problem was studied in greatest detail by Costantzi, who at the beginning of his paper mentioned the history of this research ("Aer" 1914), the experiments performed in Italy by Anastasi in 1916

* Giulio Costantzi, "A proposito di superaviazione" (On the Subject of Superaviation), L'ala d'Italia, 1926, p.426.

** E.Garuffa, "Le grandi velocità di translazione cogli aeroplani" (Large Translational Velocities with Airplanes), L'ala d'Italia, 1926, p.523.

† Enzo Bambino, "Vola ad alta quota" (Flight at High Altitudes), L'ala d'Italia, 1926, p.529.

†† H.Roeder, "Zur Frage des Höhenfluges im Luftverkehr" (On the Problem of High-altitude Flight in Aerial Communications), Flugwoche, 1927, p.344. See also M.Walter, "Superaviation," Aero Digest, 1927, p.289.

and in France by Rateau in 1917 and 1918, as well as Crocco's (1923) and Guidoni's work. Constantzi proposed a flight altitude of 22–24 km in superaviation. Computations were performed by him for a "CC" airplane weighing 4,500 kg, equipped with an engine developing a constant power of 1,000 hp. He determined the speeds obtainable with this airplane at different altitudes and angles of attack. Extrapolating experimental data, he tried to determine the law according to which lift and drag vary with speed; he found that lift and drag are no longer proportional to the square 46 of the flight speed when the latter is high.

Table 6 contains the results of his computations for constant engine power. Lift and drag were assumed to depend on a variable power of the speed.

TABLE 6

Altitude, m	0	4,000	8,000	12,000	16,000	20,000	24,000	26,000
Speed, km/hr	480	500	650	780	950	1,200	1,540	1,750

We shall now present a schematic computation of the influence of the flight altitude on the speed of the airplane.

Assume that the airplane flies at a constant angle of attack, i.e., that the coefficients K_y and K_x are constant.

The lift R_y and the drag R_x will then be respectively:

$$R_y = K_y\, \rho . v^2, \qquad (1)$$
$$R_x = K_x\, \rho . v^2, \qquad (2)$$

where ρ is the density of the air and v is the flight speed.

Let the lift be constant. We than find from (1) that v is proportional to $\sqrt{\frac{1}{\rho}}$. However, R_x is then also constant. The flight speed will be $\sqrt[2]{\frac{1}{0.16}} = 2.5$ times larger at an altitude of 15 km, where the density of the air is 16% of that at sea level, than at zero altitude (1,000 km/hr instead of 400 km/hr). The power required will then also be 2.5 times larger [since the drag remains constant].

Ing. Pomiglio considered (Aeronautica No. 4, p. 200 ?) flight at high altitudes from Paris to New York, assuming it possible to cover a distance of 6,000 km in 10 hrs at a speed of 600 km/hr. For instance, an airplane developing a speed of 250 km/hr at sea level will develop a speed of 600 km/hr at an altitude of 18–20 km if the engine produces a constant power at any height. The fuel consumption per km will be the same, but there will be a considerable saving in time. (Cf. "Le document aéronautique," 1928, p. 624).

This approach is faulty due to the assumption that the propulsion plant can produce a constant power. This depends on the development of new, still unknown designs of turbochargers, combined with high propeller rpm involving considerable centrifugal forces. This would require the propellers to be very strong and probably very heavy. Some progress is possible in this direction, but it will hardly be considerable.

b) **The second group** includes the methods proposed by the French engineer René Lorin, the Italian engineer Crocco, the German scientists Robert Hohmann and Valier, the Russian scientists Tsiolkovskii and Tsander, and many others who proposed that the engines driving propellers be replaced by reaction engines. Operation of the latter is not only unaffected by the altitude, but, according to Goddard, is even better in the absence of an atmosphere. This involves the rocket principle and motion due to the recoil or reaction caused by the discharge of gas during the combustion of propellant in the rocket. Part of the energy contained in the propellant is used to heat the gas, while part is converted into kinetic energy.

Theoretical considerations show that only 60–80% of the energy contained in the propellant is converted into kinetic energy during combustion, while
47 the remainder serves to heat the gas.*

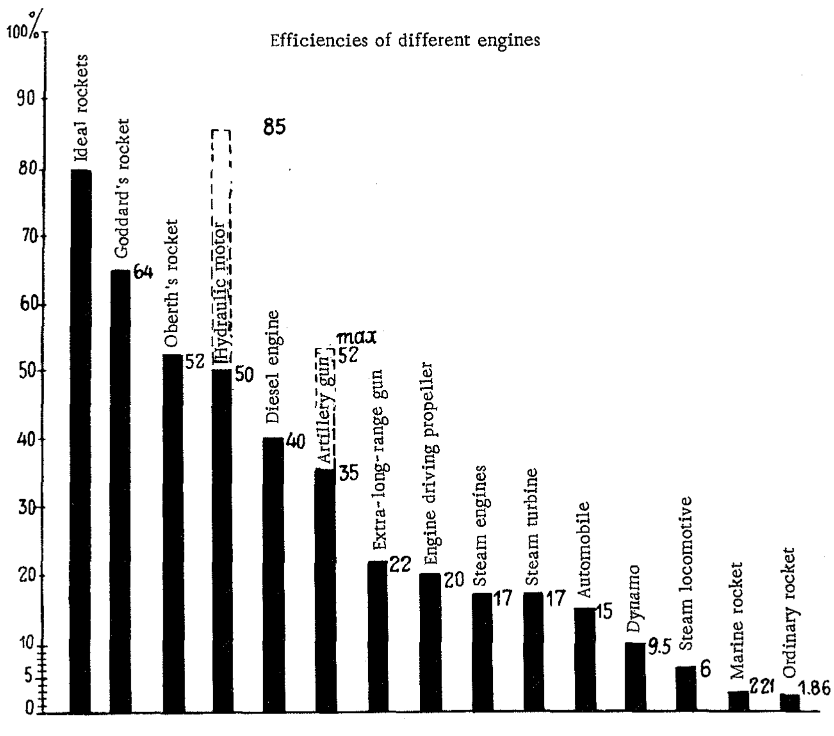


FIGURE 58

Conditions will be optimum, according to Oberth, if approximately 64% of the total energy contained in the propellant is used to propel the rocket (more correctly, if 64.7% of the kinetic energy of the gas is used to propel the rocket). Thus, only 80% × 64.7% = 52% of the total energy contained in the gas is converted into work. The ideal case would be to vary the gas

* Cf. N. Rynin, "Teoria Reaktivnogo Dvizheniya" (Theory of Rocket Propulsion), 1929, Leningrad.

outlet velocity [relative to the rocket] so as to equal the flight speed. The column of gas discharged from the rocket would then be at rest [relative to space], and the coefficient, which we had previously taken as 64.7%, would thus increase to 100%. The overall utilization of the energy contained in the propellant would then be 80%. This ideal case can be approximated if the propellant used initially provides a small gas outlet velocity; the propellant must then, however, have a high specific weight. A propellant providing a gas outlet velocity equal to the rocket speed should be used when the latter has increased and the rocket weight decreased.

Figure 58 shows the efficiencies of different engines, including rockets. The broken lines indicate the respective efficiencies at optimum operating conditions.

We shall now present the basic principles and theorems relating to superaviation, as given by the French engineer Lorin* and the Italian engineer Crocco.**

48 **b) Principles of superaviation**

Flight in the upper layers of the atmosphere is possible at speeds which increase as the air becomes more rarefied. Lift and drag are proportional to the product of the air density by some function of the speed. Let the speed increase during the climb in such a way that the above-mentioned product remains constant. Both lift and drag will then also be constant. The statics and dynamics of airplane flight thus remain almost as under ordinary flight conditions, if we neglect the peculiarities of flight at high speeds and small angles of attack, which are still unknown. The position of the resultant drag is then also altered. However, in a first approximation we assume that its position does not change. We may then write down the conditions for superaviation:

$$\left.\begin{aligned} P &= y\,\delta\,F(v) \\ R &= x\,\delta\,F(v) \end{aligned}\right\} \qquad (1)$$

where P is the lift; R is the drag; y and x are numerical coefficients which depend on the shape of the wings, fuselage, and tail; δ is the ratio of the air density at the altitude considered to that at sea level; $F(v)$ is some function of the speed. When the latter is subsonic it may be assumed to be proportional to the square of the flight speed, especially less than 240 m/sec (a speed which [at the time of writing] had not yet been attained in aviation). Data obtained from ballistic experiments should, however, be used to determine this function for supersonic speeds.

Crocco assumes as principal law of superaviation

$$\delta \cdot F(v) = \text{const.} \qquad (2)$$

* René Lorin, "Note sur la popularisation des véhicules aériens" (Note on the Popularization of Aerial Vehicles), L'aérophile, 1907, p.321.

** G. A. Crocco, "Possibilitá di superaviazione" (Possibility of Superaviation), Notiziario Tecnico, 1926, No.6, p.1.

Table 7 gives values of $\frac{1}{\delta}$ at various altitudes H, computed by Hann in 1906 from Laplace's equation.

Table 8 contains values of $F(v)$, computed for different speeds v by means of Chapel's* formula; the Table also gives the altitudes v corresponding to these speeds H when (2) is satisfied.

TABLE 7

Altitude, km	10	15	20	25	30	35	40
Ratio of air densities at sea level and altitude H, $\frac{1}{\delta}$ = ...	3.4	7	15	32	70	154	354

49 TABLE 8

v m/sec	$F(v)$	$\frac{1}{\delta}$	H, m
100	1,200	1	0
200	4,800	4	11,500
300	13,500	11.25	19,500
400	50,000	41.60	25,700
500	86,500	72	30,000
600	123,000	102.50	32,500
700	159,500	133	34,200
800	196,000	163	35,500
900	232,500	194	36,500
1, 000	269,000	225	37,400
1, 200	342,000	285	38,800
1, 400	416,000	347	39,900

Equation (2) implies that y in (1) depends only on the weight P of the airplane. Hence, x is also a function of P since for a given wing the value of x depends on the value of y, selected on the corresponding polar.

The power required for flight is

$$\Pi = x \cdot \delta \cdot F(v) \tag{3}$$

Dividing this equation by the first equation (1), we obtain

$$\frac{\Pi}{P} = \frac{x}{y} v = \varepsilon \cdot v, \tag{4}$$

where $\varepsilon = \frac{x}{y}$ is also a function of P. From (4) we deduce the first theorem: "The power required for moving unit weight horizontally is proportional to the speed and does not depend on the flight altitude."

Let c be the fuel consumption (in kg) necessary to obtain unit effective power. Assume that this specific fuel consumption does not depend on the altitude. Furthermore $v = \frac{ds}{dt}$, where ds is an element of distance, traversed

* G.Granz, "Lehrbuch der Ballistik" (Textbook of Ballistics), 1910, Vol.I, p.46. V.Mechnikov, "Soprotivlěnie Sredy" (Resistance of Media), 1912, p.182.

during time element dt ; also, $p=c.\Pi$ is the fuel consumption in unit time. We may then rewrite (4) in the form

$$\frac{p.dt}{P} = c \cdot \varepsilon \cdot ds. \qquad (5)$$

Since ε depends only on P, while $p \cdot dt$ is the fuel consumption during the time element dt, which is equal to the decrement dP, of the airplane weight, we obtain from (5):

$$\frac{dP}{P} = -c \cdot \varepsilon \cdot ds. \qquad (6)$$

This equation contains neither δ nor v. We obtain from it the second theorem (valid for $c = \text{const}$):

50 "The energy necessary to cover a given distance in horizontal flight is independent of the altitude and speed when the initial weight is given and the specific fuel consumption is constant."*

In the particular case when ε is constant but P, varies, or if an average value of ε is taken, we can integrate (6) and obtain the distance traveled at any altitude and speed, with c =const:

$$s = \frac{1}{c\varepsilon} \lg \frac{P_0}{P_1}. \qquad (7)$$

Here P_0 is the initial weight, and P_1 the final weight of the airplane.

Setting $c = 1.54 \cdot 10^{-7}$ kg · m [This is incorrect, since the dimensions of c are sec · kg/kg · m · sec =1/m, so that $1/c$ has the dimension of a distance, as follows also from (7), since ε' is dimensionless by definition], referred to the effective power, at a fuel consumption of 0.25 [?] per hr and a propulsion-plant efficiency of 60%, we obtain

$$s = 4\,500 \cdot \lg \frac{P_0}{P_1}. \qquad (8)$$

The coefficient 4,500 was obtained as follows:

One kg of fuel yields mechanical work equal to 424 · 2536 kg · m. [1 kcal is equivalent to 427 mkg of mechanical work. It is quite unclear what fuel is supposed to have heating value of only 2,536 kcal/kg]. At a propulsion-plant efficiency of 60% the work obtained is 0.6 · 424 · 2536 kg · m/kg.

This is equivalent to 1 kg . m work per 1.55 . 10 kg of fuel. Setting $\varepsilon = \frac{1}{7}$, we obtain:

$$\frac{1}{c\varepsilon} = \frac{1 \cdot 7}{1.55 \cdot 10^{-6}} = 4\,500\,000 \text{ m} = 4\,500 \text{ km}.$$

Let the fuel constitute 50% of the gross weight at takeoff. In other words, the initial weight is twice the final weight. Thus, the range is 3,100 km.

The two theorems mentioned above indicate the way in which superaviation may be developed, and lead to the conclusion that, since the amount of fuel

* It is mentioned in "Flugsport," 1927, p.15, that neither theorem is new, having been established in Germany during World War I.

necessary to cover a given distance depends only on the power and not on the speed, it should be burnt the faster, the higher the speed to be attained. This means that an engine of maximum possible power should be fitted to an airplane of given weight. It follows that the engine weight should be reduced and its cooling improved.

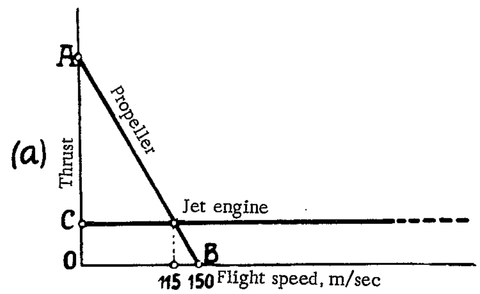


FIGURE 59. Comparison of propeller and jet engines

It is difficult to satisfy these conditions with conventional engines at high-altitude flights, since the power and heat transfer decrease with increasing altitude. Furthermore, the propeller rpm would have to be excessive, thus affecting the strength and weight of the airscrew. It is therefore necessary to use jet engines, since only these can operate satisfactorily at supersonic speeds.

In fact, the use of propellers for airplane propulsion has certain limitations. An airscrew ceases to develop thrust when the flight speed
51 (m/sec) approximates the translational propeller velocity (pitch (m) multiplied by revolutions per sec). The propeller rpm and pitch thus have certain limits in practice, beyond which the airscrew becomes useless. A jet engine extends these limits since the gas outlet velocity is very high. The limit of applicability of such engines is the flight speed, which may even exceed the gas outlet velocity as the fuel is burnt and the airplane becomes lighter. Figure 59 is a schematic diagram in which the line *AB* indicates how the propeller thrust decreases with increasing flight speed. Thus at point *B*, which corresponds to 60 revolutions per sec at a propeller pitch of 2.5 m, the limiting translational propeller velocity is approximately $60 \times 2.5 = 150$ m/sec.

The line *CD* represents the thrust developed by a jet engine. Jet engines should be employed beyond a certain flight speed, corresponding to the intersection *P*, of lines *CD* and *AB* (e.g., 115 m/sec).

An interesting paper was published in 1929 by B. V. Korvin-Kroukovsky in the U.S.A. on this subject. Below we reproduce almost the complete paper, omitting generalities.

Legends of the Far East refer to an Apus which never touched the ground during its life. The male offered his back to the female which laid its egg on it and remained there.

V.Bel'she, "Dni Tvoreniya" (Days of Creation)

c) The high-altitude airplane

Two articles,discussing engineering problems in the design of an airplane to navigate in the stratosphere,were published by B. V. Korvin-Kroukovsky (Krukovskii) in "Aviation," of 20 April and 18 May, 1929.

An airplane is a machine capable of taking off from the earth and climbing to various altitudes. The density of the air at the surface facilitates takeoff and landing. The density of the air is low at high altitudes; this reduces the drag so that the flight speed can be high.

In this paper we shall consider the prospects of high-altitude flight, ignoring tempting hypotheses (gas turbines and jet engines), but basing our discussion on the present tate of technology.

An airplane used as means of communication has a low speed at takeoff, using a large angle of attack. It thus climbs to a low altitude where the angle of attack is reduced, the drag becomes smaller, and the speed increases. A comparatively small part of the power is needed to maintain the airplane aloft when the angle of attack is large, and most of the power is used for gaining altitude. This excess power is used to overcome the drag, which is considerable at high speed, during horizontal flight at a small angle of attack. The maximum speed attained depends on the weight/power ratio and on the minimum speed. The smaller the minimum
52 speed, the larger must be the wing area, the greater will be the drag, and the smaller will be the maximum speed. The latter is given by Diehl's formula (NACA Technical Report No. 173):

$$v_{\max} = k \cdot \eta^{1/3} v_{\min}^{2/3} \left(\frac{W}{P}\right)^{-1/3}, \tag{1}$$

where $v_{\max}$ is the maximum speed in horizontal flight; $v_{\min}$ is the minimum or landing speed; W is the gross weight of the airplane; P is the engine power; η is the propeller efficiency; k is a numerical coefficient.

For modern commercial airplanes we have

$$k \cdot \eta^{1/3} = 21$$

so that (1) becomes

$$v_{\max} = 21 \cdot v_{\min}^{2/3} \left(\frac{W}{P}\right)^{-1/3}. \tag{2}$$

For the sake of determinancy we shall consider a typical six-seater plane having the following characteristics: gross weight, 4,250 lbs; power, 425 hp; wing area, 380 ft^2; maximum lift coefficient, 1.40; minimum speed, 55 miles/hr.

The maximum speed is then:

$$v_{max} = 21 \cdot 55^{2/3}, 10^{-1/3} = 141 \text{ miles/hr.}$$

In this article we shall consider various methods of flight, assuming the initial large angle of attack to be maintained. The speed, however, will increase since the flight takes place at a high altitude where the air density is low. We shall also assume that a turbocharger or similar device makes it possible to maintain a constant engine power irrespective of the flight altitude. We shall later investigate this and determine whether this assumption is permissible. Our plane can fly at sea level at a certain speed v_0, which is little higher than the minimum speed v_{min} and corresponds to a minimum power P_0. The wings provide a lift L equal to the weight of the airplane, while its drag is determined by the ratio $\frac{L}{D}$.

The ratio $\frac{L}{D}$ depends only on the angle of attack, not on the flight speed or the density of the air. In our case the angle of attack is constant throughout the climb, so that the ratio $\frac{L}{D}$ does not vary. The lift is always equal to the weight of the airplane, irrespective of the altitude, so that we shall assume it to be constant. It follows from this that the drag must also be constant. The air pressure decreases with increasing altitude, so that the speed must increase for the lift to remain constant. This speed increase ensures constancy of the drag.

53 For an airplane flying at a speed corresponding to the minimum power at sea level we may write

$$L = \frac{1}{2} \rho_0 v_0^2 C_L A \tag{3}$$

while at a different altitude

$$L = \frac{1}{2} \rho \cdot v^2 C_L A, \tag{4}$$

where L is the lift which is equal to the airplane weight; C_L is the lift coefficient, which is constant when the angle of attack is constant; A is the wing area; v is the speed at the altitude considered; v_0 is the speed at sea level; ρ is the density of the air at the altitude considered; ρ_0 is the density of the air at sea level.

Dividing (3) by (4), we obtain

$$\rho_0 v_0^2 = \rho \cdot v^2$$

or

$$\frac{v}{v_0} = \left(\frac{\rho_0}{\rho}\right)^{1/2}, \tag{5}$$

i. e., if the airplane climbs, with the angle of attack being constant, its speed is inversely proportional to the square root of the air density. We obtain the power required by multiplying the drag by the speed. However, the drag is constant in our case. The power is therefore directly proportional to the speed and inversely proportional to the square root of the air density, i. e.,

$$\frac{v}{v_0} = \frac{P}{P_0} = \left(\frac{\rho_0}{\rho}\right)^{1/2}. \tag{6}$$

Equations (5) and (6) enable us to determine the ceiling and the maximum speed of a plane equipped with some kind of turbocharger which enables the power to be maintained constant at any altitude, when the angle of attack is constant.

In this case, the speed is directly proportional to the power, and not to its cubic root, as in the case of flight at constant altitude but variable angle of attack.

This conclusion enables us to expect very high speeds during flights at large angles of attack and high altitudes, as compared with flight at a small angle of attack at sea level.

According to P. E. Warner (Aerodynamics, p. 311) the minimum power necessary for horizontal flight at sea level is

$$P_0 = \frac{W\sqrt{\frac{W}{A}}}{138}. \tag{7}$$

Substituting this value of P_0 in (6) and taking the propeller efficiency as 0.85, which is permissible in the case of the large pitch necessary at high altitudes, we obtain

$$\frac{v}{v_0} = \frac{P}{P_0} = \frac{117}{\frac{W}{P}\sqrt{\frac{W}{A}}}. \tag{8}$$

54 Writing

$$\frac{W}{A} = \frac{1}{2}\rho_0 v_0^2 C_L.$$

and taking $C_L = 1.20$ at the minimum power, $\rho_0 = 0.00237$ [slugs/ft^3], and introducing the conversion factor $(22/15)^2$ in order to obtain the speed v in miles/hr [instead of ft/sec], we obtain:

$$\frac{v}{v_0} = \left(\frac{\rho_0}{\rho}\right)^{1/2} = \frac{2{,}120}{\frac{W}{P}v_0} \tag{9}$$

and finally:

$$v = \frac{2{,}120}{\frac{W}{P}} \tag{10}$$

This very important and interesting result shows that the maximum speed does not depend on the minimum speed if the airplane flies at a constant angle of attack, and the high speed is attained at an altitude where the density of the air is low.

An airplane with a small wing area, a high landing speed, and a high speed v_0, at the minimum power cannot climb to a high altitude. This follows from the value of $\frac{\rho_0}{\rho}$ obtained from (9); its speed will be higher [than at sea level], but still low.

An airplane with a large wing area will have low values of v_{min} and v_0 but will be able to climb higher and develop large speeds. However, the *maximum speed at the ceiling is the same in both cases and depends only* on the weight/power ratio. A typical 425 hp plane with a landing speed of 55 miles/hr can develop a maximum speed of 141 miles/hr at sea level. Since $\frac{W}{P} = 10$ lbs/hp, we find from (10) that at a large angle of attack the maximum speed at the ceiling will be 212 miles/hr, i. e., 50% more than at sea level. Taking the speed at sea level at maximum power as 60 miles/hr, corresponding to a landing speed of 55 miles/hr, we obtain from (9):

$$\frac{\rho_0}{\rho} = \left(\frac{2{,}120}{10 \times 60}\right)^2 = 12.5,$$

which corresponds to an altitude of 63,000 ft.

It is of interest to investigate the properties of an airplane, which enable it to attain a higher speed at a high altitude than at sea level.

We divide (10) by (2), writing v_g for the maximum speed at sea level, v_a for the maximum speed at the altitude considered, and v_m for the minimum speed at sea level. This yields

$$\frac{v_a}{v_g} = \frac{101}{\left(\frac{W}{P} v_m\right)^{2/3}}. \tag{11}$$

This shows that the gain in speed at a high altitude varies inversely
55 with the landing speed and the weight/power ratio.

Attempts to reduce the weight/power ratio have been made for a long time; from year to year we build more powerful engines and obtain smaller weight/power ratios. The possible gain in speed due to altitude therefore increases constantly; this will continue until the gain becomes negligible. We can at present expect a 50% gain.

At the same time there is a tendency to increase the landing speed; this is, however, at the expense of the maximum speed, an unnecessary sacrifice. Consider now the influence of a high speed on the range. We have to distinguish between a record range and the normal range.

To attain a record range the airplane flies at a constant angle of attack, at which $\frac{L}{D}$ is maximum. This case corresponds to a high-altitude plane. The difference consists only in that consumption of part of the fuel entails a reduction in the weight; this provides excess power which a high-altitude plane utilizes to gain height; in an ordinary plane the throttle is closed in order to save fuel.

In both cases the value of $\frac{L}{D}$ and the angle of attack remain constant, so that the range is determined by Bréguet's equation.

$$\text{Range in miles} = 863.5 \left(\frac{L}{D}\right) \cdot \left(\frac{\eta}{C}\right) \lg_{10}\left(\frac{W_0}{W_1}\right). \qquad (12)$$

where W_0 is the initial weight, lbs; W_1 is the landing weight, lbs; η is the propeller efficiency at the cruising speed; C is the specific fuel consumption.

It will be shown later that high-altitude planes will probably use engines geared to the propeller whose efficiency will be constant up to 80%. The engine will operate at fully open throttle, the specific fuel consumption being approximately 0.55 lbs/hp • hr.

We thus obtain:

$$\frac{\eta}{C} = \frac{0.80}{0.55} = 1.45.$$

Airplanes flying at low altitudes at variable throttle openings have directly coupled propellers whose efficiency does not exceed 70%. The specific fuel consumption in this case varies between 0.55 lbs/hp • hr at the beginning of the flight, when the throttle is fully open, to 0.75 lbs/hp • hr at the end of the flight, when the throttle is half open. The mean fuel consumption is 0.65 lbs/hp • hr.

We then obtain:

$$\frac{\eta}{C} = \frac{0.70}{0.65} = 1.08.$$

We thus see that while the same law governs the record ranges of ordinary and high-altitude planes, the differences in the propeller efficiency and specific fuel consumption give the high-altitude plane an advantage of 1.45/1.08 or 35% in the range.

56 We have assumed that the airplane is designed to attain record ranges and flies at the maximum $\frac{L}{D}$ value and a low speed. However, an airplane usually flies at the cruising speed, which spares the engine when distances between 300 and 600 miles have to be traversed. This speed is usually 75—85% of the maximum speed. The cruising speed varies with the maximum speed; a high-altitude plane whose maximum speed is increased by 50% at the same engine power will have a 50% larger range in ordinary flight. We shall show that high-altitude flight enables us to increase the speed at the same weight/power ratio as obtained with modern commercial airplanes, and that this gain in maximum speed at sea level will increase further when the weight/power ratio is reduced through the construction of more powerful and lighter engines.

We have already shown that this speed increase is accompanied by an increase in the range. This entails a reduction in the number of intermediate stops, so that the [commercial] speed of the plane is increased even more. We have also seen that this speed increase is achieved not at the expense of the landing speed, as is the case now; the landing speed may be selected in accordance with safety requirements, allowing especially for the limited size of airfields. This is a particular advantage of high-altitude planes; in addition, they provide greater safety, passenger comfort, ease of navigation, and several other advantages whose discussion we shall postpone until we have shown that such an airplane is possible. This depends on two factors, namely the availability of airtight cabins in which normal air density and temperature are maintained despite the low temperature and density of the outside air, and of engines designed to operate in rarefied and cold air.

We shall consider the latter problem first, since we must show that it is technically possible to attain high altitudes, and then that equipment ensuring convenient working conditions for the crew can be designed.

The power of a nonsupercharged engine decreases rapidly with increasing altitude. The weight of the air aspirated into the cylinders decreases with the air density. However, the frictional losses remain the same, so that the torque decreases more rapidly than the density of the air, i.e., faster than the propeller rpm at a given rotational speed [sic!].

The engine rpm thus decrease with increasing altitude and this leads to a loss of power.

Some engines were tested at low pressures and temperature in the high-altitude chamber of the Bureau of Standards; it was found (c.f. E. P. Warner, Aerodynamics, p. 261) that the relationship between the power and the density of the air is approximately [This should obviously read hp/(hp)]

$$\mathrm{hp} = \left(\frac{\rho}{\rho_0}\right)^{1.4}.$$

The variation of the power with the altitude, obtained from this formula, is represented in Figure 60 [lower curve]. Approximately 41% of the power is used at an altitude of 20,000 ft, but only 10% at 45,000 ft. Engine power can be maintained better if the density of the air entering the carburetor is increased by means of a supercharger. We mention in this article: 1) the Roots blower which is driven at a medium speed from the engine shaft via two pairs of gears, 2) the gear-driven high-speed centrifugal compressor, and 3) the turbocharger, which uses the energy of the exhaust
57 gas. The first two types are similar in that the power necessary for compressing the air is obtained from the engine shaft; this reduces the engine power with increasing altitude, although not to the same extent as when no supercharger is provided.

The work necessary for adiabatic compression of the air is

$$\mathrm{hp}_s = \frac{144}{33\,000} \cdot \frac{n}{n-1}\,\frac{1}{K}\,pv\left[\left(\frac{p_0}{p}\right)^{\frac{n-1}{n}} - 1\right], \qquad (13)$$

where hp_s is the power necessary for driving the supercharger; n is the ratio of the specific heat of the air at constant pressure to the specific heat of the air at constant volume (for air $n = 1.405$; K is the supercharger efficiency, including that of the transmission; p is the air pressure at the altitude considered, lb/in^2; p_0 is the air pressure at sea level, lb/in^2; v is the volume of the air aspirated at pressure P, ft^3/min.

Aero engines require on the average about 0.5 lb of gasoline per hp and hr. The weight of air required is theoretically 12 times, in practice, 15 times as much.

Assuming the density of air to be 0.07608 lb/ft^3 at normal conditions, we obtain the volume of air necessary per hp and min:

$$0.5 \cdot \frac{15}{0.07608} \cdot 60 = 1.64 \text{ ft}^2.$$

The weight of the air necessary for the combustion of 0.5 lb of gasoline and for developing a power of 1 hp does not vary with the altitude.

The volume v is inversely proportional to the pressure p, since the product pv remains constant as long as the temperature is constant.

Let the temperature of the air at sea level be 60°F or 288°K; in the standard atmosphere the air temperature decreases by 1.98°C for every 1,000 ft up to an altitude of 35,000 ft, after which it remains constant at 218°K or -68°F. [The limits given by R. V. Mises, Theory of Flight, p. 8, Dover Pubs., 1959, are 59 and -67°F respectively.]

The quantities p, v, and T are related by the equation:

$$pv = RT.$$

At sea level (normal conditions) we have:

$$P = P_0 = 14.7 \text{ lb/in}^2;$$
$$v = 1.64 \text{ ft}^2, T = 288°\text{K};$$
$$pv = 0.084 \ T.$$

Inserting these values into (13), we obtain the power required by the blower:

$$\text{hp} = \frac{0.00127}{K} T\left[\left(\frac{p_0}{p}\right)^{0.29} - 1\right]. \qquad (14)$$

All superchargers have until now been designed for purely military purposes and flight at an altitude of 20,000 ft, where the pressure ratio is $\frac{p_0}{p} = 2.18$. For this a single-stage compressor with an efficiency of 70% is adequate. However, in this article we are considering the possibility of flying at altitudes between 50,000 and 100,000 ft, where the pressure ratio varies from 9–95. Such a pressure ratio cannot in practice be obtained with a single-stage compressor; we therefore propose a three-stage compressor with a correspondingly lower maximum efficiency of 40%.
58 Substituting this value for K, multiplying the coefficient 0.00127 by three, and dividing the exponent 0.29 by three, we obtain the energy required by a three-stage compressor:

$$\text{hp}_{,} = 0.0096 \ T\left[\left(\frac{p_0}{p}\right)^{0.10} - 1\right]. \qquad (15)$$

The engine power must be reduced by this value in order to obtain the net* power. However, the engine power is slightly increased, since the resistance to the discharge of the exhaust gas/counterpressure is less than at sea level.

* [Russian text says "total," which is wrong.]

Let the compression ratio be 5:1, the exhaust gas being discharged into vacuum. The volume of the fresh charge in the cylinder is equal to the volume displaced by the piston during its stroke, together with the volume of the combustion space, i. e., altogether 120% [of the volume displaced by the piston during its stroke]. Tests carried out with engines exhausting into a very rarefied atmosphere* have shown that the power increases by 1.3% per lb/in^2 difference between the inlet and exhaust pressures. In the limiting case of exhaust to vacuum we have

$$\Delta \text{hp} = 1.3 \times 14.7 = 19.1\%,$$

which satisfactorily confirms the theoretical results obtained.

The net power of an engine equipped with a three-stage compressor, driven via a mechanical transmission and maintaining a pressure of 14.7 lb/in^2 in the carburetor, is given by:

$$\frac{\text{hp}_{\text{altitude}}}{\text{hp}_{\text{sea level}}} = 1 + 0.013\,(p_0 - p) - 0.0096\,T\left[\left(\frac{p_0}{p}\right)^{0.10} - 1\right] \qquad (16)$$

when the exhaust gas is discharged to atmosphere.

The upper curve in Figure 60 has been plotted from this formula. The net power is 25% of the gross power at an altitude of 50,000 ft, and becomes zero at 100,000 ft. We therefore conclude that a mechanically driven compressor is excellent for military airplanes at an altitude of 20,000 ft, but useless for flight at altitudes between 50,000 and 100,000 ft. Consider now the operation of a turbocharger. A Rateau turbocharger is shown in Figure 31; it was later improved in France. Figure 32 shows a U.S. Air Service and General Electric Co. of America turbocharger. Operation of this engine was excellently characterized by S. A. Massia, an expert in this field, as follows:

". . . . the chamber containing the exhaust valve is connected by a branch pipe *s* (Figure 32), through which the combustion gas flows, to a nozzle directing it against the turbine rotor ***T***. The latter drives centrifugal blower ***W***, which aspirates air, compresses it, and discharges it into the carburetor from where the air is sucked into the cylinder.

The atmospheric pressure at an altitude of 20,000 ft is one half of that at sea level. The pressure at an altitude of 35,000 ft is one quarter of that at sea level. A normal pressure [in the carburetor] can be maintained at these high altitudes with the aid of a blower . . . . "

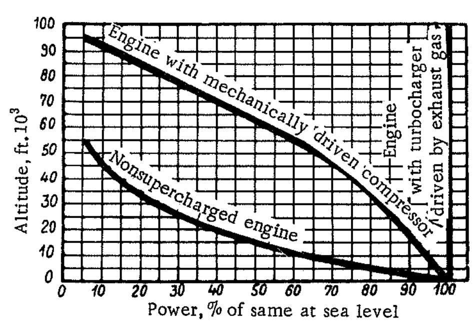


FIGURE 60

* A.C.A.Internal.Comb.Engine Subcommittee Report No.35.

59 We shall now determine the power which can be obtained through the expansion of the hot combustion gas from a pressure of 14.7 lb/in² in pipe (*s*) to the outside pressure at a larger altitude.

The work done during expansion is also given by (13). We only have to find the values of v and pv for the exhaust gas.

The mixture entering the cylinder consists on the average of one part by weight of gasoline vapors, three parts by weight of oxygen, and 12 parts by weight of nitrogen. The exhaust gas discharged from the cylinder consists* of one part by weight of water vapor, three parts by weight of CO_2 and CO, small amounts of free hydrogen, and 12 parts by weight of nitrogen. This means that the mass of the gas passing through the engine remains constant, while only $^1/_5$ is converted into gas whose specific weight slightly exceeds that of air. We stress "slightly" because CO_2 is heavier than the oxygen, part of which it replaces, but CO and hydrogen are lighter than oxygen. We therefore do not make a large error by assuming that the exhaust gas, cooled to normal temperature, has the same volume of 1.64 ft³ per hp and min, as the fuel mixture, and that the equation

$$pv = 0.084\,T,$$

derived for the former, is also valid for the exhaust gas.

For the same reason the expansion exponent $n = 1.405$ is also valid for the exhaust gas.

The power obtained through the expansion of the exhaust gas under these conditions is given by (14). The ratio of the power hp_s, necessary for compressing the air in a single-stage blower, to the power hp_e obtained through the expansion of the exhaust gas in a turbine is

$$\frac{hp_s}{hp_e} = \frac{T_a}{T_e},$$

where T_a is the absolute temperature of the outside air, equal to 218°K at an altitude of 35,000 ft, while T_e is the absolute temperature of the exhaust gas in pipe (*s*) (1,000°K).

Inserting these values, we obtain

$$\frac{hp_s}{hp_e} = 0.218,$$

i.e., a turbocharger is able to compress air to a pressure of 14.7 lb/in² at any altitude, provided that its efficiency is not less than 21.8%. This value may be reduced if a three-stage compressor is used.

The power necessary for operation of the latter at 100% efficiency [K = 1 in (14)] is

$$hp_s = 0.00381\,T_a\left[\left(\frac{p_0}{p}\right)^{0.10} - 1\right]$$

* S.A.E. Transactions, Vol. 20, Part 2, p. 105.

and the ratio of this power to the power hp_e obtained from the exhaust gas is [cf. (14)]:

$$\frac{hp_s}{hp_e} = 3\,\frac{T_a}{T_e}\,\frac{\left(\frac{p_0}{p}\right)^{0.10} - 1}{\left(\frac{p_0}{p}\right)^{0.20} - 1}. \qquad (17)$$

60 This ratio depends both on $\frac{p_0}{p}$ and on $\frac{T_a}{T_e}$. At the same temperature ratio $\frac{T_a}{T_e}$ as before we obtain at the various altitudes the following values of $\frac{hp_s}{hp_e}$:

50,000 ft	0.182
75,000 "	0.159
100,000 "	0.138

This reduction in the ratio $\frac{hp_s}{hp_e}$ with increasing altitude is very advantageous, since it compensates for the increased air consumption due to the lower oxygen content. Air is a mixture of oxygen and nitrogen [and some other constituents]; oxygen is slightly heavier than air, while nitrogen is slightly lighter. The difference of their densities is small, and the gases are well mixed in the lower layers of the atmosphere (below an altitude of 31,000 ft), due to perturbations caused by temperature variations. However, the air temperature is uniform above an altitude of 35,000 ft, so that no perturbations occur. This causes stratification of the air, so that the heavier oxygen accumulates at the bottom. The oxygen content at the various altitudes is as follows:*

above	35,000 ft.	21%
at	50,000 "	20%
"	75,000 "	17%
"	100,000 "	15%

Since the oxygen content of the air decreases, the amount of air needed by the engine increases; the power required for compression will increase proportionally.

We thus obtain the following values for the ratio $\frac{hp_s}{hp_e}$:

at	50,000 ft.	19.1%
"	75,000 "	19.6%
"	100,000 "	19.3%

The turbocharger thus must have an efficiency of not less than 20% if air is to be supplied to the engine at a pressure of 14.7 lb/in^2 at any altitude. This is not difficult to achieve in practice. The fact that above 35,000 ft

* Mc.Adie "The Principles of Aerography," p.23.

(in the stratosphere) the air becomes stratified, leads to an interesting speculation. Air contains not only oxygen and nitrogen, but also considerable amounts of hydrogen, which is almost absent in the lower layers of the atmosphere, but predominates in its upper layers. Since the oxygen content of the air decreases with increasing altitude, while the hydrogen content increases, their proportions at an altitude of about 200,000 ft yield a combustible mixture.

Indeed, the huge explosions caused by meteors could not occur if this mixture, extremely rarefied at the altitude of 200,000 ft were not combustible. If compressed, this mixture might thus serve as fuel for airplanes once they succeed in gaining this altitude. It would therefore be possible to fly at this altitude over unlimited distances at speeds of up to 3,000 miles/hr, using the atmosphere as fuel.* However, to achieve this a power is necessary
61 which is at least 40 times larger than that required for horizontal flight at sea level.

Up to now we have only considered the power necessary for driving the compressor and the engine power. Let us now discuss the size and weight of the compressor. Existing superchargers for 400 hp engines, capable of delivering air at normal pressure at 20,000 ft, weigh 150 lbs each. The pressure ratio is only 218 at this altitude. A mechanically-driven single-stage supercharger requires 60–80 hp. We have found in this article that the excess power permits modern planes to fly at an altitude of 75,000 ft.

We see from (15) that the compressor necessary to charge a 400 hp engine at this altitude requires 336 hp, i. e., almost four times more than existing compressors. The general rule of engineering problems states that reducing the size of any mechanism makes the problem easier, while increasing the size makes it more and more difficult.

There is no doubt that powerful gas turbines will be designed in the future; however, it is difficult to expect the appearance of 336 hp gas turbines in the near future. We may, by the way, do without them, using instead three or four compressors of existing size.

It is quite possible to combine four superchargers in such a way that the air discharged from one enters the next, and that the exhaust gas expands successively in all four turbines. The pressure ratio at an altitude of 75,000 ft is 29.2, and the expansion ratio in each turbine is:

$$\sqrt[4]{29.2}=2.32,$$

which is quite acceptable.

The gas will expand almost adiabatically if the pipes connecting the turbines are short and well insulated in order to prevent heat losses, so that we may use (14).

Combining four superchargers, each of which has a compression ratio of 2.32, we obtain an overall compression ratio of $2.32^4=29.2$.

In contrast to the turbines, the compressors should be connected in such a way that all surfaces which have to be cooled are exposed to air currents, so that the heat produced during the compression is dissipated.

In talking about compressors of existing types we assume that the vital parts, such as turbine rotors, shafts, and vanes, have present-day dimensions.

* See epigraph to this article (p.49).

There will be certain differences in detail, particularly in the number of nozzles through which the exhaust gas is directed against the turbine rotor. The number or the size of the nozzles in successive turbines increases as the gas expands. The dimensions of the compressor intakes will increase similarly, and the vanes in the low-pressure stages will probably be wider. It may be possible in time to reduce the number of compressors to three by increasing the compression ratio in each.

Existing turbochargers, weighing 150 lbs *for a* 400 hp *engine, are thus* arranged in a series of four, weighing 600 lbs altogether. We may expect
62 the weight of such a combination to be reduced, so that the entire turbocharging equipment will, for an altitude of 75,000 ft, weigh not more than 1 lb/hp.

Up to now we have discussed the development of full power in the engines at any altitude. We shall now consider engine cooling. About one third of the heat supplied by the fuel to the engine cylinders is transmitted to the cooling water or the cooling fins (in air-cooled engines) and from there to the air. The amount of this dissipated heat is a very important factor in the operation of the engine, as is excessive heating of the cylinders, causing detonations [knocking], which reduces the power and may result in mechanical damage. The water temperature in liquid-cooled engines must be below 212°F at sea level; it usually varies between 160 and 180°F.

The temperature of the cylinder heads in air-cooled engines must not exceed 600°F during continuous operation.* The amount of dissipated heat depends on the difference between the temperatures of the cylinder or radiator and the air, on the flow velocity of the cooling air, and on its pressure.

All these quantities vary with the altitude; the low temperature at large altitudes improves cooling, as does the increase in the air flow velocity. The reduced pressure of the cooling air, i. e., the lower weight of the air flowing through the radiator or past the fins on the cylinders in the case of an air-cooled engine, reduces the cooling effect and thus counterbalances the advantage gained by the low temperature and high speed.

It has been shown** that the heat transmitted by the cylinder to the air flowing past is equal to

$$W_c = KLD^{1/4} p^{1/2} V^{1/2} \frac{\Delta t}{T^{0.123}},$$

where W_c is the quantity of heat transferred in unit time; K is a numerical coefficient; L is the cylinder length; D is the cylinder diameter; p is the absolute static air pressure; V is the air velocity; Δt is the difference between the temperatures of the cylinder walls and the air stream; T is the mean absolute temperature of the surface and the surrounding air.

The amount of dissipated heat also depends on the power developed in the cylinders, which is constant at all altitudes in the case of an engine with turbocharger. *Furthermore, we are not interested in the absolute* quantity of this heat, but only in the ratio of the heat dissipated at a given altitude to that dissipated at sea level from the same cylinder, i. e., at the same values of L and D. Let p_0, V_0, Δt_0 and T_0 be the values of the

* S.D.Heron.– S.A.E. Journal, January, 1923.

** Chester W.Rice. "Forced Convection of Heat in Gases and Liquids." – Industrial and Engineering Chemistry, May, 1924.

respective quantities at sea level; we may then write:

$$\left(\frac{p}{p_0}\right)^{1/2}\left(\frac{V}{V_0}\right)^{1/2}\frac{\Delta t}{\Delta t_0}\left(\frac{T}{T_0}\right)^{0.123}=1. \tag{18}$$

63 In this article we are dealing with flight at a constant angle of attack, assuming that the speed increases with decreasing air density. We may therefore write:

$$\frac{V}{V_0}=\left(\frac{p_0}{p}\right)^{1/2}\left(\frac{T}{T_0}\right)^{1/2}.$$

Inserting this into (18), we obtain:

$$\left(\frac{p}{p_0}\right)^{1/4}\left(\frac{T}{T_0}\right)^{0.127}\frac{\Delta t}{\Delta t_0}=1.$$

The factor $\left(\frac{T}{T_0}\right)^{0.127}$ does not differ by more than 3% from unity for the limits considered; we may therefore take it as equal to unity and write:

$$\frac{\Delta t}{\Delta t_0}=\left(\frac{p_0}{p}\right)^{1/4}. \tag{19}$$

Before analyzing this expression for the case of high altitudes we shall consider under what flight conditions these results are valid. The Vought UO-1* airplane tested had a Wright Whirlwind engine with a Roots blower; the temperatures of all the cylinders were measured at the heads directly behind the ignition plugs. The temperatures differed for the various cylinders, but their mean was approximately 415°F at sea level and 465°F at an altitude of 20,000 ft; the corresponding air temperatures were 75 and 10°F respectively. From actual test results we thus obtain:

$$\frac{\Delta t}{\Delta t_0}=\frac{465-10}{415-75}=1.34.$$

Using the theoretical formula, we must take into account that the power developed in the cylinders and the heat dissipated increase by approximately 1.3% per lb/in^2 pressure difference when the blower is driven by the engine (the case we are considering) and the exhaust gas is discharged to a rarefied atmosphere. Equation (19) thus becomes:

$$\frac{\Delta t}{\Delta t_0}=[1+0.013\ (p_0-p)]\left(\frac{p_0}{p}\right)^{1/4}. \tag{20}$$

At an altitude of 20,000 ft we have:

$$\frac{p_0}{p}=2.18;$$

$$p=\frac{14.7}{2.18}=6.75\ \text{lb/in}^2;$$

$$p_0-p=7.95;$$

* N.A.C.A.Report, No.282.

whence:

$$\frac{\Delta t}{\Delta t_0} = 1.03 \cdot 2.18^{1/4} = 1.34$$

There is complete agreement between the theoretical conclusions and the experimental results; we may therefore use (19) to obtain correct
64 data on the operation of air-cooled engines equipped with turbochargers.

Assuming (19) to be valid, we shall try to determine the conditions at an altitude of 75,000 ft. The ratio of the differences between the temperatures of the cylinder walls and the air at this altitude and at sea level is:

$$\frac{\Delta t}{\Delta t_0} = \left(\frac{p_0}{p}\right)^{1/4} = 29.2^{1/4} = 2.32.$$

We mentioned that the maximum cylinder temperature is 600°F. The air temperature in the stratosphere is assumed to be -73°F. The maximum permissible temperature difference thus is 673°F. For this difference to be obtained at an altitude of 75,000 ft the temperature difference at sea level must be:

$$\Delta t_0 = \frac{673}{2.32} = 290°\text{F}.$$

The observed temperature difference* for the Wright Whirlwind engine installed in the Vought UO-1 airplane was 340°F; the cooling effect must therefore be increased by only 15%, which is possible in various ways. It was found that fitting an oil cooler reduces the temperature of the cylinder heads by 45°F, i.e., by 13% of the entire temperature difference.* Another way is to reduce the temperature of the mixture before it enters the cylinder by providing intermediate coolers between the supercharger and the cylinders. This reduces the compression temperature to below the detonation limit.**

Engine cooling by means of air is not yet based on scientific research, so that there is much room for improvement. The last method of cooling an enclosed engine, investigated by NACA, is very effective and entails low drag. However, even this method can be improved by adding fins and increasing the velocity of the air flowing past the cylinders, so that the additional cooling effect necessary at large altitudes is obtained. The slight projection of rounded-off cylinders into the air stream does not ensure adequate cooling. Theory and experiments show that the air velocity in front and behind the cylinders is low, but is considerably higher at their sides.

Measurements of the heat in a cylinder of an air-cooled engine indicated a degree of cooling of the same order.†

The results of the experiments are given in Figure 61. Guide vanes fitted between the cylinders might induce a more uniform air flow past the latter, so that cooling is improved (Figure 62).

* Report No.283.

** S.A.E. Transactions, Vol.21, Part I, page 276.

† M.Schrenk. "Probleme des Höhenflugs." (Problems of High-altitude Flight), Z.F.M., 19th year, 1928.

Consider now the influence of the propeller on the high-altitude flight.

The operation of a propeller depends mainly on the pitch/diameter ratio and on the engine power, propeller rpm, flight speed and air density. We thus have four variables which depend on the altitude. An airplane able to fly at any altitude up to 80,000 ft passes through air whose density varies by a factor of 25; its propeller must be able to absorb the power of the engine under all conditions. Much theoretical and experimental data are available which permit the design of propellers for high-altitude planes. It was found that the aerodynamic properties of propellers of any type depend on the three coefficients: $\frac{v}{ND}$, C and η.

65 The advance ratio $\left(\frac{v}{ND}\right)$ is usually taken as the independent variable, while the other two are functions of the first.

Here v is the flight speed, ft/sec; N is the number of propeller revolutions per sec; D is the propeller diameter, ft.

The second coefficient (the power coefficient C) is defined by the equation:

$$P = C\rho v^5 N^{-2}, \tag{21}$$

where P is the engine power, ft · lbs/sec; ρ is the air density, slugs*/ft^3 or lbs/ft^3, divided by the gravitational acceleration.

The coefficient C depends only on the shape of the propeller (but not on its dimensions) and on the advance ratio $\frac{v}{ND}$. As long as the advance ratio is constant, C is also constant irrespective of the propeller dimensions.

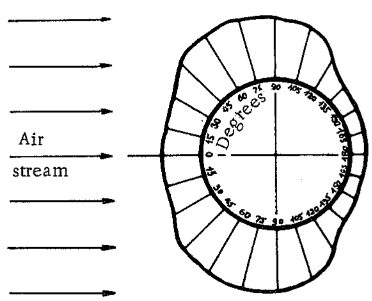


FIGURE 61

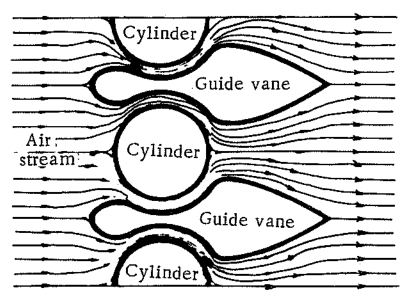


FIGURE 62

The third coefficient is the propeller efficiency η which depends on the advance ratio $\frac{v}{ND}$ and on the shape of the propeller, but not on its dimensions. The values of C and η are obtained as functions of $\frac{v}{DN}$ when a propeller or its model is tested in a wind tunnel. Many data on propellers were obtained by W. F. Durand in the laboratory of Stanford University and published by NACA. Figure 63 gives one of the diagrams pertaining to this

* Slug = 14.594 kg.

work,* which shows variation of C with $\frac{v}{ND}$ for No. 96 propellers having different blade settings. This propeller type was initially designed for a pitch/diameter ratio of 0.7; the blades were later set at angles between -6° and (maximum pitch) 20°. The maximum efficiency η is indicated on the curves for the various blade-setting angles; the broken line passing through these points shows the variations of C with $\frac{v}{ND}$ at the maximum efficiency.

66 Consider first a propeller with fixed pitch, assuming the advance ratio remains constant at all altitudes. Let P be the power, C the power coefficient, ρ the air density, v the air speed, N the rpm at any altitude, and P_0, C_0, ρ_0, v_0 the corresponding magnitudes at sea level. Using (21), we can write down the ratio of the power at any altitude to that at sea level:

$$\frac{P}{P_0}=\frac{C}{C_0}\,\frac{\rho}{\rho_0}\left(\frac{v}{v_0}\right)^5\left(\frac{N_0}{N}\right)^{1/2}. \qquad (22)$$

The diameter D of the propeller is the same at any altitude. From:

$$\frac{v}{ND}=\frac{v_0}{N_0D}=\text{const.},$$

we thus obtain:

$$\frac{N_0}{N}=\frac{v_0}{v}=\left(\frac{\rho}{\rho_0}\right)^{1/2} \qquad (23)$$

The power coefficient C is constant when $\frac{v}{ND}$ is constant, so that $\frac{C}{C_0}=1$.

Noting this and inserting (23) into (22), we obtain:

$$\frac{P}{P_0}=\frac{\rho}{\rho_0}\left(\frac{\rho_0}{\rho}\right)^{5/2}\cdot\quad\frac{\rho}{\rho_0}=\left(\frac{\rho_0}{\rho}\right)^{1/2},$$

or, by (23),

$$\frac{P}{P_0}=\frac{N}{N_0}=\frac{v}{v_0}=\left(\frac{\rho_0}{\rho}\right)^{1/2}. \qquad (24)$$

The first equation states that during the climb the engine power is proportional to the propeller rpm, as is the case with a supercharged engine. The other two equations are identical with (6) for the power required during horizontal flight at any altitude. It follows from (24) that the power obtained from an engine equipped with an exhaust-driven turbocharger, transmitted to a fixed-pitch propeller, is proportional to the power required during flight at any altitude, as long as the power is proportional to the propeller rpm.

The propeller must be large enough to permit sea-level flight at low rpm (e.g., 400 rpm). A small excess over the power required for horizontal

* Report 141.

flight is necessary in this case. Let this excess power required for the climb amount to 20%. The density of the air decreases when the plane gains altitude; while the air speed, the rpm, and the net and required power increase proportionally. The excess power remains 20%. The absolute excess power will thus increase; the ability to climb therefore increases with altitude, since the power available for this varies inversely as the square root of the air density. At an altitude of approximately 68,000 ft, where $\frac{\rho_0}{\rho}=16$, the air speed will be four times as high as at sea level, with the engine developing its normal power at the normal rpm.

67

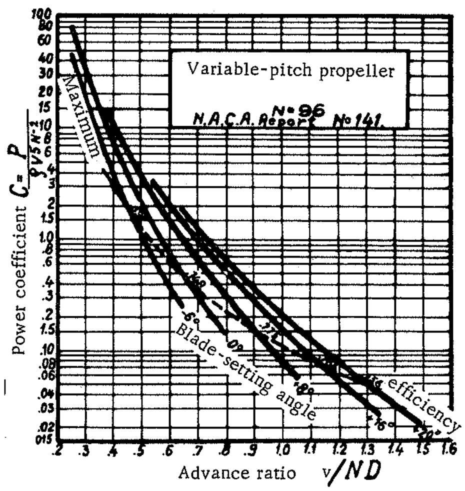


FIGURE 63

The torque begins to decrease when the plane climbs higher and the propeller rpm increase; the power is then no longer proportional to the rpm, and (24) loses its validity. The ceiling of a plane with fixed-pitch propeller is thus determined by the engine speed at which the power ceases to be proportional to the rpm.

The necessity of reducing the rpm at low altitudes is detrimental to the performance of a fixed-pitch propeller in a high-altitude plane. The engine can develop a power four times as high as that required for horizontal flight at sea level, but the propeller absorbs power only at a very low rotational speed, so that little advantage is gained from the excess power, and the rate of climb is low. Increasing the propeller rpm and the rate of climb by suitably designing the propeller causes the maximum rpm to be attained at a small value of $\frac{\rho_0}{\rho}$, so that the ceiling is low.

The engine rpm may be varied as desired by the pilot when a controllable-pitch propeller is used, the throttle being kept fully open. In analyzing the operation of such a propeller we must make the rather arbitrary assumption that the propeller rpm depends on the altitude or the air density.

Let this variation be according to the law:

$$\frac{P_0}{P}=\frac{N_0}{N}=\left(\frac{\rho_0}{\rho}\right)^n, \tag{25}$$

where n may assume any value. The advance ratio $\frac{v}{ND}$ varies in this case, so that C will not be constant. We obtain from (21):

$$\frac{C}{C_0}=\frac{\rho}{\rho_0}\cdot\frac{P_0}{P}\left(\frac{v}{v_0}\right)^5\left(\frac{N_0}{N}\right)^2$$

Inserting into this (25) and

$$\frac{v}{v_0}=\left(\frac{\rho_0}{\rho}\right)^{1/2},$$

we obtain:

$$\frac{C_0}{C}=\frac{\rho}{\rho_0}\left(\frac{\rho_0}{\rho}\right)^n\left(\frac{\rho_0}{\rho}\right)^{5/2}\left(\frac{\rho_0}{\rho}\right)^{2n}=\left(\frac{\rho_0}{\rho}\right)^{3n+1.5} \tag{26}$$

The required rpm remains constant in the ideal case, and the engine
68 develops its full power at any altitude, so that $n=0$.
Hence

$$\frac{N_0}{N}=\left(\frac{\rho_0}{\rho}\right)^0=1=\text{const},$$

and by (26):

$$\frac{C_0}{C}=\left(\frac{\rho_0}{\rho}\right)^{3/2}.$$

Thus, at an altitude of 68,000 ft,

$$\frac{\rho_0}{\rho}=16;\ \frac{v}{v_0}=16^{1/2}=4,$$
$$\left(\frac{v}{ND}\right):\left(\frac{v_0}{N_0D}\right)=4;\ \frac{C_0}{C}=16^{3/2}=64.$$

It is seen from Figure 63 that a variation of C by a factor of 64 does not correspond to a fourfold change in $\frac{v}{ND}$.

Let $\frac{v}{ND}=1.25$ at the maximum altitude and speed. We then obtain $C=0.06$ at the maximum propeller efficiency when the blade-setting angle is 20°. At sea level we have $\frac{v}{ND}=1.25/4=0.31$. The power coefficient should then be $C=0.06\times 64=3.84$ whereas Figure 63 gives a limiting value of 18 at a blade-setting angle of -6°. This large difference shows that the assumptions made are inadmissible. We therefore conclude that even with variable-pitch propellers it is hardly possible to maintain constant rpm and engine power at all altitudes.

We shall now consider the case where the pitch is varied at low altitudes; the rpm are then reduced, but not to such an extent as with a fixed-pitch propeller. For the latter we had $n=-\frac{1}{2}$; for a variable-pitch propeller we found that $n=0$ is impossible. We therefore assign to n the intermediate value $-\frac{3}{8}$ and insert this into (26):

$$\frac{C_0}{C}=\left(\frac{\rho_0}{\rho}\right)^{3/8}.$$

At the altitude of 68,000 ft we have:

$$\frac{\rho_0}{\rho}=16;\ \frac{v}{v_0}=16^{1/2}=4;\ \frac{N}{N_0}=16^{3/8}=2.82;\left(\frac{v}{ND}\right):\left(\frac{v_0}{N_0D}\right)=1.41,$$

$$\frac{C_0}{C}=2.82.$$

At a high speed and a blade-setting angle of +20° the values corresponding to the maximum propeller efficiency are:

$$\frac{v}{ND}=1.25;\ C=0.06,$$

whereas at sea level:

$$\frac{v_0}{N_0D}=\frac{1.25}{1.41}=0.89;\ C_0=0.06\times 2.82=0.17.$$

It is seen from Figure 63 that the point $\left(\frac{v_0}{N_0D}=0.89;\ C_0=0.17\right)$ exactly
69 corresponds to the maximum efficiency at a blade-setting angle of +8°. This shows that the rpm should vary with the air density, and that even the rpm of a variable-pitch propeller decrease considerably at low altitudes. We have already found that the rotational speed at sea level is 1,600/4 = 400 rpm for a fixed-pitch propeller and 1,600/2.82 = 568 rpm for a variable-pitch propeller if the maximum propeller speed is 1,600 rpm at 68,000 ft, where $\frac{\rho_0}{\rho}=16$. Even this reduction of the propeller rpm ensures better utilization of the engine power at sea level, namely 50% more with a variable-pitch propeller than with a fixed-pitch propeller. The rate of climb is also larger, without a lowering of the ceiling.

These results can be obtained by varying the blade-setting angle between 8° and 20°. Far better results are possible than those given in Figure 63 which refers to propellers with large variations of the blade-setting angle and a pitch/diameter ratio of 0.7. Their efficiency is therefore also much lower than is desirable for high-altitude planes.

We have repeatedly mentioned a maximum propeller speed of 1,600 rpm. To the reader this may appear very low, since modern engines frequently operate at more than 2,000 rpm. Such high-speed engines are at present installed in airplanes flying at intermediate altitudes where the air density makes it possible to use propellers with small diameters; the stresses in the blades thus do not exceed the permissible limits, and the maximum blade-tip speed does not exceed the velocity of sound.

The rarefied air at an altitude of 70,000 ft necessitates propellers of much larger diameters; the stresses in the blades may then become excessive if the same rotational speed is maintained. The blade-tip speed increases in proportion to the propeller diameter.

However, maximum permissible blade-tip speed, which is slightly below the velocity of sound, decreases since the velocity of sound varies inversely with the altitude. The velocity of sound is 1,110 ft/sec at sea level, but only 970 ft/sec at an altitude of 36,000 ft.*

The necessity of reducing the blade-tip speed requires engines with reduction gears; some way must be found to reduce the propeller diameter, e. g., by fitting propellers with four wide blades. A large pitch/diameter ratio, necessary for high speeds at high altitudes, does not significantly influence the efficiency. The reader may gain a better idea on the dimensions of the propeller from the fact that utilization of 400 hp at 1,600 rpm, at an altitude of 68,000 ft $\left(\frac{\rho_0}{\rho}=16\right)$ and a speed of 280 miles/hr requires a two-blade propeller of 15 ft diameter or a four-blade propeller of 13 ft diameter at the ordinary blade width and shape. The advance ratio $\frac{v}{ND}$ is approximately 1.2 for a four-blade propeller, while the pitch/diameter ratio is 1.3.

We thus arrive at the following conclusions regarding propellers for high-altitude planes:

1. It is possible to use fixed-pitch propellers having a constant advance ratio $\frac{v}{ND}$. Such propellers have a high efficiency at all altitudes. The
70 propeller rpm and the engine power will be proportional to the air speed, the percentage of the excess power available for climbing will be constant at all altitudes. This percentage is not large, so that the rate of climb will be low, unless the ceiling and speed are altered.

2. The use of variable-pitch propellers does not permit a constant propeller speed and full engine power to be maintained at low altitudes. Furthermore, such a propeller will have an unsatisfactory blade shape and a low efficiency at large altitudes.

3. The best solution is to use a controllable-pitch propeller; this involves a reduction of the rpm at low altitudes, although not to the same extent as with a fixed-pitch propeller. Varying the propeller rpm at constant torque according to the law

$$\frac{N}{N_0}=\left(\frac{\rho_0}{\rho}\right)^{3/8},$$

yields a propeller having a maximum efficiency at all altitudes. The variation of the blade-setting angle is in this case small, and not detrimental to the efficiency.

Utilization of the power at sea level is in this case 50% better than with a fixed-pitch propeller. It is desirable to fit a propeller with four wide blades, driven via gears by a low-speed engine. This eliminates the danger of excessive deflections and centrifugal forces encountered in large-diameter propellers; furthermore, the blade-tip speed will not exceed the velocity of sound.

* N. A. C. A. Technical Memorandum, No.412.

d) Theory of the rocket plane

We first present Crocco's computation.*

Examples of reaction engines are the rocket and the turbojet. The velocity of the gas streams discharged from them is approximately 2,000 m/sec, or even more at high altitudes. Let $m=\frac{P}{g}$ be the mass of the products of combustion discharged from the engine in unit time. Their [relative] velocity V is assumed to be constant, and to have a direction opposed to that of the flight (Figure 64b). Let the rocket, whose mass if M, have a velocity v at some instant t. R denotes the resultant of the external forces opposing the motion of the rocket. The momentum of the rocket $t+dt$ is $(M-mdt)\,(v+dv)$, whereas the momentum of the discharged mass of gas is mdt $(v-V)$. The momentum equation thus is

$$(M-m\,dt)\,(v+dv)+mdt\,(v-V)+R\,dt=Mv, \qquad (1)$$

whence

$$mVdt=Mdv+Rdt\,, \qquad (2)$$

or, dividing by dt:

$$mV=M\frac{dv}{dt}+R. \qquad (3)$$

The reaction caused by the gas discharge is thus equal and opposite to the sum of the inertia force of the rocket and the resistance. Furthermore, the reaction does not depend on the flight speed, but only on the outlet velocity of the products of combustion, on the external resistance, and on the mass of the rocket.

We shall show that the rocket may attain a speed exceeding the outlet
71 velocity of the gas.

Noting that $mdt = -dM$ we obtain from (2):

$$-\frac{dM}{M}\left(1-\frac{R}{mV}\right)=\frac{dv}{V}\,. \qquad (4)$$

Assuming $R=\text{const.}$ we integrate between the limits 0 and v_1, and M_0 and M_1, respectively:

$$\left(1-\frac{R}{mV}\right)\lg\frac{M_0}{M_1}=\frac{v_1}{V}\,. \qquad (5)$$

Since $mV>R$, we may find a value $\frac{M_0}{M_1}$, at which $v_1>V$, as was to be proved.

Let the rocket continue to fly at a constant speed $(dv=0)$, when a certain altitude has been reached.

Setting $\frac{x}{y}=\varepsilon$ in (1) on p. 45, we obtain $R=\varepsilon P=\varepsilon . Mg$, and by (2):

$$mVdt=\varepsilon\,Mgdt. \qquad (6)$$

* G. Crocco. Possibilita di Superaviazione (The Possibility of Superaviation), Notiziario Technico, 1926, No. 7, p. 1.

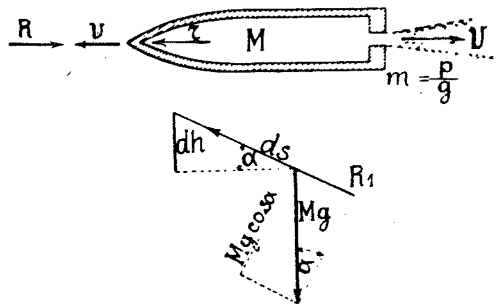


FIGURE 64

Since $mdt = dM$ and $v_1 dt = dS$, we obtain, setting $\varepsilon = \text{const.}$

$$\frac{dM}{M} = -\frac{\varepsilon g}{V v_1} dS. \tag{6'}$$

whence:

$$S = \frac{v_1 \cdot V}{\varepsilon \cdot g} \lg \frac{M_0}{M_1}. \tag{7}$$

This expression is a modification of the integral of (6) [on p. 47] if we replace the weight P by the mass M and assign a variable value to C, assumed to be constant in the case of ordinary engines.

Since C is referred to the effective power Π, which in our case is mVv_1, while the fuel consumption in unit time is $p = mg$, where m has been determined earlier, we obtain

$$c = \frac{p}{\Pi} = \frac{g}{v_1 V} \tag{8}$$

and (6') goes over into (5) when $\varepsilon = \text{const.}$

The second theorem given on p. 47 for an engine driving a propeller
72 thus becomes (in accordance with (6')):

"The fuel consumed in moving unit load in horizontal flight at uniform speed over a given distance is inversely proportional to the product of the flight speed by the gas outlet velocity." This proves the advantage of reaction engines at high speeds, and their shortcomings at the speeds usual in aviation.

The conclusions drawn on the basis of (8) are valid only when the flight speed and altitude are constant. The work performed in attaining this speed and altitude has been neglected. Consider now a body moving along a curved path, using the following notation:

M — mass of body; v — horizontal distance; v — flight speed; h — altitude; $f = m.V$ — thrust; α — inclination of [tangent to] flight path to horizontal; s — distance traveled along flight path at time t; $(\varepsilon \cos\alpha + \sin\alpha) Mg$ — resistance.

Furthermore,

$$dh = ds \sin\alpha; \qquad R_1 = Mg \sin\alpha;$$
$$dS = ds \cos\alpha; \qquad R_2 = \varepsilon Mg \cos\alpha \quad \left(\varepsilon = \frac{P}{R}\right);$$
$$\Sigma R = R_1 + R_2.$$

The equation of work is:

$$f.ds = Mv.dv + Mg\,ds \sin\alpha + Mg\varepsilon \cos\alpha\, ds$$

or

$$\frac{mV}{M}.ds = vdv + gdh + g\varepsilon.dS. \qquad (9)$$

Division by v yields:

$$-V\frac{dM}{M} = dv + g\,\frac{dh}{v} + g\varepsilon.\cos\alpha\, dt. \qquad (10)$$

We may in this equation set $\cos\alpha = 1$, since the flight path is very flat.

In considering (9) and (10) we must remember that $\delta.F(v) = \mathrm{const.}$ These equations give a relationship between v and h or between dv and dh.

We can integrate (10) for the entire flight path when $\cos\alpha = 1$, since the initial and final values of the speed and altitude are then respectively equal. The definite integrals of the first and second terms on the right-hand side of (10) vanish under these conditions, so that the result of the integration is

$$V \lg\frac{M_0}{M_1} = g\varepsilon t. \qquad (11)$$

We shall now integrate (9), assuming $\frac{m}{M} = \mathrm{const} = \mu$. The definite integrals of the first two terms on the right-hand side vanish under the above conditions. The integral of the left-hand side represents the work done by the engine during the climb:

$$\mu.V s_1 = g\varepsilon S, \qquad (12)$$

73 where s_1 is the distance traveled during the ascent; however,

$$\mu\, dt = \frac{mdt}{M} = -\frac{dM}{M}. \qquad (13)$$

Integrating this equation over the duration t_1 of the climb, we obtain

$$\mu t_1 = \lg\frac{M_0}{M_1}. \qquad (14)$$

Inserting this into (11) and dividing (12) by (11), we obtain

$$\frac{s_1}{t_1} = \frac{S}{t} = v_m, \qquad (15)$$

i. e., the average speed during the climb is equal to the average horizontal flight speed.

Multiplying (11) by v_m and noting that $tv_m = S$, we obtain

$$S = \frac{Vv_m}{g\varepsilon} \lg \frac{M_0}{M_1}. \qquad (16)$$

This equation is similar to (7), except that v_m enters it instead of v.

Let us now determine the average speed v_m.

If the flight path during the climb is flat, we may take s_1 as equal to its horizontal projection S_1; the unknown average speed is then equal to the average speed during horizontal travel. We therefore take it as equal to the corresponding magnitude during the descent.*

Setting $m=0$ and $\alpha=1$, in (9), the relationship between the maximum speed v_1 and the corresponding altitude H_1 becomes:

$$g\varepsilon S_1 = \frac{v_1^2}{2} + gH_1. \qquad (17)$$

Analogous integration of (10) requires a knowledge of the integral of $g\frac{dh}{v}$, where dh depends on dv, as shown above. Carrying out these computations for small increments of v, we obtain the results given in Table 9.

TABLE 9

v_1	$g\varepsilon t_1$	$g\varepsilon S_1$	v_m	$\frac{v_m}{v_1}$
100	–	–	–	–
200	898	126, 500	141	0.705
300	1,301	243, 000	187	0.624
400	1,560	337, 000	216	0.540
500	1,747	424, 000	243	0.486
600	1,895	504, 000	266	0.444
700	2,021	587, 000	290	0.415
800	2,138	676, 000	317	0.394
900	2,250	772, 000	343	0.381
1,000	2,358	874, 000	371	0.371

74 The values of v_m, given in this Table, are inserted into (13). To use these formulae it is necessary to determine the flight path. The latter may in theory even be vertical, but in practice it will not be very steep. We have therefore assumed a flat path whose length is taken as equal to the horizontal distance traveled.

We shall now consider a numerical example, assuming that the horizontal distance traveled during takeoff is in a certain relationship to the maximum altitude attained, e. g., that their ratio is 4:1. Setting $\varepsilon=0.1$, $g\varepsilon=\infty 1$, $v=2{,}000$ m/sec, we obtain Table 10.

* This is true at a speed of 100 m/sec at sea level for a certain relationship between the speed and the altitude. For the sake of generality the computations are performed for speeds between 0 and v_1.

TABLE 10

v_1	H_1, km	S	$\frac{M_0}{M_1}$	$\frac{M_0-M_1}{M_0}$ — %	kg/t·km $\frac{P_0-P_1}{P_1 S}$
100	0	0	0	—	—
200	11.65	173	1.84	45.6	4.85
300	19.8	322	2.35	57.5	4.30
400	25.7	440	2.76	63.7	3.95
500	29.9	544	3.05	67.2	3.75
600	32.4	634	3.27	69.4	3.58
700	34.2	729	3.47	71.2	3.35
800	35.6	819	3.61	72.3	3.20
900	36.6	918	3.80	73.7	3.05
1,000	37.5	1,024	3.96	74.8	2.90

The following conclusions may be drawn from these results:

1. The distance traveled (Table 10) is almost proportional to the maximum speed. During the descent this distance is independent of the gas outlet velocity. The distance covered during the ascent, however, is a multiple of the maximum altitude, and depends on the maximum flight speed but not on the gas outlet velocity.

2. The ratio of the average to the maximum speed (Table 9) varies inversely with the latter and does not depend on the gas-outlet velocity.

3. The mass of fuel carried (in % of the initial mass) varies inversely with the gas outlet velocity. Table 10 gives the fuel mass to be carried; it increases with the maximum speed.

4. The fuel consumption per ton · km (Table 10) varies inversely with the maximum speed, but to a smaller extent than in the case of horizontal flight, due to the smaller ratio of the average to the maximum speed.

We herewith conclude the description of Crocco's work.

During flight in vacuum space $R=0$ and (3) becomes

$$mV = M\frac{dv}{dt} \quad \text{or} \quad mdt\, V = Mdv$$
$$\text{or } -VdM = Mdv,$$

whence

$$v = V \lg \frac{M_0}{M_1}. \tag{18}$$

75 Here M_0 is the initial mass of the rocket; M_1 is the final mass of the rocket when all the fuel is burnt.

If $V=v_1$, we have $\lg\frac{M_0}{M_1}=1$, or $M_0 = 2.71\ M_1$.

The propulsive efficiency is

$$\rho = \frac{M_1 \cdot v^2}{(M_0-M_1)V^2}. \tag{19}$$

Substituting for v from (18), we obtain

$$\rho = \frac{M_1}{(M_0 - M_1)} \lg^2 \left(\frac{M_0}{M_1}\right) \qquad (20)$$

For $v = V$, we have $\rho = 0.58$.

We obtain the maximum propulsive efficiency by differentiation (20) with respect to M_1 and setting the result equal to zero:

$$\lg \frac{M_0}{M_1} = 2\frac{M_0 - M_1}{M_0}.$$

Hence $\frac{M_0}{M_1} = 5$; $v = 1.61$ V; $= 0.645$. Thus, $V = 2000$ m/sec for $v_1 = 3200$ m/sec. When the propulsive efficiency is not maximum but, e.g., 0.46, we obtain $\frac{M_0}{M_1} = 20.62$ and $v = 3\,V$.

We have computed the values of $\frac{v}{V}$ and ρ for different ratios $\frac{M_0}{M_1}$ using (18) and (19). The result are given in Table 11 and presented graphically in Figure 65. It is seen that flight is most economical when $\frac{M_0}{M_1} = 5$, $v = 1.61\,V$, and $\rho = 0.645$.

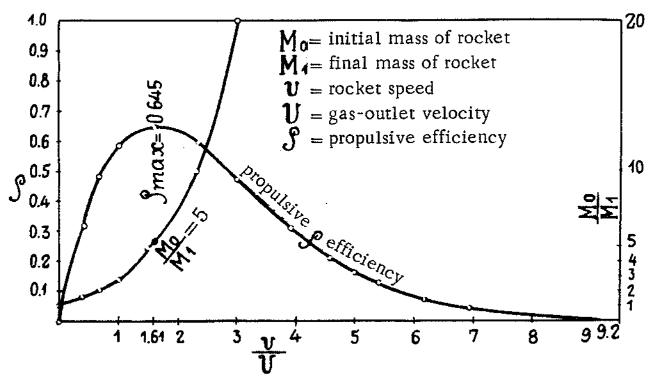


FIGURE 65

The circles in Figure 65 correspond to the following values:

TABLE 11

$\frac{M_0}{M_1}$	1	1.5	2.0	2.71	5	10	20	50	100	150	200	500	1,000	10,000
$\frac{v}{V} = \lg \frac{M_0}{M_1}$	0	0.406	0.69	1.0	1.61	2.3	3.0	3.91	4.6	5.01	5.3	6.21	6.9	9.2
ρ	0	0.32	0.48	0.58	0.645	0.59	0.47	0.31	0.21	0.17	0.14	0.07	0.047	0.004

e) Rocket installation for high-altitude plane

Consider a rocket using smokeless powder. We assume that the gas formed during the combustion, which has a temperature of approximately 3,000°C and a high pressure, expands in a Laval nozzle to the outside pressure. The gas thus attains a certain outlet velocity w_a. The thermal efficiency of the expansion (Figure 66) increases with the expansion ratio.

The effective power of the engine depends on the mass discharged [per unit time], at a velocity w_a.

It follows from this that the efficiency of a rocket increases with the ratio of the absolute rocket speed to the absolute gas velocity (Figure 67). The rocket efficiency becomes unity when the absolute gas velocity is zero.

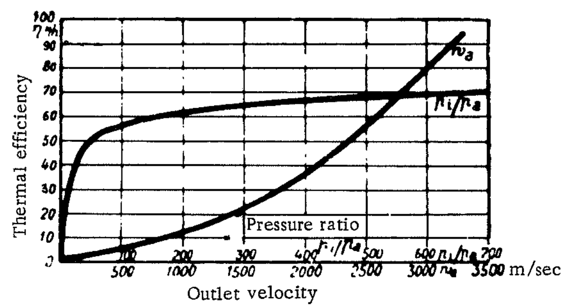


FIGURE 66

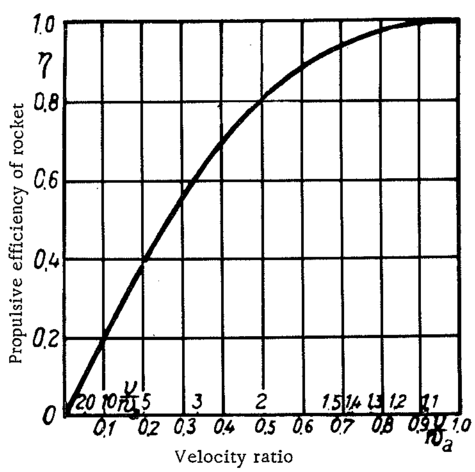


FIGURE 67

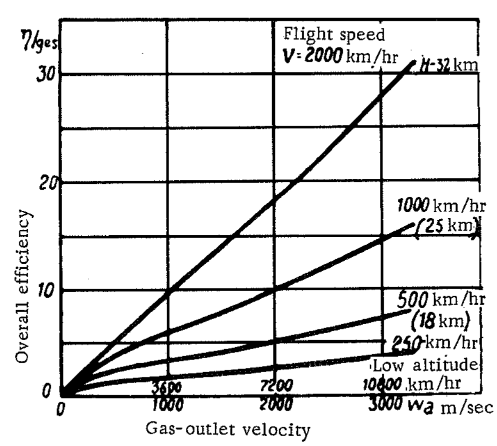


FIGURE 68

The overall efficiency is the product of the thermal and propulsive efficiencies, if heat and eddy losses inside the rocket are neglected. Figure 68 represents the overall efficiency as a function of the velocity w_a for different flight speeds v. It is seen that the overall efficiency increases with increasing gas outlet velocity. The corresponding pressure ratio increases rapidly (Figure 66). The altitudes indicated on the curves refer to flight at the maximum lift/drag ratio and a maximum speed of 150 km/hr at sea level.

The calorific value of rocket fuel is small. Gunpowder has a calorific value of 1,100 kcal/kg. The best conceivable propellant, namely oxygen + hydrogen, has a calorific value of only 3,200 kcal/kg. Gasoline has a calorific value of 10,000 kcal/kg. Taking the latter value, we find
77 that the overall efficiency at an altitude of 30 km is 3–10% at a thermal efficiency of 33.3–50%. On the other hand, an internal-combustion engine, including the propeller, has an overall efficiency of approximately 20%. A rocket engine thus gives worse results than a gasoline engine at high altitudes, without mentioning the difficulty of attaining these altitudes. At present one can envisage only recording rockets with automatic instruments for flights above 30 km altitude.

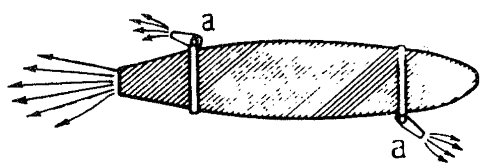


FIGURE 69

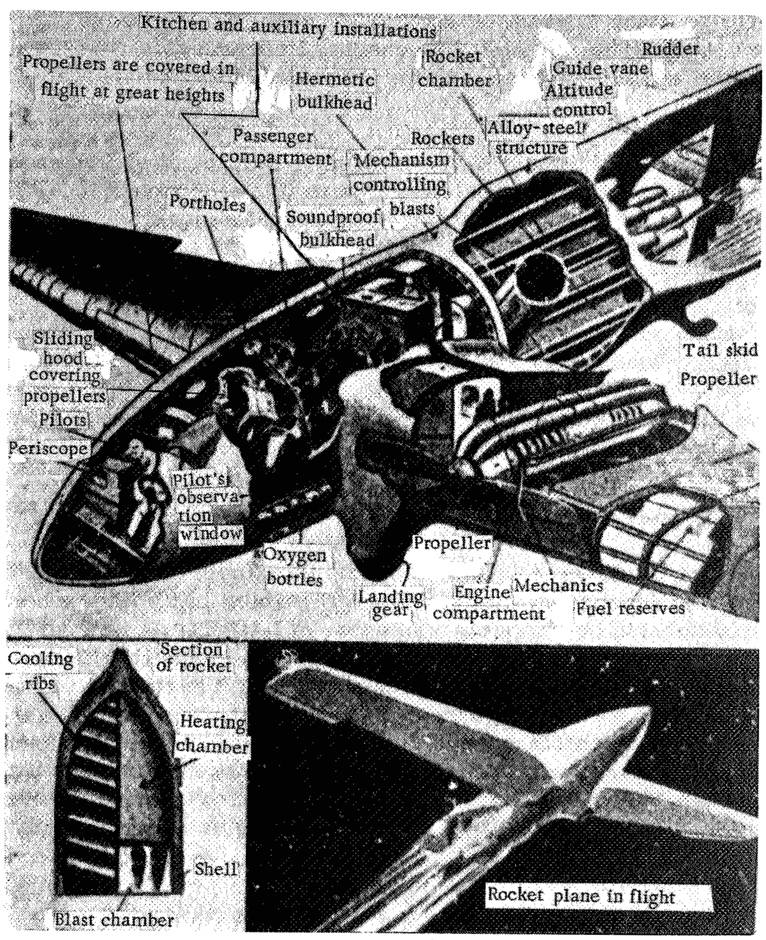


FIGURE 70. Future rocket plane

On rocket controls

The direction of motion of a rocket (Figure 69) traveling in space can be changed by means of small adjustable rockets (a), which may be moved along the main rocket and rotated like artillery guns. By firing these rockets we can turn the main rocket and thus change the direction of the
78 gas jet discharged from it.

The space surrounding the rocket may be observed during such turns by means of a special optical system similar to an inverted praxinoscope.

Figure 70 shows a future rocket plane with some details, according to the Czechoslovak periodical "Letec." The retractable engines with propellers are an original feature.

f) Rocket flight in the stratosphere

(Hans Lorenz)

A paper by Hans Lorenz, "Rocket Flight in the Stratosphere," was published in the Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt (Yearbook of the Scientific Society for Aviation) in 1928. Its translation is given below.

The properties of the air in the stratosphere, i. e., at altitudes above 20 km, differ from those of the lower layers (the troposphere). The oxygen content is lower, while the hydrogen content is higher. According to Humphrey the oxygen concentration is 15–20% at altitudes between 30 and 50 km, while the hydrogen concentration is 0.2–3%. The temperature is almost constant (-50°C) at altitudes between 10 and 35 km, but the velocity of sound increases from 300 to 360 m/sec. We shall henceforth assume this velocity to be $c = 334$ m/sec, as at sea level. The air density is 0.29, 0.059, and 0.011 kg/m^3 at an altitude of 10, 20 and 30 km, respectively.

Let an airplane with wing area F fly at a speed v. Its lift A, induced drag R, and parasite drag W are respectively

$$A=\zeta_1.\rho.\frac{v^2}{2}F;\quad R=\zeta_2.\rho.\frac{v^2}{2}F;\quad W=\zeta_3.\rho.\frac{v^2}{2}F. \qquad (1)$$

Here ζ_1 and ζ_2 (referred to the principal axes) depend on the angle of attack, while ζ_3 depends on the frontal area f; which in its turn depends on the atmospheric conditions. The value of ζ_3 is almost constant up to $v=300$ m/sec $< c$. at a given value of f and normal air densities (Figure 71). The maximum of ζ_3 occurs at $v = 490$ m/sec; thereafter it asymptotically approaches the value $\zeta'_3 = 3\zeta_3$. We may therefore assume that the drag W trebles at supersonic speeds, and that all three magnitudes A, R, and W are proportional to $\rho.v^2$.

Let the airplane have a speed of 75 m/sec (270 km/hr) at an altitude of not more than 5 km, where the air density is ρ_0. Its speed at an altitude of 20 km, where the air density is $0.06\,\rho_0$ is then given by the equation

$$v^2\rho = v_0^2\rho_0. \qquad (2)$$

so that at the same drag

$$v = \frac{v_0}{\sqrt{0.06}} = 4.08\,v_0 = 306\,\text{m/sec} = 1100\,\text{km/hr}.$$

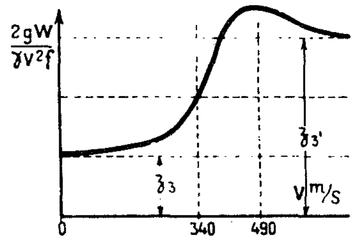


FIGURE 71

The propeller rpm and the engine power increase in the same proportion. The low air density necessitates a cylinder volume which is 4.08 times larger, in order to charge the engine with the required amount of oxygen;
79 the engine weight increases in almost the same proportion. This is excessive for an airplane, so that this method cannot be employed for flight in the stratosphere.

Consider now the rocket principle and the reaction caused by the discharge of a mass at a velocity w. Let the energy of the hot gas be approximately two-thirds of the total energy contained in the propellant.

Let m be the instantaneous mass of the airplane with the payload; the lift is therefore

$$A = mg, \tag{3}$$

while the thrust is

$$-\frac{w\,dm}{dt} = \frac{m\,dv}{dt} + R + W. \tag{4}$$

The drag/lift ratio (of (1)) is

$$\varepsilon = \frac{\zeta_2 + \zeta_3}{\zeta_1}, \tag{1a}$$

we thus obtain

$$-\frac{w\,dm}{dt} = m\left(\frac{dv}{dt} + \varepsilon \cdot g\right). \tag{4a}$$

We multiply this formula by the elementary distance $ds = v\,dt$ and divide by m:

$$\begin{aligned} -vw\frac{dm}{m} &= v\,dv + \varepsilon \cdot g\,ds; \\ -w\frac{dm}{m} &= dv + \varepsilon \cdot g\frac{ds}{v}. \end{aligned} \tag{4b}$$

Let the acceleration be constant and equal to q over a distance s_1. Then $v^2 = 2q s_1$ and integration of (4b) yields:

$$w \lg \frac{m_0}{m_1} = v + 2 \frac{\varepsilon g s_1}{v}.$$

The distance s_2 traveled at constant speed is

$$w \lg \frac{m_1}{m} = \frac{\varepsilon g s_2}{v}.$$

Adding these two equations, we obtain

$$w \lg \frac{m_0}{m} = v + \frac{\varepsilon g}{v}(2s_1 + s_2) = v + \frac{\varepsilon g s}{v}. \qquad (5)$$

Nitroglycerin, which is the most powerful propellant, has a calorific value $Q = 1580$ kcal/kg; expressed in units of work this is

$$L = 6.7 \,.\, 10^{-5} \text{ m/sec.}$$

At an efficiency $\eta = 66.7\%$ this yields:

$$\frac{w^2}{2g} = \eta L; \quad w = \sqrt{2g\eta L} = 2950 \text{ m/sec}$$

or approximately $w = 3000$ m/sec.

We assume the airplane to consist only of wings without a fuselage. The wings are thin and the rocket nozzles are located at their trailing edges. The values of ζ_2 and ζ_3 will be very low for this shape, so that the drag/lift ratio will also be very small. Hence for $v < c$:

$$g\frac{\zeta_2}{\zeta_1} = 0.3; \quad g\frac{\zeta_3}{\zeta_1} = 0.42; \quad g\varepsilon = g\frac{\zeta_2 + \zeta_3}{\zeta_1} = 0.72.$$

We obtain from (5) for $v = 300$ m/sec:

$$\lg \frac{m_0}{m} = 0.1 + 8 . 10^{-7} . s,$$

80 where s is given in meters.

Table 12 gives the time taken for traversing various distances.

TABLE 12

s, km	1,000	2,000	3,000	5,000
$\lg \frac{m_0}{m}$	0.9	1.7	2.5	4.1
$\frac{m_0}{m}$	2.46	5.47	12.18	60.3
$t = \frac{s}{v}$	55'33''	1°51'6''	2°46'39''	4°37'45''

The mass ratios obtained are very large in this case. The term containing s in (5) has v as its denominator, so that it is advisable to increase the flight speed as much as possible. This requires that at a given lift the air density be less, so that the flight altitude should be great. The velocity of sound has therefore to be exceeded (v=500 to 600 m/sec), so that the value of ζ_3 is almost trebled. We then obtain:

$$g\frac{\zeta_2}{\zeta_1}=0{,}3;\quad g\frac{\zeta_3}{\zeta_1}=1{,}26;\quad g\varepsilon=1{,}56.$$

The air density at altitudes y between 20 and 30 km is given by the equation:

$$y^4\rho=y_0^4\rho_0. \tag{6}$$

Using (2), we obtain

$$y^2v_0=y_0^2v\,. \tag{6a}$$

Setting y_0=20 km, ρ_0= 0.06 kg/m^3, v_0= 300 m/sec, we obtain for y=40 km (sic): ρ=0.0037 kg/m^3, v=1,200 m/sec.

At a speed:

$$v_l=\sqrt{\varepsilon gs}, \tag{7}$$

which increases with the distance s, we obtain from (5) the minimum:

$$\lg\frac{m_0}{m}=2\frac{v_l}{w}=2\frac{\sqrt{\varepsilon gs}}{w}. \tag{7a}$$

For w=3,000 m/sec we obtain the values given in Table 13.

TABLE 13

s, km	1,000	2,000	3,000	5,000
v, m/sec	1,250	1,770	2,160	2,790
v, km/hr	4,500	6,370	7,770	9,980
$\lg\frac{m_0}{m}$	0.833	1.18	1.44	1.82
$\frac{m_0}{m}$	2.3	3.25	4.22	6.17
$t=\frac{s}{v}$	13'20''	18'50''	23'10''	29'50''

However, if the speed is v=1,200 m/sec = 4,320 km/hr at an altitude of y=40 km, the results are far worse, as seen from Table 14.

81 TABLE 14

s, km	1,000	2,000	3,000	5,000
$\lg\frac{m_0}{m}$	0.833	1.268	1.70	2.57
$\frac{m_0}{m}$	2.30	3.55	5.47	13.1
t	13'53''	27'50''	41'40''	h 19 30''

The twofold take-off distance $2s_1$, which depends on the acceleration q, enters in the length of the path traveled. This acceleration must not exceed a certain limit in view of the passengers carried.

We obtain $s = v^2 : 2q = 0.1 v^2$ at $q = 0.5g = 5$ m/sec^2. In the first case we have $s_1 = 9$ km at $v = 300$ m/sec, and $s_1 = 144$ km at $v = 1{,}200$ m/sec. In the second case (Table 13) we obtain $s_1 = 780$ km at $s = 5{,}000$ km, and $v = 2{,}790$ m/sec, so that the takeoff distance represents a significant part of the total distance traveled.

Lastly, we shall determine the flight conditions for an airplane equipped with an internal-combustion engine flying at high speed in the troposphere.

Let the calorific value of the fuel be $h = 427 \cdot 10^4$ mkg/kg and the overall efficiency of the propulsion plant (engine and propeller) $\eta = 0.25$. We then obtain the equation of work:

$$-dm.h\,g.\eta = m\left(\frac{dv}{dt} + \varepsilon.g\right).ds. \qquad (8)$$

Integrating and inserting the above numerical values, we obtain

$$\lg\frac{m_0}{m} = 10^{-7}\left(\frac{v^2}{2} + \varepsilon g s\right). \qquad (8a)$$

We obtain the results given in Table 15 for $v < 300$ m/sec and $\varepsilon g = 0.72$.

TABLE 15

s	1,000	2,000	3,000	5,000	7,000
$\lg\frac{m_0}{m}$	0.072	0.144	0.216	0.360	0.504
$\frac{m_0}{m}$	1.07	1.16	1.24	1.42	1.66

Comparison of these data with those in the preceding tables shows that rocket flight in the stratosphere is not worthwhile with the propellants available at present.

Despite the pessimistic views on the use of rockets as engines expressed by some scientists, many engineers have begun experiments with them, such as Messrs. Opel in Germany (near Frankfurt a. M.), who in 1929 continued their tests of rocket planes. Such a plane, equipped with eight rockets, was propelled over a distance of 20 km by the reaction caused by the gas discharge from the rockets.

Similar experiments were carried out by Messrs. Junkers. The purpose of the rockets was to assist in the takeoff of the Junkers W. 33 airplane on the River Elbe and were arranged beneath the wings. The experiments were very successful.

5. LARGE MODERN GUNS

a) Gun types

The development of artillery, which was particularly pronounced during the First World War, continues rapidly. New types of guns are appearing, due to the development of the novel kinds of warfare. Thus, the participation of airplanes in the war gave rise to the appearance of antiaircraft guns able to shoot up to 12 km high, i. e., farther than field guns were able to fire at land targets before World War I. The range of land artillery has also been increased; an example is the gun with which the Germans shelled Paris from a distance of 120 km. There is reason to believe that guns having even larger ranges exist outside the Soviet Union.

There are already means of creating a flying artillery, i. e., guns mounted on airships and airplanes.

The weight of the shells fired from guns has also increased.

Thus, the French 52 cm howitzer fires a shell weighing 1,654 kg. This considerably exceeds the known weight (930 kg) of German projectiles fired in World War I from 42 cm guns.

Novel projects of turbo-guns and guns from which the shells are ejected electrically have been designed in order to improve the performance and reduce the weight of the guns. In another design the projectile is accelerated further after leaving the gun, due to the reaction caused by the discharge of gas from the rear of the shell. It may also be possible to use spent parts of the casing for this purpose; the remaining part of the projectile with the warhead will thus acquire a higher velocity. The trend toward higher muzzle velocities led to the use of long barrels and the development of "progressive powders."

Below we shall describe several large guns which either exist already or are to be built in the future, as well as the flight characteristics of their projectiles. It will be seen that both superaviation and superartillery must solve the same problem: how to transport a given load over a maximum distance in minimum time.

Table 16 contains data on some antiaircraft guns. These data are represented graphically in Figure 72, where two curves of the same type refer to each gun.

TABLE 16. Antiaircraft guns

83

Model	Bore, mm	Projectile weight, kg	Muzzle velocity, m/sec	Range, km	
				vertical	horizontal
Russian: 3" model 1914	76.2	6.55 shrapnel	588	5.000	9.000
Russian: 3" model 1915	76.2	6.0 shell	610	5.7	9.4
Russian: 40 mm Vickers	40	0.92	610	2.200	7.160
Russian: 3" model 1900–1902	76.2	6.55 shrapnel 6.0 shell	588	5.500	9.000
75/50* naval gun	75	5.73 shell 4.91 shrapnel	741.7 823.0	5.000	10.000
German 37 mm automatic	37	0.67 (cartridge)	540	2.0	6.0
7.5 cm Rheinmetallwerke, mounted on car	75	6.5	500	–	–
French 75 mm automatic	75	7.25	525	6.0	10.0
British 3"/45	75	5.6	785	6.3	10.0
French 75 mm	75	–	700	7.0	10.6
Italian 75 mm C.K.	75	–	560	7.0	11.0
U.S. 3"/50 model 1923	76.2	6.8	792	7.5	16.2
Former German 88 mm (Belgium)	88	–	785	7.0	12.0
8.8 cm Krupp naval	88	9.5	800	–	–
10.4 cm Krupp	104	15.5	800	–	–
French 105 mm model 1926	105	–	700	9.8	15.0
French 105 mm model 1913	105	–	533	7.0	12.5
German 105 mm model 1918	105	–	720	8.0	14.0
U.S. 4.7"/42 model 1920	119	20.4	790	12.0	18.26

* [Translators note: the first figure denotes the bore, the second the barrel length in calibers.]

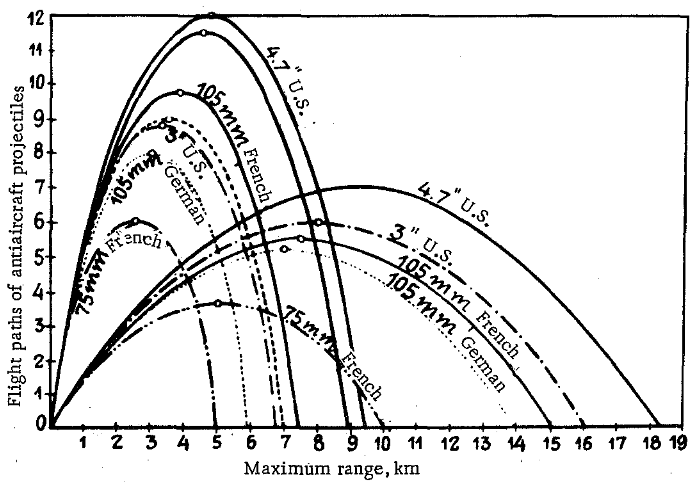


FIGURE 72. Flight paths of antiaircraft projectiles

84 One curve each refers to high-altitude firing, and the other to long-range firing. Thus, the French 105 mm gun firing to a height of 9.8 km, has a horizontal range of about 7.5 km; *during firing at the maximum range of* 15 km the shell rises only to a height of 5.5 km.

Table 17 contains data on some modern guns.

Table 18 contains data on various "large" guns in their respective periods. Figures 72—78 show some of these guns. Table 18 also gives information on some planned guns whose horizontal and vertical ranges are even greater. Their descriptions and pictures will be given later. Figure 79 shows the flight paths of projectiles fired from these guns, compared with those for antiaircraft guns, while Figure 80 shows the proposed flight path of a projectile fired from a novel gun which still appears fantastic.

TABLE 17. Particulars of guns

Particulars	Type					
	rifle	field gun	naval gun	long-range gun	coast gun	British long-range gun
Bore	0.79	7.5	21.0	21.0	40.64	50.8
Cross-section, cm^2	0.49	44.2	346.4	346.4	1,297.10	2,026.8
Inside length, calibers	101.5	26.7	50.0	150.0	50.0	100.00
Inside length, m	0.80	2.0	10.5	33.6	20.3	50.8
Barrel length, calibers	116.52	28.7	55.0	171.0	52.5	105.8
Inside length, m	0.90	2.2	11.0	36.0	21.4	53.7
Barrel weight, kg	1.00	310.0	15,450.0	142,000.0	113,100.00	550,000.0
Projectile weight, kg	0.01	6.5	125.0	100.0	920.0	2,000.0
Muzzle velocity, m/sec	900.00	600.0	940.0	1,600.0	940.0	1,340.0
Maximum range, kg	4.00	9.0	26.0	130.0	40.0	160.0
Kinetic energy of projectile leaving muzzle, m·t	0.413	119.3	5,629.0	15,360.0	41,440.0	183,000.0
Kinetic energy of projectile per unit weight of barrel, m·kg/kg	413	383.9	364.0	108.0	366.0	333.0
Mean thrust, kg	516	59,700.0	534,850.0	457,140.0	2,039,400.0	3,602,400.0
Mean barrel pressure, atm.	1,053	1,350.0	1,544	1,320.0	1,572.0	777.0
Average time required for traveling inside barrel, sec	1/563	1/150	1/46	1/23	1/23	1/13
Mean power, hp	3,100	238,600.0	3,359,500.0	4,735,200.0	12,780,000.0	32,780,000.0
Mean power per unit barrel weight, hp/kg	3,100	769.7	217.4	33.35	115.63	5,824

85 We shall give, as a historical reference, an interesting description of firing a gun into interplanetary space, performed in the 17th century.

Camille Flammarion in his book "Astronomie Populaire" (Popular Astronomy) describes an experiment, performed in the 17th century, of firing a gun upward. This experiment was carried out by Marsenne and Petit, but they did not recover the cannon ball after firing. This experiment is illustrated in Figure 81; two persons, the soldier Petit and the monk Marsenne, are seen standing beside a gun pointing to heaven. They are looking upward as if following the flight of a cannon ball just fired. The

picture has the inscription "Retombera-t-il?" (Will it fall back ?). They repeated this dangerous experiment several times, and since they did not have sufficient skill to make the cannon ball hit them exactly on the head, they concluded *that it remained in the air, undoubtedly for a long time.* Varighon in his book (Suggestions on the Causes of Gravity), an extract from which is given by Flammarion, does not dispute the fact itself,
87 but is surprised by it: "A cannon ball suspended over our heads.... this is indeed surprising! " The experimenters, if they may be called such, informed Descartes of their experiments and the results obtained. He did not doubt the accuracy of the facts, and saw in them only a confirmation of his discoveries concerning gravity. They repeated the experiment at Strasbourg and found the cannon ball at a distance of some hundreds of meters from the gun.*

(85) TABLE 18. Particulars of some large guns

Designation of gun	Barrel length		Weight of barrel with breech mechanism	Weight, t		Muzzle velocity, m/sec	Maximum range, km	Total weight of gun and mounting	Figure No.
	calibers	m		of projectile	of charge				
Dahlgren	—	—	—	41	—	—	4.56	—	—
Rodman's columbiad	—	7.62	—	500	65	730	9.65	—	73
Mahomet's gun	—	—	—	778	—	—	—	—	—
Gun of Knights of Malta	—	—	—	1,024	—	—	—	—	74
Mortar of Louis XI	—	—	—	204	—	—	—	—	—
Armstrong gun	—	—	—	205	31	—	—	—	—
12'' Vickers howitzer	—	—	—	—	—	363	10.2	—	—
16'' U.S.howitzer	25	—	88	1,001	152	678	27.4	295	75
British 12" gun	52	13.25	—	447	143	793	32.4	—	—
U.S.14'' gun	50	—	102	708	205	853	36.6	318	76
14'' gun	—	—	—	639	189	—	70	—	—
British 15'' gun	—	—	—	—	—	—	64.0	—	—
16'' naval gun	—	21.34	172	1,090	410	—	50	—	—
British 20.4 cm gun	122	—	—	109	159	1,500	110.0	—	—
German 21 cm "Big Bertha"	—	36.0	—	120	240	1,600	110.0	147	77
French 21 cm gun	110	—	—	108	160	1,450	120.0	320	—
German 42 cm mortar gun	—	—	—	930	—	—	—	—	—
French 52 cm howitzer	—	—	—	1,654	—	—	—	—	—
U.S.16'' coast gun	—	—	—	—	—	—	—	—	78
D.M.turbo-gun		—	—	—	—	850	240	—	89
F.B.electric gun		30	—	100	—	1,600	—	150	—
F.B.electric gun		120	—	—	—	3,000	600	—	95
Alco electric gun		125	—	2,000	—	5,000	3,000	1,000	101

* Concerning the point where it hit the ground cf. the computations at the end of section 6.

(86)

FIGURE 73. Rodman's columbiad

FIGURE 74. Gun of Knights of Malta

FIGURE 75. U.S.16" coast howitzer

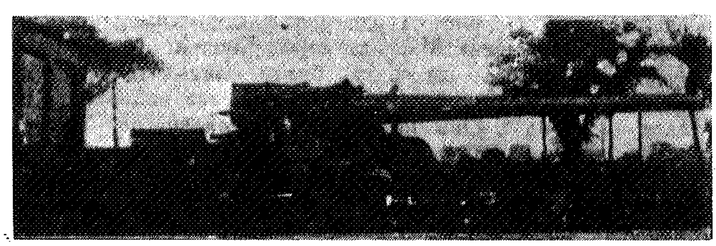


FIGURE 76. U.S. 14" gun

The "Big Bertha," a superlong-range gun

The Germans began to shell Paris on 23 March 1918, from the unprecedented ranges of 80, 100, and 120 km (from the Saint-Gobain forest, near Crépy-en-Laonnois). This was so unusual that at first it was assumed that the city was being bombed by a novel type of airplane from an altitude at which the plane remained invisible. However, the regularity of the explosions (every 15 min), study of the splinters after the bursts, sound rangings, and certain technical considerations led to the correct conclusion that this was a novel type of artillery attack at a so-called "superlong range." This shelling differs considerably from that of long-range guns employed until then. Such guns had maximum ranges of 35–40 km, while the latest British pre-World War I 15" gun had an extreme range of 60 km.

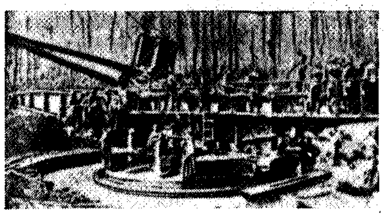


FIGURE 77. German "Big Bertha"

The reason is that ordinarily the shell encounters a large air resistance so that its velocity decreases rapidly; this also causes the range to be reduced. However, when the shell travels in vacuum along a parabolic path the range is maximum when the gun is fired at an elevation of 46°, as against 42–43° when the shell travels in air; in the latter case the range is $^1/_3$–$^1/_5$ of that in vacuum. The Germans therefore decided to exploit all the advantages offered by high-altitude flight paths. The elevation employed was 50° and the muzzle velocity, 1,600 m/sec. The height attained by the shells was 35–40 km, the duration of their flight 3 min.

For this purpose the Germans modified three 17 m-long 15" naval guns by inserting an internal barrel having a bore close to 8" (210 mm) and a
88 length of 36 m (Figures 77 and 82). When the rifling had become worn the barrel was bored out to a diameter of 240 mm. The overall length of the barrel was 36 m, of which 30 m were rifled (64 grooves at twist-angle of 4°), while the last 6 m were smooth; the relative length of the barrel thus attained the unprecedented value of 140 calibers. Deflection of the barrel during firing was prevented by holding it in position with the aid of cables fixed to a special structure. The gun mounting rested on a concrete foundation and a steel frame (Figure 82). The entire system moved on a circular rail track. The gun was mounted on a cast-steel carriage equipped with two hydraulic counterrecoil mechanisms, permitting a recoil of 1.3 m.

(87)

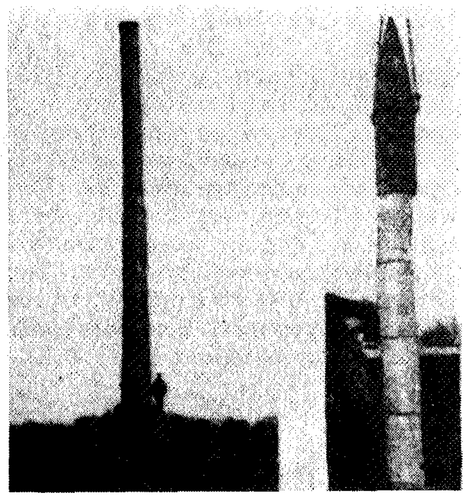


FIGURE 78. U.S.coast gun, its shell and charge

The shell weighed 120 kg, and the charge 240 kg. The shell was over 1 m
89 long, but more than half the length was taken up by a tapered, so-called "ballast" tip screwed to the forward part of the shell. This ensured optimum motion in the air by giving a streamlined form to the projectile. The shell itself consisted of a steel bomb whose interior was divided into two parts by a screwed-in diaphragm with seven holes. The copper guide rings of ordinary projectiles were in this case replaced by two steel rims with precut grooves on the shell, since copper rings would have been sheared off during firing. The shell also carried copper rings behind the grooved rims in order to reduce the leakage of gas through the grooves. Two detonators were screwed into the shell for the sake of greater reliability; one was located in the shell bottom and the other in the diaphragm. The charge was placed in a brass case which was sealed hermetically by brazing. According to one source the charge consisted of a mixture of hydrogen and oxygen, liquefied under pressure, while according to another source a special nitroglycerin powder was used.

(88)

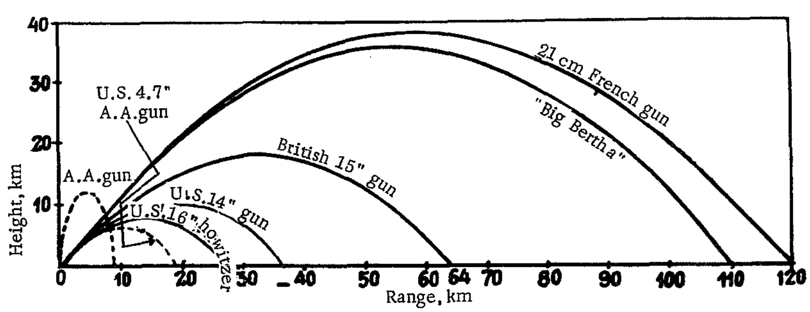


FIGURE 79. Comparison of paths of projectiles fired from antiaircraft and long-range guns

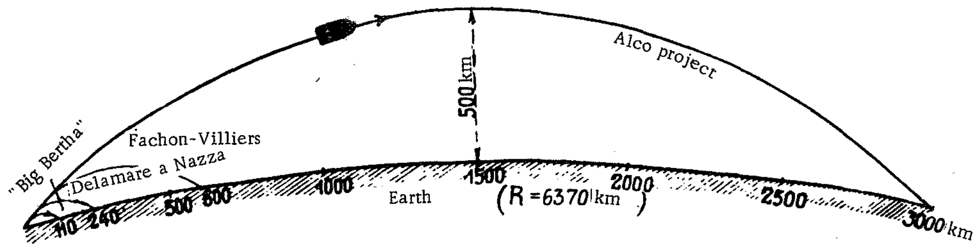


FIGURE 80. Flight paths of projectiles fired by fantastic guns

In firing from such guns it is necessary to allow for the rotation of the earth. Aiming must be very accurate, since an error of 1° gives a deviation of approximately 2 km. Initially the accuracy of hitting the target was 86%, but this dropped to 25% on the fourth day, due to deformation of the gun barrel. A hole of 5 m diameter and 3 m depth was caused by a shell hitting a road. A total of 303 shells were fired from the three guns during half a year, out of which 183 fell in Paris and 120 in the suburbs. Some structures were damaged as a result. The average losses were one killed and two wounded per shell. The overall effect was thus only psychological.

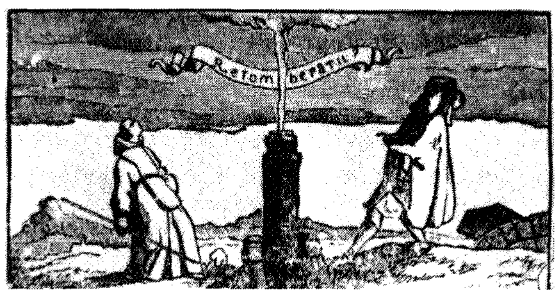


FIGURE 81. Marsenne and Petit's experiment

Studies of such large-bore guns were begun by Messrs. Krupp in Essen already in 1887. It was initially proposed to design guns having a range of 268 km. At the end of World War I the Germans built guns having a range of 200 km.

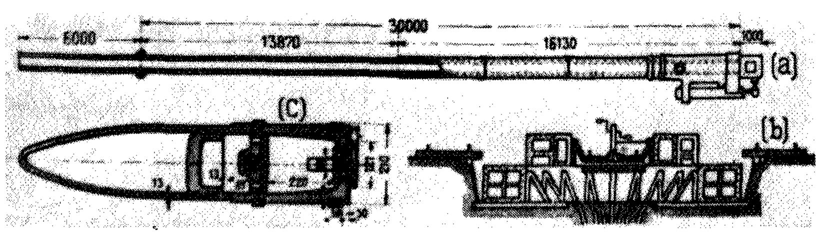


FIGURE 82. Details of "Big Bertha"

b) Superlong-range shooting

Superlong-range shooting, i. e., at ranges exceeding 100 km, is a recurrent problem of artillery, which arose during recent wars and has not yet been
90 finally solved.

Various authors have proposed methods of determining the flight path of projectiles fired at such ranges.*

Below we give a schematic description of the method, proposed by Oppokov, of computing the flight path of a projectile. We shall give the results obtained with this method for the flight path of a shell fired at Paris from a distance of 110 km, as begun by the Germans on 23 March 1918.

α) Basic assumptions

In computing the flight path of a projectile Oppokov makes the following assumptions:

1. The surface of the earth is flat.
2. The earth itself does not move.
3. The atmosphere remains at rest while the projectile travels through it.
4. The force of gravity acting on the projectile is constant in magnitude and is directed vertically downward.
5. The resistance of the air acts along the tangent to the flight path in a direction opposite to that of the projectile.
6. The resistance of the air is:

$$\rho = \lambda . A . \pi . R^3 . \frac{\Pi}{\Pi_0} . v^n , \qquad (1)$$

where ρ is the resistance of the air, kg; R is the radius of the cylindrical part of the projectile, m; Π is the density of the air at the altitude considered; Π_0 is the density of the air at sea level; λ is the projectile form factor which is constant and independent of the flight velocity; v is the velocity of the projectile; A and n are coefficients which were determined at the Krupp plant for velocities below 1,100 m/sec, and are given in Zabudskii's Tables (Vneshnayaya Ballistika (External Ballistics), p. 55). $A = 0.03$ and $n = 2$ for $v > 1{,}100$ m/sec.

7. The density of the air at altitudes up to 10 km is determined from Bjerknes' formula (V. Bjerknes, Dynamische Meteorologie (Dynamic Meteorology), Statik (Statics), p. 51):

$$\frac{\Pi}{\Pi_0} = (1 - 0.0000125 y)\, e^{-0.0001 y} \qquad (2)$$

where y is the altitude of the point considered, m, above the point from where the projectile is fired, which is assumed to be at sea level.

* Trofimov. Vychislenie traektorii snaryada, broshennogo s bol'shoi nachal'noi skorost'yu i pod bol'shim uglom vozvysheniya (Computation of the Flight Path of a Projectile Fired at a Large Muzzle Velocity and a High Elevation). Mechnikov. O razlichnykh sposobakh dlya vychisleniya traektorii snaryada pri sverkhdal'noi strel'be (On Various Methods of Computing the Flight Path of a Projectile in Superlong-range Shooting). Sparr. Sur le calcul des grandes trajectoires des projectiles (On the Computation of Large Flight Paths of Projectiles), 1923. Oppokov, F.V. Vychislenie traektorii snaryada pri sverkhdal'noi strel'be (Computation of the Flight Path of a Projectile in Superlong-range Shooting), Leningrad, 1924.

For altitudes exceeding 10 km,

$$\frac{\Pi}{\Pi_0} = 0.3\, e^{-\frac{y-1000}{6340}} \tag{3}$$

91 8. The entire flight path is divided into separate arcs. The abscissa, ordinate, velocity and inclination of the projectile to the horizontal are determined for the end of each arc.

9. The whole space traversed by the projectile is divided into two zones, one below and the other above an altitude of 30 km (referred to sea level). Flight in the upper zone is assumed to differ little from flight in vacuum.

β) Fundamental equations of projectile motion

It is assumed that the center of gravity of the projectile moves like a material point at which the entire mass of the shell is concentrated, and on which forces equal and opposite to the resultant external force act. We then obtain the following differential equations for the motion of the center of gravity of the projectile (Figure 83a):

$$\frac{P}{g} \cdot \frac{d^2x}{dt^2} = -\rho \cdot \cos\theta \text{ and } \frac{P}{g} \cdot \frac{d^2y}{dt^2} = -\rho \cdot \sin\theta - P, \tag{4}$$

where P is the weight of the projectile; g is the gravitational acceleration; x and y are the coordinates of the point considered; ρ is the resistance of the air; θ is the angle between the tangent to the flight path at the point considered and the horizontal.

Substituting for ρ from (1) and for $\frac{\Pi}{\Pi_0}$ from (2) and (3), Oppokov integrated (4) and determined the shape of the flight path, the coordinates of its different points, as well as the projectile velocity, its horizontal projection and the inclination of the flight path at various points. For these computations he used tables of the various coefficients specially prepared by him. We shall not solve the above equations but only give the results obtained by Oppokov for the flight path of a shell fired by the Germans at Paris from a distance of 110 km.

γ) Shooting at Paris from a distance of 110 km

Basing ourselves on Miller's article "The German Long-range Gun" (Engineering, 2 and 9 July, 1920), we have

		According to Tekhn. i Sh.Kr. Armii No.100, p.31
Weight of shell	120 kg	100 kg
Diameter = $2R$	0.21 m	22 cm
Muzzle velocity	v = 1,600 m/sec	1,600 m/sec
Elevation	50°	—
Vertical range	36.09 km	50 km

The form factor is taken as $\lambda = 0.5$, since the radius of the ogival part is quite large (approximately 7 calibers). The results of the computations and the shape of the flight path are shown in Figure 83a. It is seen that the summit of the flight path lies at a height of 36 km.

Assuming the earth to be flat, we have at the point of impact:

Range	109.6918 km	120 km
Height	0..........	—
Velocity	706.6 m/sec	—
Horizontal projection of velocity	386.5 m/sec	—
Angle of fall	56°50'........	—

92 Introducing a correction for the curvature of the earth (Charbonnier, Ballistique Exterieure (External Ballistics) problèmes secondaires (secondary problems), p. 125), we obtain the values indicated in the diagram.*

c) The stability of the projectile

The horizontal and vertical ranges of a projectile depend on the accuracy with which its axis is maintained on the flight path; the projectile thus has to be stable. This problem increases in importance with the density of the air layers traversed by the projectile.

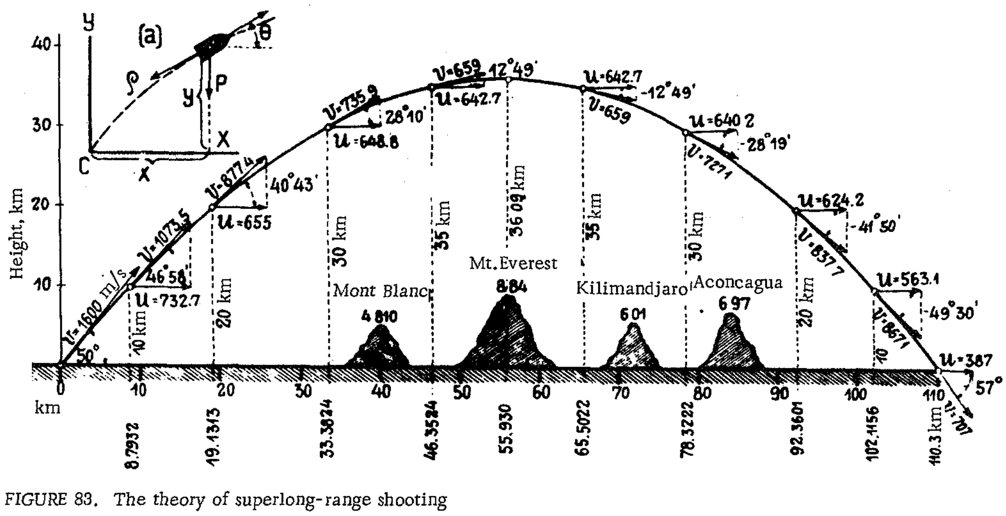


FIGURE 83. The theory of superlong-range shooting

* For details on computating the length of a superlong-range gun see also Guido v. Pirquet's paper "Die ungangbaren Wege zur Realisiering der Weltraumschiffahrt" (The Impossible Ways of Realizing Space Travel) in Die Möglichkeiten der Weltraumfahrt" (Possibilities of Space Travel), Leipzig, 1928, p. 300.

A projectile which does not rotate about its longitudinal axis is unstable in flight. This causes a phenomenon similar to that observed on revolving bodies in aerodynamical laboratories, described by D. Ryabushinskii in his paper "Sur l'autorotation des projectiles" (On the Autorotation of Projectiles), published in the Bulletin of the Kuchino Aerodynamic Institute, Vol. IV, Paris, 1920.

Rotation of the projectile about its transverse axis after firing can be prevented if the forces acting on it do not give rise to a couple or torque, i. e., if the vector of the air resistance passes through the center of gravity of the projectile. It is, however, very difficult to achieve this, particularly when the flight path of the projectile is inclined.

Consider in general the flight of spherical and oblong projectiles.

a) Spherical projectiles. Gases escape when a cannon ball leaves the muzzle, due to the clearance between the projectile and the barrel. The leakage of gases is not uniform on all sides, so that the cannon ball is rotated. This gives rise to the Magnus effect, so that the cannon ball is deflected to one side, depending on the sense of rotation. Figure 84a illustrates the case when the cannon ball rotates about an axis perpendicular to the plane of the paper, the elevation of the gun being less than 45°. The upper curve shows the flight path when the cannon ball rotates counter-clockwise; the lower curve corresponds to clockwise rotation, while the center curve represents the flight path when the cannon ball does not rotate.

93

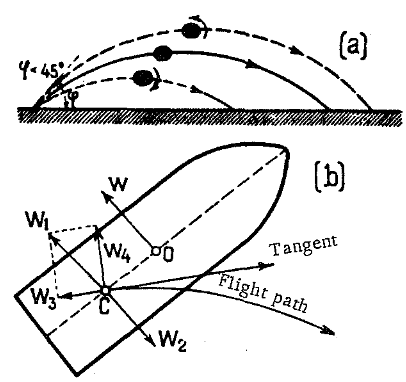


FIGURE 84

b) Oblong projectile. Attempts to increase the mass of the projectile without increasing the bore of the gun necessitated lengthening the shell. Figure 84b shows a projectile traveling along some flight path; the longitudinal axis of the projectile does not coincide with the tangent to the flight path [at the point considered]. Let the resultant $\mathbf{W}$ of the air resistance act at point 0 . We apply two forces, $\mathbf{W}_1$ and $\mathbf{W}_2$, at the center of gravity of the projectile; these forces are equal and opposite to each other, and parallel and equal to $\mathbf{W}$. We resolve the force $\mathbf{W}_1$ into two components, namely $\mathbf{W}_3$ along the tangent to the flight path, and $\mathbf{W}_4$ normal to it. These forces cause the projectile 1) to decelerate (force $\mathbf{W}_3$),

2) to turn back (couple W_1, W_2), and 3) to move upward or downward, depending on its inclination (force W_4).

The influence of the resistance W on the flight stability is reduced by imparting to the projectile a rotation with the aid of the rifling; this, however, gives rise to other effects which are no less objectionable. They cause the projectile to deviate from its path to different sides during ascent and descent, so that the flight path acquires a double curvature. The deviation may in certain cases amount to 5% of the range. It depends mainly on the following three factors:

1) The gyroscopic effect. Due to its rotation the projectile tends to keep its axis parallel to the initial direction. However, at various points of the flight path, the resistance of the air differs in direction and magnitude, while the position of its point of application changes in relation to the center of gravity of the projectile. This causes the projectile to deviate from its initial direction, while its axis describes a conical surface.

Let an external force tend to change the position of the axis of a rotating projectile; the latter will then resist this action. This phenomen is called the gyrostatic effect of rotation.

Assume that the perturbation force overcomes the gyrostatic effect and causes the axis of rotation of the projectile to change its position; the projectile will then tend to turn in a direction perpendicular to that of the external force. This phenomenon is called the gyroscopic effect. It causes swaying of the projectile or the so-called nutational motion of the projectile axis about the tangent to the flight path.

2) Furthermore, the axis of the projectile does not coincide with the tangent to the flight path, so that the air pressure at the front will be larger than that at the rear. Friction at the front will exceed that at the back, thus causing further deviation during rotation.

3) Lastly, the Magnus effect is superimposed on the above two perturbations.

Figure 85a and b shows curves characterizing the unstable motion of a projectile in the air (according to "The Coast Artillery Journal," May 1924)
94 at the beginning of the flight. These curves were obtained by processing the results of firing a 3" projectile of the mortar-shell type at a row of cardboard screens, and indicate that the axis of the projectile deviated by almost 10° from the tangent to the flight path. Figure 85a shows the path taken by the end of the projectile axis, in relation to the center of gravity of the shell, during the first 250 ft from the muzzle. The center of the circles corresponds to the position of the tangent to the flight path at different instants; the axis of the projectile coincided with this tangent at the beginning of the flight. Figure 85b shows similar curves for distances between 900 and 1,200 ft from the muzzle. The nutational oscillations have been largely suppressed by the resistance of the air. However, an angle of about 4° in the mean obviously remains between the axis of the projectile and the tangent to its flight path. The flight stability depends mainly on the ratio of the equatorial and axial moments of inertia, which should not exceed 10 for satisfactory results to be obtained. This ratio increases rapidly with the length of the projectile, so that this dimension has to be limited.

Another factor influencing the flight stability of the projectile is its gyroscopic inertia, which depends on the rotational speed. Lengthening the projectile necessitates a greater twist of the grooves. It can be predicted that twist angles of 10—12° will become necessary. Experiments were carried out at the Harfleur firing range in France with a 106-mm gun having a large twist angle; the projectiles were ogival at the front and rear. Flight was satisfactory at small elevations, but a curious phenomenon was observed: motion of the projectiles was perfectly silent. However, projectiles with excessive gyroscopic stability maintained their axis of rotation constantly in the same direction at large elevations, which is a drawback. To avoid this it was found desirable to reduce the rotational speed of the projectile gradually during the flight. This was achieved by means of fins fitted to the forward ogival part of the projectile, which slowed down its rotation. However, they also reduced its flight velocity.

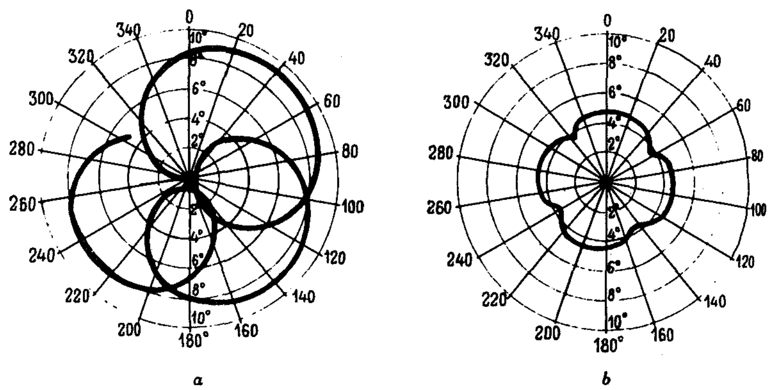


FIGURE 85. Experiments to determine flight stability of projectile

The French general Charbonnier proposed that grooves be cut on the surface of the projectile itself and not in the guide rings, so as to increase the range while maintaining the flight stability of the projectile. Experiments showed that this is advisable.

95 **d) Energy requirements and efficiency of guns**

Table 19 gives comparative data on the energy required at various ranges (according to M. Illyukevich, Problema ognya i gazov v vozdushnom flote (The Problem of Fire and Gases in the Air Fleet)). The letters denote the following magnitudes: V — muzzle velocity, m/sec; D — range, m; P — potential energy of charge, kg · m (equal to weight of charge × calorific value of propellant × mechanical equivalent of heat); W — energy of projectile in muzzle section $= \frac{m \cdot v^2}{2} = \frac{\text{weight}\, v^2}{2 \cdot g}$; S — energy required per km of range $= \frac{P}{D}$; r = efficiency of gun $= \frac{W}{P}$, %.

TABLE 19

Number	Type of gun	V	D	P	W	S	r	According to A. Korol'kov
1	6" field gun	335	7,681	701,860	229,787	91,377	32.7%	–
2	6" fortress gun.................	381	8,747	833,340	297,224	95,272	35.6	–
3	6" gun with reduced charge......	213	3,840	286,534	92,890	74,618	32.4	–
4	6" Schneider gun...............	640	12,347	2,876,600	838,676	232,471	29.1	27
5	6" with reduced charge	349	7,681	758,860	243,602	98,693	32.0	–
6	6" Canet gun	793	13,228	4,123,760	1,303,666	311,743	31.0	–
7	12" Vickers howitzer	363	10,237	4,697,280	2,084,106	459,048	44.3	–
8	12"/52 gun....................	762	32,431	49,780,000	13,676,521	1,427,574	27.4	23
9	French 75 mm field gun (model 1897)	525	8,500	300,000	106,000	35,300	35%	52
10	Superlong-range gun	1,500	120,000	60,800,000	14,400,000	506,666	22%	14.6 4.9

The energy values of S given in this table are represented graphically in Figure 86. One curve refers to the 6" guns, and the other to the 12" guns. The value of S for gun No. 10 is shown separately. It is seen that with every type of projectile the energy required per km of the range increases with the range itself. The small energy required for firing the superlong-range gun is due to the fact that the projectile flies at a great height where the resistance of the air is small. The curves indicated by numbers with apostrophes represent the variation of the efficiency r for the guns considered. It seems that in general the efficiency of a gun varies inversely with the range. Prof. A. Korol'kov, in his paper "K voprosu o sverkhdal'nykh orudiyakh" (On the Problem of Superlong-range Guns), published in Voina i Tekhnika, 1925, No. 218, p. 27, gives other values for the efficiency, proceeding from the fact that 1 kg of gunpowder releases 1,000 kcal or 427,000 kg · m of
96 energy during combustion and that a superlong-range gun can be fired also with a mixture of liquid hydrogen and oxygen, having a calorific value of 3,000 kcal/kg.

The data in Table 20 were obtained on the basis of these assumptions.

This table shows how the efficiency decreases with increasing energy required to impart 1 kg · m kinetic energy to the projectile. The efficiencies are generally similar to those given in Illyukevich's table, except for the
97 French 75 mm gun. This may be due to the fact that Illyukevich assumed for it the excessive value $P = 300{,}000$ [no. 9, Table 19], whereas Korol'kov used the value $13{,}300 \times 7.4 = 98{,}420$ kg · m.

Table 21 contains comparative data on the power required in the same guns per kg of the projectile weight.

This table is of interest in considering the replacement of the chemical energy of the propellant by electric energy. In this case the power required for firing a 7.4 kg shell from a 75 mm gun is $7.4 \cdot 20{,}000 \cong 150{,}000$ hp.

(96)

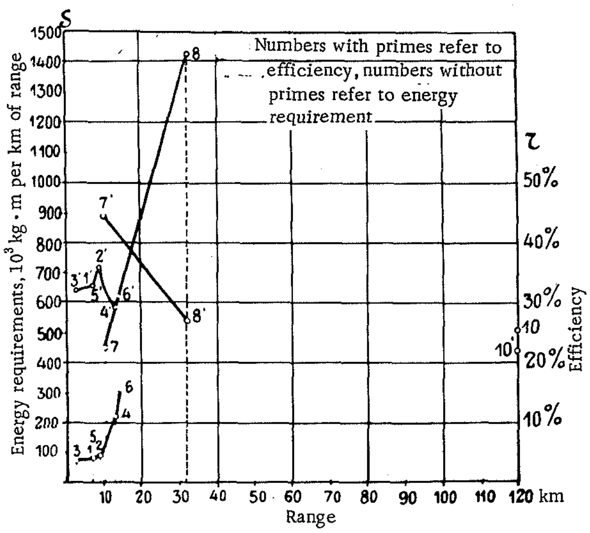


FIGURE 86. Energy requirements of guns

TABLE 20. Energy requirements per kg of projectile weight in different guns

Gun	Weight: of projectile P	Weight: of charge Q	$\frac{Q}{P}$	Muzzle velocity V_0 m/sec	Energy, kg·m/kg: of projectile	Energy, kg·m/kg: of charge	Efficiency
75 mm	7.4 kg	0.44 kg	0.06	–	13,300	25,600	52%
6" Schneider	100 lb (42.7 kg)	18.5 lb (7.58 kg)	0.185	640	20,500	75,900	27%
12"/52	1,090 lb (447 kg)	345 lb (143 kg)	0.317	793	31,500	137,000	23%
22–24 cm superlong-range	120 kg	240 kg	2.0	1,600	128,000	854,000 2,562,000	14.6% (4.9%)

(97)

TABLE 21. Power requirements per kg of projectile weight in different guns

Gun	Barrel length, m	Duration of projectile travel in barrel, sec	Power, 10^6 kg · m/sec	hp · 10^3
75 mm	2.3	0.0082	1.5	20
6" Schneider	3.66	0.0114	1.8	24
12"/52	13.25	0.033	0.95	12.6
22–24 cm superlong-range	36.0	0.045	2.78 (3.34)	37 (44)

e) Comparison of the economics of carrying a bomb by airplane and firing a gun

We shall perform an approximative computation of the costs of carrying a bomb by airplane and firing a shell from a gun. We shall consider a commercial airplane, namely the Dornier Komet III, to be more economical than the British 12"/52 gun, and also the German "Big Bertha."

The cost of the fuel needed by a Dornier Komet III plane for carrying a 983 kg (60 pud) bomb over various distances is determined as follows:

Given data: engine power, 360 hp; speed, 140 km/hr; specific fuel consumption, 0.22 kg/hp·hr; fuel cost, 35 kopeks/kg. Taking the distance as D, we obtain the total length of the flight path as $2D$. The flying time is $\frac{2D}{140}$ hrs. We add another quarter hour for checking the engine and gaining the required altitude. The total running time of the engine is thus $\left(\frac{2D}{140}+0.25\right)$ hrs, while the fuel consumption is $\left(\frac{2D}{140}+0.25\right)360\cdot 0.22$ kg.

The cost of this fuel is

$$q=\left(\frac{2D}{140}+0.25\right)\cdot 360\cdot 0.22\cdot 0.35=0.396D+6.93\ \text{rub.} \qquad (1)$$

Thus, for D = 32.5 km we obtain q = 19.80 rubles.

The maximum distance D is determined from the condition that the total weight of fuel, pilot (70 kg), and bomb must not exceed the payload of the airplane, which is 1,200 kg.

Thus,

$$70+983+\left(\frac{2D}{140}+0.25\right)360\cdot 0.22=1.200, \qquad (2)$$

98 whence D = 113.4 km.

The cost of firing a 447 kg shell from a 12"/52 gun to a distance of 32.5 km, using a charge weighing 143 kg, which costs 3.75 rubles per kg of gunpowder, is

$$q'=143\cdot 3.75=536.25\ \text{rub.}$$

Referred to a weight of 983 kg, this is

$$q'=\frac{983}{447}536.25=2.2\,.\,536.25=1180\ \text{rub.}$$

This exceeds the cost of carrying a bomb of the same weight by plane by a factor of

$$\frac{1180}{19.8}=\infty 60$$

The cost of firing a 120 kg shell from the German superlong-range "Big Bertha" is, at a charge weighing 240 kg and a range of 110 km,

$$q'=240\cdot 3.75=800\ \text{[should read 900] rubles.}$$

Carrying eight bombs of the same weight by plane over a distance of 110 km costs: $q=0.396 \cdot 110+6.93=50.50$ rubles, or 6.30 rubles per bomb.

Shooting thus is $\frac{800.8}{50.5}$ [should read 900. . .] = 127 times more expensive than bombing.

We shall now compare the respective costs by allowing for depreciation and operating expenses.

We use as basis the highest costs of air transport, namely U. S. $3.87 per ton • km (New York — San Francisco). For other routes the costs are as follows:

London-Paris	3.77 U. S. $
French lines	3.52 "
German lines	0.80 "
Deruluft (1922)	0.70 "

The cost of carrying a 983 kg bomb by plane over a distance D is thus (at 1 U.S. $ = 2 rubles):

$$p=2D \cdot 3.87 \cdot 2 \cdot \frac{983}{1000}=15.2D \text{ rub.}$$

Thus, for $D=32.5$ km we obtain $p=15.2 \cdot 32.5=494$ rubles.

For $D=110$ km and eight 120 kg bombs we obtain per bomb:

$$p=2 \cdot 110 \cdot 3.87 \cdot 2 \cdot \frac{120}{1000}=204.33 \text{ rub.}$$

Firing a British 12"/52 gun at a range of 32.5 km involves the following expenses: the cost of the gun is 250,000 rubles, and 200 shells can be fired from it. Thus,

Depreciation per shell fired = 250,000/200	1,250 rubles
Cost of charge, as computed before	536 "
Total	1,786 rubles

Referring this to a weight of 983 kg, we obtain:

$$\frac{983}{447} \cdot 1786=2.2 \cdot 1786=3930 \text{ rub.},$$

99 i. e., the cost of firing a gun is $\frac{3930}{494}=8$ times as high as carrying the equivalent bomb load by plane. Operating costs of the gun, such as crew, transportation, etc., have been neglected.

We make the following assumptions concerning the "Big Bertha": the gun costs 1,000,000 rubles, while 150 shells can be fired from it. Thus,

Depreciation per shell fired, approximately	6,700 rubles
Cost of charge	800 "
Total	7,500 rubles

This is $\frac{7\,500}{204.33} = 36.7$ times more expensive than carrying a corresponding bomb load by plane (if all other operating costs of the gun are neglected).

The results are summarized in Table 22.

TABLE 22

	Dornier Komet III airplane		Guns	
			British 12"	"Big Bertha"
Cost of energy per missile	$D = 32.5$ km, missile 983 kg, 19 rub. 80 kop.	$D = 110$ km, missile 120 kg, 6 rub. 30 kop.	$D = 32.5$ km, missile 983 kg, 1,180 rub.	$D = 110$ km, missile 120 kg, 800 rub.
Total cost	494 rub.	204 rub. 33 kop.	3,930 rub.	7,500 rub.

For comparison we also give the results obtained by M. Illyukevich and published in the paper mentioned above (Table 23).

TABLE 23

Method	Missile weight, kg	Energy cost per missile		Total cost per missile	Range, km
		rubles	kopeks	rubles	
F.50 airplane	500	19	40	1,028	37
Scout plane	166	–	–	676	–
Bomber	500	–	–	1,726	–
12"/52 gun.............	447	491	–	1,741	32.5
"Big Bertha"	120	600	–	–	110

6. METHODS OF INCREASING THE VERTICAL AND HORIZONTAL RANGES

The vertical and horizontal ranges can be increased by:

a) reducing the drag of the projectile,

b) increasing the energy of the projectile.

Consider these two methods in general outlines.

100
a) Reducing the drag of the projectile

The resistance of the air is a great obstacle to the flight of artillery projectiles. It is overcome by the following means:

1. The projectile is fired as high as possible, so that most of its flight path lies in rarefied layers of air.

2. The load per unit cross-sectional area (the weight of the projectile) is increased.

3. The shape of the projectile is improved so as to minimize its drag.

4. The flight stability of the projectile is increased, so that its axis is as far as possible tangential to the flight path.

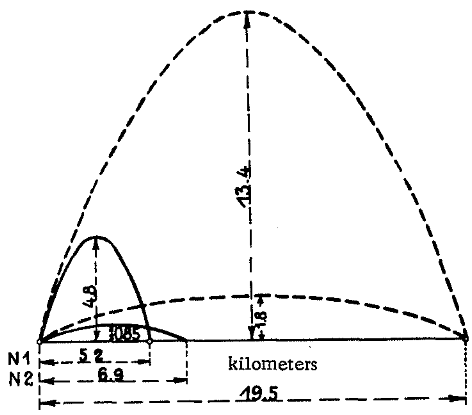


FIGURE 87. Flight paths of projectiles in air and in vacuum

The first method does not require special explanations and is illustrated in Figure 87, which shows the horizontal and vertical ranges of projectiles in air and in vacuum (Lorenz's Ballistics, 1917). The broken lines represent the parabolic flight paths of a projectile in vacuum fired at a muzzle velocity of 550 m/sec at elevations of 20 and 70°. The full lines represent flight paths in air, No. 1 at an elevation of 70°, and No. 2 at an elevation of 20°. The 77 mm projectile weighs 6.9 kg in either case.

The second method consists in increasing the weight of the projectile per unit cross-sectional area (g/cm^2).*

The third method consists in giving the projectile an optimum form either by streamlining its outside or by introducing an axial bore.

The bore is rounded off (ogival), so that it has a form factor of 0.45—0.55. Great expectations are put on such projectiles, in which the harmful effects of eddies formed behind the shell are eliminated as far as possible.

An example is the Wagner projectile (Figure 88) which has a central bore shaped like a turbine nozzle. This bore is closed during firing by an insert
101 which separates from the projectile when it leaves the muzzle.

Such hollow projectiles were designed because the drag increases much faster than the flight velocity. This greatly slows down the flight, as shown in Table 24, due to Cranz, established for the pointed German *S* projectile.

* This magnitude is called the absolute projectile loading, in contrast to the relative loading, which is the ratio of the shell weight to the cube of the bore (g/cm^3) and depends on the design and material of the projectile.

The air resistance decreases when a bore is made in the projectile. Two cases have to be distinguished here: 1) The velocity at which the air enters the bore is less than the speed of sound. The air velocity thus increases in the convergent part, while the pressure decreases. This case does not interest us, since the projectile flies at supersonic speed. 2) The velocity w at which the air enters the bore (Figure 88) is higher than the speed of sound. The velocity w_m of the air in the throat section (f_m) is then less than w, while the pressure is higher than at the inlet. Figure 88b represents the variation of the pressure between the intake section (f) and the throat section (f_m). The larger the ratio $\frac{w}{w_0}$ where (w) is the velocity of sound, the greater may be the convergence of the bore at a given air flow rate. The outlet should diverge slightly. Table 25 gives the ratio of the intake section area (f) to the throat-section area (f_m) at different ratios $\frac{w}{w_0}$. In designing such a bore it is necessary to prevent flow separation from the walls, which increase the drag.

100

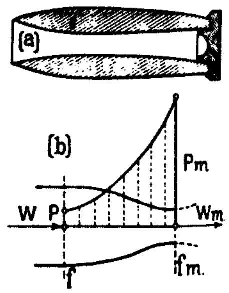


FIGURE 88. Wagner projectile

TABLE 24

Projectile velocity, m/sec	350	400	500	750	1,000
Deceleration, m/sec^2	163	234	346	607	886
Deceleration, % of flight velocity	50	60	70	80	90

TABLE 25

Velocity ratio $\frac{w}{w_0}$	1.1	1.25	1.5	1.75	2.0	2.5	3
Area ratio $\frac{f}{f_m}$	1.1	1.3	1.8	2.4	3.4	6.6	12.7

b) Increasing the energy of the projectile

The energy of the projectile may be increased in three ways: a) by increasing the energy of the propellant, using new gunpowder compositions which release more energy during combustion than those used at present, in particular, so-called progressive powders; 2) by means of turbo-guns; 3) by means of electric guns.

α) The Delamare-Mazza turbo-gun

The Delamare-Mazza turbo-gun is based on the following idea (Figure 89, bottom):

Projectile C is inserted into the gun barrel. The charge is fired in the breech chamber. The gases formed are discharged through a nozzle similar to those used in turbines. After the projectile has been launched
102 the gases escape through openings (a), so that the recoil is reduced. A different version is shown in Figure 89, top. In this case the orifices a are in the projectile itself, which is placed in the barrel.

Such a gun was first tried near Paris in 1917 and then in 1922, the muzzle velocity being 850 m/sec. It is suggested that ranges of up to 240 km can be obtained with such guns.

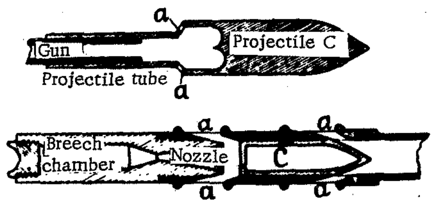


FIGURE 89. Delamare-Mazza turbo-gun

β) Electric guns

Prof. A. Korol'kov in his paper "Elektrifikatsiya i elektricheskie orudiya" (Electrification and Electric Guns), which appeared in "Voina i Tekhnika," 1927, No. 4, expresses some interesting views and gives computations referring to electric guns. We present a short abstract of this paper.

Classification of electric guns

a) Direct-current gun utilizing the interaction of the magnetic fields of the gun and the projectile in which a current flows (French project).

b) Direct-current gun in which a magnetic field moves rapidly toward the muzzle, pulling the steel projectile with it (Swedish project).

c) Alternating-current gun utilizing the interaction of a variable magnetic field and an induced field in the projectile (E. Thomson's principle).

d) Multiphase a. c. gun, in which a rapidly moving magnetic field carries a steel projectile with it (magnetofugal gun).

Elementary computations

We introduce the following notation: P — weight of projectile; v — muzzle velocity; L — barrel length; T — time during which projectile travels in barrel; W — energy required.

We assume that the velocity of the projectile in the barrel increases uniformly from 0 to v.

The mean velocity of the projectile is then $\frac{v}{2}$.

The length of the barrel is thus $L=\frac{v}{2}T$.

The energy of the projectile is $W=\frac{P}{g}\,\frac{v^2}{2}$ in the muzzle.

Assuming the efficiency of the gun to be 50%, we obtain the energy supplied by the generator as $W_1=2W$.

103 Let the muzzle velocity be 800 m/sec and the projectile weight 36.5 kg.

The energy required will thus be $W=\frac{36.5}{10}\,\frac{800^2}{2}=1{,}170{,}000$ kjoule.* The energy which has to be delivered by the generator is $W_1=23{,}400$ kjoule. Let the gun be supplied with current by an electric power plant developing 100,000 kw or 100,000 kjoule per sec. The generator then supplies the energy required in $\frac{1}{4}$ sec. The barrel length must therefore be

$$L=\frac{0+800}{2}\times\frac{1}{4}=100\text{ m}.$$

In his paper A. Korol'kov came to the following conclusions:

1. Electric guns with generators designed for accumulating (e. g., by means of a flywheel or a battery) the energy required for operating the gun during a certain time interval are hardly of practical value, since it is difficult to utilize the stored energy within the short time available for the shooting, and because of the low efficiency at which the energy is released rapidly.
2. Generators developing powers of the order of 100,000 kw may supply electric guns shooting projectiles weighing about 40 kg, and having muzzle velocities of approximately 800 m/sec.
3. A 100,000 kw generator can be used for shooting about 200 projectiles per min, each of which weighs approximately 40 kg.
4. Electric guns using a constant generator power must be very much longer than ordinary guns.
5. An electric gun may be located at a distance of 100 km from the generator, being connected to it via lines and transformers.
6. At the present [i. e., 1929] state of electrification in Russia it would be opportune to build an experimental electric gun having a bore of approximately 150 mm (6") and a muzzle velocity of 500–800 m/sec.

γ) The Fachon-Vilplet electric gun

A plan to build an electric gun was presented to the French government in 1916, but was not carried out although a model was tested. The inventor

104 submitted two designs, data on which are contained in Table 26.

We shall describe the second type, since the first type was built as a model.

An overall view of the gun is given in Figure 90. It consists of a steel electromagnet with coils of copper strip located inside a steel girder frame and connected to it by springs taking up the recoil. The frame can be turned about a horizontal axis so that the gun is given the required

* For the sake of simplicity we have taken $g=10\text{ m/sec}^2$ instead of 9.82 m/sec^2, and $1\text{ kg}\cdot\text{m}=10$ joule instead of 9.82 joule.

elevation. A cross section of the gun is shown in Figure 91 on the left, while the projectile is shown in Figure 91 on the right. The projectile carries four longitudinal fins sliding in corresponding grooves in the gun barrel. The latter (Figure 91) consists of the magnetic circuit formed by steel blocks aa with four poles whose length is equal to that of the gun. Tracks bc, bc and copper strips de, de, in which the magnetizing current flows, also have this length. Strips de, de are insulated from one another and from the magnetic circuit. The tracks are insulated on three sides but are bare on the fourth side along which the fin slides. The fins have small recesses (g) so that they act like springs while sliding.

TABLE 26

(103)

Characteristic \ Type	I	II
Projectile weight, kg	0.050	100
Muzzle velocity, m/sec	200	1,600
Barrel length, m	2.0	30
Energy in barrel, kg · m	–	$340 \cdot 10^6$
Power in barrel, kw	–	$3.4 \cdot 10^6$
Current intensity in barrel, amp	–	$3.55 \cdot 10^6$
Weight of gun, t	–	150
Efficiency	–	25%

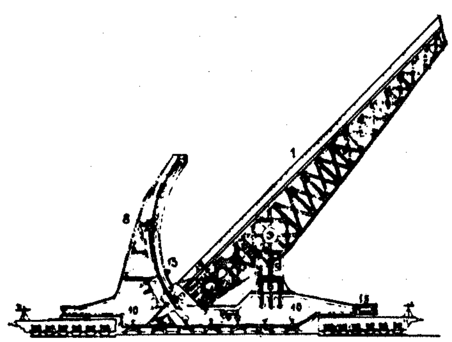


FIGURE 90. Fachon-Vilplet electric gun

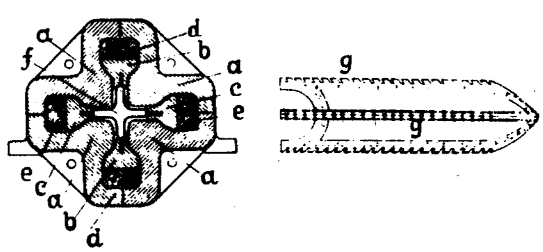


FIGURE 91. Cross section and projectile of Fachon-Vilplet electric gun

An engine driving a generator and a dynamo driven by a heavy flywheel are set into motion for the shooting. When the required power is attained, the current is made to act on the projectile, forcing its fins to slide in the track grooves and shooting it out of the gun.

Historical information

1) Experiments were carried out a long time ago in Sweden on a gun whose operating principle was based on the pull exerted by an iron core inside a coil. However, due to the insufficient development of electrical engineering at that time these experiments did not yield encouraging results.

2) E. Thomson proposed an electric gun whose operating principle was based on the repelling action of two coils energized by a. c.

3) Guns operated with multiphase currents were also proposed. A magnetic field moving along the barrel was to pull the projectile with it, inducing in it currents as in a three-phase motor.

4) The Austrian Heft in 1895 proposed a solenoid gun for launching interplanetary spaceships.

105 A shortcoming of the Fachon-Vilplet gun was the necessity to have available, for the size mentioned, a large electric power which, at an overall efficiency of 25% and with one shot per 20 min, would have amounted to 450 kw (900 kw with a shot every 10 min). This figure is based on the assumption that a current of $3.55 \cdot 10^6$ amp flows as long as the projectile is in the gun barrel ($^3/_{80}$ sec). In fact, the current does not attain this intensity instantaneously, so that the power required is tens of times higher.

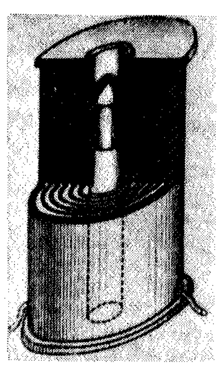


FIGURE 92. Solenoid gun (schematic).

A detailed criticism of this project is found in Prof. A. A. Korol'kov's paper "Elektricheskoe orudie dal'nego boya" (Electric Long-range Gun), published in "Tekhnika i Snabzhenie Krasnoi Armii," 1923, No. 68, p. 1, from where the above description was copied.

He stated that guns like that described, but having a length of 120 m and a muzzle velocity of 3,000 m/sec, may have ranges of 600 km and can be built at the present state of technology.

б) Solenoid guns

Several plans for so-called solenoid guns were made public after World War I. They were intended to shoot projectiles at high velocities over large distances. The operating principle of such a gun is illustrated in Figure 92. The gun consists of a coil formed by a large number of turns of a long wire. The projectile is located in a hollow space inside the coil and is ejected at high speed when a circuit is closed.

Fantasy, working faster than technology, has devised guns able to launch missiles at speeds exceeding 11,800 m/sec, i. e., at velocities sufficient to reach the interplanetary space.

Figure 93 shows such a gun.

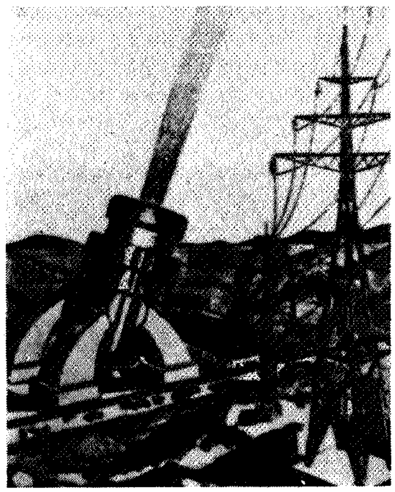


FIGURE 93. Planned fantastic electric gun

A noiseless and smokeless electromagnetic gun was also proposed by
106 Prof. Birkeland.*

A model of a rocket ship and space station was shown in spring 1929 by G. A. Polevoi (Figure 94) at the Moscow Exhibition of Interplanetary Machines. We present a description of this machine, as it was given in the album of the exhibition.

FIGURE 94. G.Polevoi

G. A. Polevoi (Russia, 1913) proposes the use of electric energy. The interplanetary station is characterized by a compressor-solenoid tunnel (Figure 95) in which a well-streamlined carriage slides on guides. The carriage is surrounded by an iron armor which is also well streamlined (Figure 96). The compressor-solenoid station imparts a speed of 1,600 m/sec to the armored rocket carriage and ejects it through a tunnel in the rock to an altitude of 150 km. The speed is maintained by burning part of the rocket tube, which protrudes through the rear of the armor. The armor is opened automatically at the altitude of 150 km, and ignition takes place in all tubes. The rocket craft escapes into space after gradually developing a speed of 11,000 m/sec; the armor descends on a parachute and is

* Details on the computations concerning electric guns are given in Guido von Pirquet's paper "Die ungangbaren Wege zur Realisierung der Weltraumschiffahrt" (The Impossible Ways of Realizing Space Travel), published in "Die Möglichkeiten der Weltraumfahrt" (Possibilities of Space Travel), Leipzig, 1928, p.301.

returned to the station. The initial energy supplied by the electric power plant enables the carriage to rise by its inertia to a high altitude where the
107 resistance of the air is negligible. Less propellant is therefore required to impart a cosmic speed to this craft, than to a rocket which rises only due to the reaction caused by the discharge of gases formed during combustion. Furthermore, the absence of propellers, etc. reduces the weight and size of the craft; this facilitates maneuvering and return to earth, effected in a glide and with the aid of retro-rockets. A cross section of the rocket carriage is shown in Figure 97.

(106)

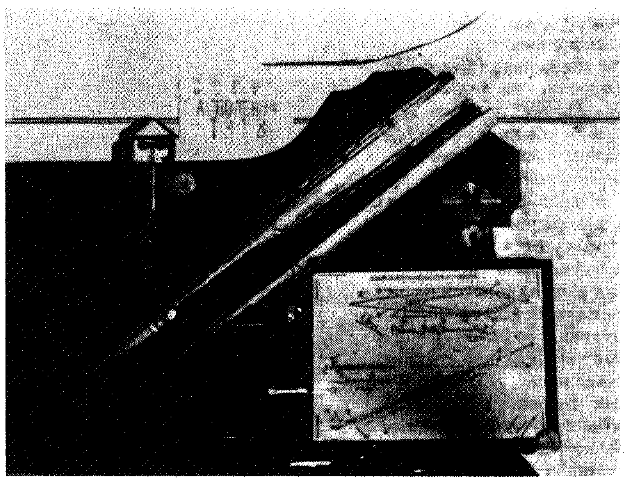
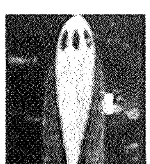


FIGURE 95. Polevoi's electromagnetic gun

A similar idea was developed in Russia from 1911 to 1913 by Prof. B. P. Veinberg of the Tomsk Technological Institute. He carried out experiments with a carriage weighing 10 kg, traveling in an annular tube from which the air had been exhausted in order to reduce the resistance. A longitudinal section of the tube is shown in Figure 98. Friction between the carriage and the inner surface of the tube was eliminated by means of electromagnets arranged at certain distances, which imparted a wavelike motion to the carriage by overcoming its weight. The beginning of the tube was surrounded by a series of solenoids capable of attracting the carriage, so as to impart an initial velocity to it. B. Veinberg developed a plan for a 3 verst-long line of solenoids at the starting station with which a speed of 800—1,000 km/hr would easily be attained.



108 FIGURE 96. Polevoi's rocket carriage

Figure 99 is an overall view of the curved part of the tube with the solenoids, while Figure 100 shows schematically the starting station. Small carriages loaded with "passengers" are introduced on a platform into chamber II. The latter is then shut off from the outside by means of a gate and the air exhausted from it.

The platform is then transferred into the "empty" chamber I and brought to the opening of the tube into which the carriages are launched at intervals of 5 sec. Solenoids S, S, S and electromagnets E_1, E_2, E_3 are shown in plan. The platform is then transferred to Chamber V in order to be loaded again, and so forth.

(107)

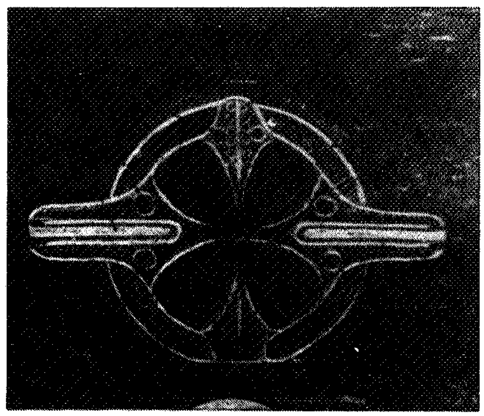


FIGURE 97. Cross section of Polevoi's rocket carriage

107

FIGURE 98. Veinberg's electric carriage

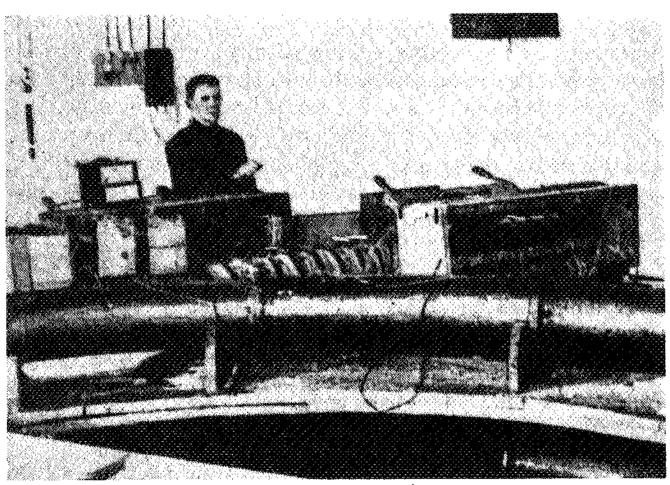


FIGURE 99. Overall view of Veinberg's tube

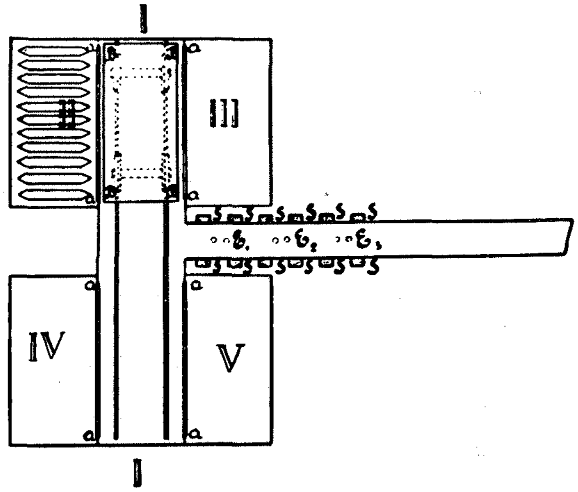


FIGURE 100. Veinberg's starting station

ϵ) *The Alco electric gun*

Issue No. 100 of "Tekhnika i snabzhenie Krasnoi armii" of 1924 contained a description of a future electric gun, a fantastic plan published under the pseudonym of Alco.

The author stated that as a secret agent he was able to discover a gigantic electric gun built by the French at Cappel in the forests of the French Ardennes. This gun could shoot 2-t projectiles of 0.5 m diameter and 2 m length to a distance of approximately 3,000 km (allowing for the curvature of the earth) at a muzzle velocity of 5,000 m/sec. The steel projectile
109 had the form of a Whitehead torpedo, being tapered at the front and rear. It did not revolve [during flight], its stability being ensured by two gyroscopes started during launching by rack and pinion inside the gun barrel. The projectile carried a series of copper rings arranged flush with its surface, which increased the magnetic effects of the current.

The gun (Figure 101) was 125 m long and had a fixed elevation of 36°. It rested on a viaduct-like metal girder by means of three platforms. The latter traveled on rails arranged lengthwise, so that the gun could be moved back and forth. The girder itself was carried by wheeled trucks on a circular track, and could thus turn about a vertical axis together with the gun.

The electric machine developing the magnetic force ejecting the projectile consisted of ten 4,000-kw dynamos. It could launch the projectile in $\frac{1}{20}$ sec and develop a force of $2 \cdot 10^7$ kg at an energy output of $2 \cdot 10^7 \cdot 125$ kg · m. This energy was stored by means of flywheels driven for 11 min and then braked rapidly, thus releasing the energy required. Allowing for unavoidable losses, the author of the description thought that one shot was possibly every 25 min.

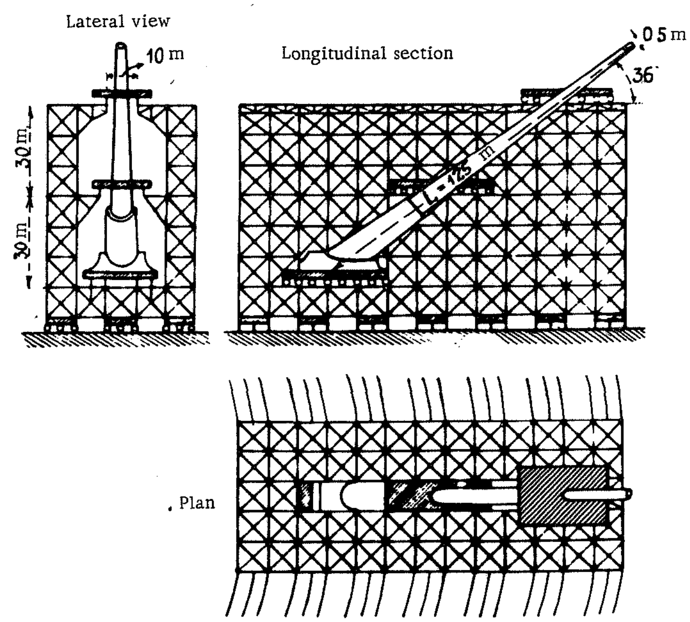


FIGURE 101. Alco electric gun

The force moving the projectile inside the barrel was obtained in the following way: the steel gun barrel was wound with copper strips carrying an electric current. These strips formed several independent windings
110 of electromagnets. A separate current flowed from an alternator through each winding; the various currents had very small phase differences between one another. The electromagnets were therefore energized successively, so that a magnetic-force wave carrying the projectile with it traveled along the barrel.

The range was adjusted by varying the generator voltage. The recoil was absorbed by electric braking. After being launched the projectile was to travel for only ten seconds at heights below 15 km, the remainder of the flight taking place in almost perfect vacuum. The total flight duration was to be 10 min. A maximum height of 500 km was to be attained after 330 sec (Figure 102).

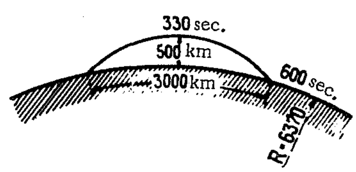


FIGURE 102. Flight path of Alco projectile

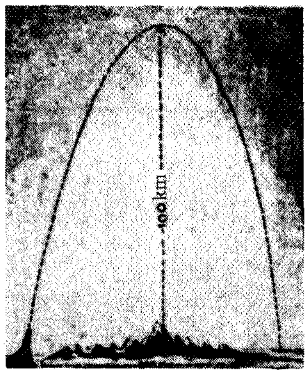


FIGURE 103. Flight path of planned projectile

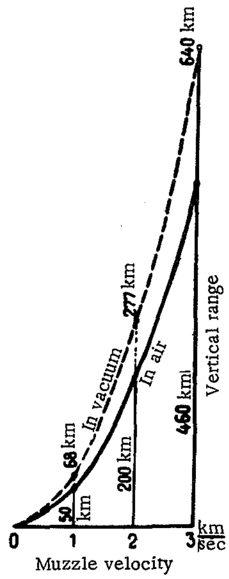


FIGURE 104. Vertical ranges attained by projectiles in air and in vacuum

The author concluded this description by mentioning some possible targets for this gun, if fired in France: 1) a plant in Mannheim, at a distance of 250 km, 2) the fort of La Spezia (Italy) at a distance of 600 km, 3) Touggourt in the Sahara (1,800 km), and 4) the Krematorsk plant near Kharkov (2,400 km). [This would place the location of the gun somewhere between Sarreguemines and Metz, the location of a village named Cappel.]

A large range can be obtained if the projectile is fired to a great height. Figure 103 shows the flight path of a projectile fired at a horizontal range of 120 km and rising to an altitude of 100 km.

Figure 104 shows the approximate relationship between the muzzle velocity (km/sec) and the vertical range, allowing for the resistance of the air. This vertical range is less than that attained by the projectile in vacuum.

111 **c) The influence of the rotation of the earth** *

Consider now the influence of the rotation of the earth on the flight of an artillery projectile. This problem was already discussed by I. M. Kirillov-Gubetskii in his paper "Vliyanie vrashcheniya zemli na polet snaryada" (The Influence of the Rotation of the Earth on the Flight of a Projectile), published in "Voina i Tekhnika," 1925, No. 244, p. 21. However, we have arrived at different conclusions in certain particular cases, which we

* This article was first published by us in "Voina i Tekhnika, 1927, No.1.

shall present here. We assume the same linear speeds for the diurnal rotation of the earth as Kirillov-Gubetskii, namely 465 m/sec at the equator and zero at the poles.

Like Kirillov-Gubetskii, we shall consider the influence of four factors on the flight of the projectile. These are α) the rotation of the earth, β) the altitude at which the projectile flies, γ) the variation of the force of gravity, and δ) the resistance of the air. We shall only deal with the northern hemisphere, since analogous considerations apply to the southern hemisphere.

α) The rotation of the earth

a) **Shooting along a tangent to a parallel.** Let the projectile move in an easterly direction along the tangent to a parallel (Figure 105). The meridian Na on which the gun is located will assume a position Na_1 after a time interval t: the projectile, however, moves in the plane aSo passing through the center of the earth, and will be deflected from the parallel considered toward the south, falling at some point a_1 to the right of the parallel.

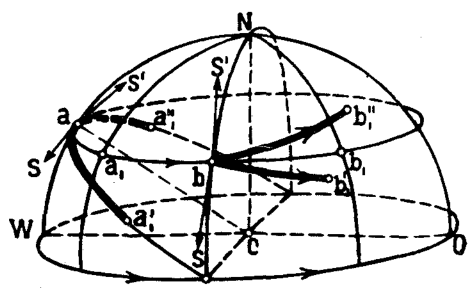


FIGURE 105. The theory of superlong-range shooting

Let the projectile now move in a westerly direction along the tangent to the parallel. Considerations similar to the above show that the projectile will be deflected to the left, i.e., south of the parallel, and will fall at some point a_1''.*

b) **Shooting along a tangent to a meridian.** In this case our conclusions are the same as those of Kirillov-Gubetskii: A projectile fired in a southerly direction along bS will be deflected from its meridian toward the west, since its diurnal rotation is slower than that of the target, and will therefore fall at some point b_1', behind (to the west of) the meridian Nb_1 on which the gun is located after the time interval t. Similarly, the projectile will fall at some point b_1'' to the east of the meridian when it is fired in a northerly direction.

* [The deflection is always to the right in the northern hemisphere, due to the Coriolis force.]

β) The influence of the flight altitude (Figure 106)

c) **Shooting vertically upward on the equator (in direction cZ).** In this case, as also assumed by Kirillóv-Gubetskii, the projectile velocity can be resolved into a vertical component cZ, representing the relative projectile motion, and an equatorial component ZE, corresponding to the diurnal rotation of the earth. The projectile will thus describe a path differing from that of the gun. For instance, let the projectile rise to a height c_1'. Due to its smaller angular velocity it will then be to the west of
112 (behind) the gun position c_1, and will fall to the earth at some point c_2' when the gun is already at c_2.

d) The projectile will not be deflected at all when fired **at the pole** (point d).

e) **Shooting vertically upward on a parallel (in direction eZ).** The velocity of the projectile can again be resolved into a component representing the relative projectile motion and a component nE corresponding to the diurnal rotation of the earth.

The effect of the latter motion is the same as in case a) (Figure 105), i. e., the projectile will be deflected to the south, while the first component will cause the projectile to lag behind the meridian on which the gun is located. The projectile will therefore fall to the right of the parallel at point e_1' which lies to the west of the meridian Ne_1 of the gun.

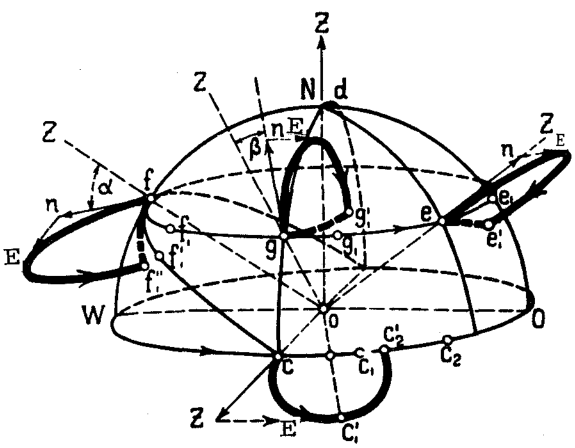


FIGURE 106. The theory of superlong-range shooting

f. **Shooting at an angle α to the vertical in a southerly direction.** In this case the axis fn of the gun barrel lies in a meridional plane and is directed toward the south. The projectile velocity has a component fn, corresponding to the relative motion of the projectile, and a component nE along the tangent to the parallel at the point of firing, which represents the diurnal rotation of the earth. The component fn in its turn can be resolved into two components, one vertical at point f, the other along the tangent to the meridian and directed to the south. The vertical component and the diurnal rotation of the earth deflect the projectile to the south and cause it to lag behind the gun (f_1), as in case e). The velocity component along the tangent to the meridian causes the projectile to move toward the south and to lag behind the gun (case b). The projectile will thus be deflected toward the south to a larger extent than in case a); it will fall to the south of the great circle fOf_1' which is similar to aOa_1' in Figure 105, and to the west of the gun.

g) **Shooting at an angle β to the vertical in a northerly direction.** In this case the axis gn of the gun barrel lies in a meridional plane and is directed toward the north. The projectile velocity has a component gn, corresponding to the relative motion of the projectile, and a component nZ along the tangent to the parallel at the point of firing, which represents the diurnal rotation of the earth. The component gn in its turn can be resolved into two components, one vertical at point g, the other along the tangent to the meridian and directed to the north.

The vertical component and the diurnal rotation of the earth deflect
113 the projectile to the south and cause it to lag behind the gun (case e). The velocity component along the tangent to the meridian causes the projectile to move toward the north and to overtake the gun (case b).

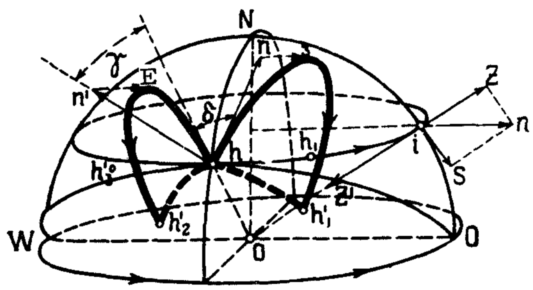


FIGURE 107. The theory of superlong-range shooting

The influence of the velocity component along the tangent to the meridian varies directly with the angle β. However, when the angle β is small, the projectile may fall to the south and west of the gun, although it is fired toward the north. Consideration of cases f) and g) shows that the range to the south will be larger than to the north. In the former case, the influence of all three velocity components is added while in the latter, one of the velocity components has an effect counteracting that of the two other components.

h_1) **Shooting at an angle δ to the vertical in an easterly direction** (Figure 107). In this case the axis hn of the gun barrel lies in the plane of the great circle hOW and is directed toward the east. The projectile velocity has a component hN, corresponding to the relative motion of the projectile, and a component hZ along the tangent to the parallel at the point of firing, which represents the diurnal motion of the earth. The velocity component nZ in its turn can be resolved into two components, one vertical, at point h, the other along the tangent to the parallel and directed to the east. The three velocity components deflect the projectile not only from its parallel toward the south, but also from the great circle hOW. The projéctile will overtake the gun when δ is large, but will lag behind it if δ is small.

h_2) **Shooting at an angle γ to the vertical in a westerly direction.** The projectile velocity hn' in this case has a vertical component and a component along the tangent to the parallel, directed to the west. The vertical component and the component representing the diurnal motion of the earth will deflect the projectile from the parallel toward the south and cause it to lag behind the gun (case e), Figure 106). The velocity component along the tangent to the parallel, directed toward the west, will deflect the projectile toward the south.

When γ is small, the projectile will move to the west and be deflected to the south of its parallel and the great circle hOW, so that it falls at h_2'. However, when γ is large, the projectile may be deflected from the parallel toward the south* and east, but it will lag behind the gun.

γ) The influence of the variation of the gravitational
114 force (Figure 107)

The motion of a projectile is also affected by gravity, apart from the rotation of the earth, the flight altitude, and the direction of firing (Figure 107). The gravitational force would be directed toward the center of the earth (along the line iz') if the latter did not rotate. However, the rotation of the earth gives rise to a centrifugal force in normal to the axis of rotation. This force can be resolved into a radial component iz, and a component is along the tangent to the meridian, and directed toward the south. The magnitudes of iz and is vary with the latitude of the point of firing. At the pole $iz=0$; it gradually increases toward the equator where it attains its maximum, The component is vanishes at the pole and at the equator, attaining its maximum at some intermediate latitude. The resultant of iz' and iz is always directed toward the center of the earth and reduces the flight altitude of the projectile. The lag of the projectile behind the gun thus decreases (case e), Figure 106), so that the range is increased during firing toward the east but reduced during firing toward the west. The deflection of the projectile toward the south is also diminished. The component is, however (case b), Figure 105), causes the projectile to lag behind the gun, so that the range decreases during firing toward the east and increases during firing toward the west; the deflection toward the south also increases.

Lastly, the latitude and the flight altitude affect the component iz, which decreases with increasing distance from the center of the earth. This increases the flight altitude and the effects related to it.

δ) The influence of the diurnal motion of the atmosphere

We assume that the atmosphere rotates uniformly with the earth from west to east. We can then determine its effects at different directions of firing:

* Kirillov-Gubetskii writes (p.24) that in case h) the projectile is deflected toward the north, irrespective of the magnitude of the angle γ.

a) When the direction of firing is along a tangent to the parallel and toward the east (***aS***, Figure 105), the projectile will gain speed so that the range and the deviation toward the south will increase. The opposite happens when the direction of firing is toward the west (*aS*) .

b) When the direction of firing is along a tangent to the meridian and toward the north (***bS'***, Figure 105), the projectile will be deflected toward the west, since [the air] velocity at the higher latitudes will be less than at ***b***, giving rise to a relative east wind. The motion of the projectile toward the north will be reduced. During firing toward the south (***bS***) the projectile will be deflected toward the east while its motion toward the south is increased.

c) The deflection toward the east will increase if the projectile is fired vertically upward at the equator (Figure 106).

d) No change will occur when the projectile is fired vertically upward at the pole (d).

e) The velocity toward the east will be increased if the projectile is fired vertically upward at some intermediate latitude (Figure 106, case e).

f) and g). The effects noted under b) and e) are superposed if the projectile is fired in the meridional plane at some angle α or β to the vertical (Figure 106, cases f) and g)).

h) The effects noted under a) and e) are superposed if the projectile is fired in the plane of the great circle ***WhO*** at an angle γ or δ to the vertical (Figure 107, case h)).

ϵ) Computation of the height attained by a projectile fired vertically upward at the equator

Figure 108 represents an equatorial section of the earth. We assume that a projectile is fired vertically upward from point A at a velocity
115 $v_c = 1$ km/sec.

At the start it also has a velocity $v_e = 0.465$ km/sec along the tangent to the equator. This velocity is equal to that of a point on the equator [due to the diurnal rotation of the earth].

Neglecting the resistance of the air, we find that the maximum height attained by the projectile is:

$$h = AE = \frac{v^2}{2g} = \frac{1}{2 \cdot 0.01} = 50\ \text{km}.$$

The mean vertical velocity is $v_{cm} = 0.5$ km/sec.

The duration of the ascent is $t = 50/0.5 = 100$ sec.

During this time the projectile moves parallel to the tangent AC over a distance $0.465 \cdot 100 = 46.5$ km, and reaches point B.

During the descent from point B the projectile motion can be resolved into a component $BF \parallel AC$ and a component BD directed toward the center of the earth (O). We draw $BC \parallel AE$ and determine the angle $DBC = \alpha$.

$$\operatorname{tg}\alpha=\frac{BE}{EO}=\frac{46.5}{6370+50}=0.00724;\ \alpha=0°24'53''.$$

We set $BD=50$ km; then $DG=46.5$ km and the diagonal BG will be equal to

$$BG=\sqrt{50^2+46.5^2-2\cdot 50\cdot 46.5\cdot\sin\alpha}=68.03.\ \text{km}.$$

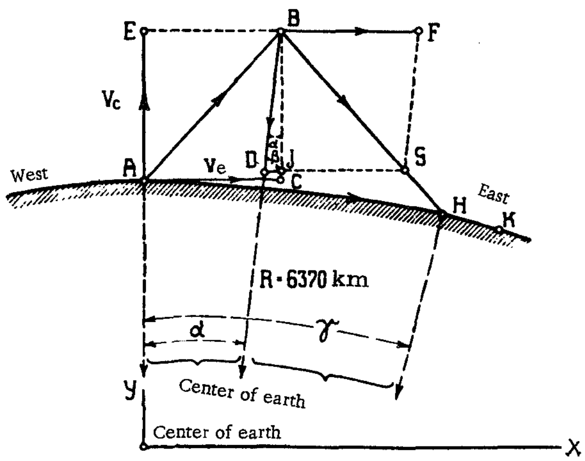


FIGURE 108. The theory of vertical shooting

We determine the position of point H, where the projectile hits the earth by establishing the equations of the equatorial arc and of the straight line BG, and then finding their intersection referred to the OX and OY axes. Here OY is the prolongation of AE, while OX is perpendicular to AE.

The equation of the equatorial arc is $x^2+y^2=6{,}370^2$, where 6,370 is the radius of the earth; the equation of the straight line BG: is $y=ax+b$; we must therefore determine the parameters a and b from the coordinates of points B and G.

For B, $x_B=46.5$; $y_B=6{,}370+50=6{,}420$.

For G: $x_G=EF-DJ=46.5\cdot 2-50\ \sin\alpha=92.638$ km.

$y_G=6370+IC=6370+(50-IB)=6370+50-$

$-50\cdot\cos\alpha=6370.0025.$

116

Inserting these values into the equation of the straight line and solving for a and b, we finally obtain this equation in the form

$$y=-1.08x+6470.22.$$

Solving this equation simultaneously with that for the equatorial arc, we obtain the coordinates of point H:

$$x_H=92.9;\ y_H=6369.9.$$

The length BH is 68.3 km, the vertical distance through which the projectile falls is $6{,}420-6{,}369.1=50.1$ km, the duration of the descent is $50.1/0.5=100.2$ sec. The total flight duration is thus $100+100.2=200.2$ sec; during this time point A on the equator moves to K; the length of arc AK is 93.1 km. The length of arc AH is found from the angle γ. We have

$$\mathrm{tg}\,\gamma=\frac{92.9}{6369.9}=0.01459, \text{ hence } \gamma=0^\circ 50'9''.$$

The length of arc AH is thus 92.88 km. The projectile thus lags behind the gun, which is at point K, falling at a distance $AK-AH=93.1-92.9=0.2$ km to the west of it.

CONCLUSION

During shooting in any direction the projectile velocity can be resolved into three components directed 1) vertically from the point of firing, 2) along the tangent to the parallel, and 3) along the tangent to the meridian. The influence of each component can then be analyzed as above. Superposing on these effects those of the diurnal rotation of the earth, gravity, and atmosphere, we obtain the resultant velocity which determines the flight path of the projectile.

Consideration of the methods described of shooting with existing or planned guns shows that the projectile is fired into space with the aid of energy imparted to it only during its motion inside the gun barrel. The energy required at a large range is enormous since the projectile has a high velocity and must overcome the resistance of the air, which is proportional to a certain power of the velocity.

7. SUPERARTILLERY AND ROCKETS

Superartillery is the firing of projectiles to heights where the air is greatly rarefied, so that the projectile can move at a high speed over long distances.

A great height may be attained by firing the projectile from a big ordinary gun, as described in the preceding sections. However, the noted shortcomings of this method make it necessary to consider another method of firing projectiles, namely the use of "rocket shells." Such a projectile is fired from the gun by means of a charge located in the latter. After attaining a certain height the rocket continues to fly, utilizing the reaction caused by the discharge of gases formed during the combustion of a propellant carried in the projectile itself. Trials of firing rockets from
117 guns were performed already a long time ago. Thus, Congreve in about 1805 suggested such a gun (Figure 109, top). Even before World War I trials were carried out of firing at airplanes from antiaircraft guns with rockets leaving behind them a luminous trace at night or smoke during daytime (Figure 109, right). Winged rockets were also launched (Figure 109, bottom).

Experiments on the flight and operation of rockets were carried out a long time ago.

a) Rocket tests

Some tests have been described in our book "Rakety" (Rockets), Leningrad, 1928. We built a device for testing rockets in 1929 at the Leningrad Institute of Communication Engineers. Here we shall describe two further recent testing installations.

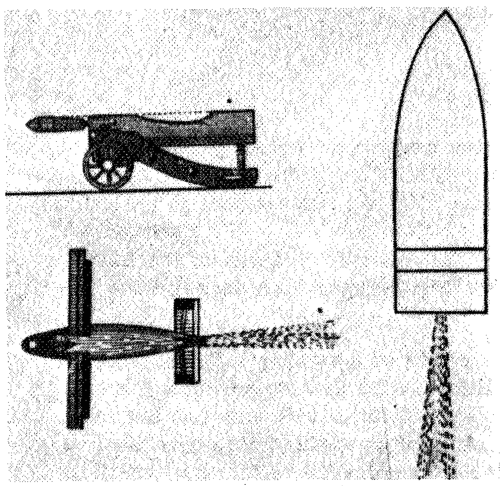


FIGURE 109. Congreve's gun. Smoke shell. Winged rocket

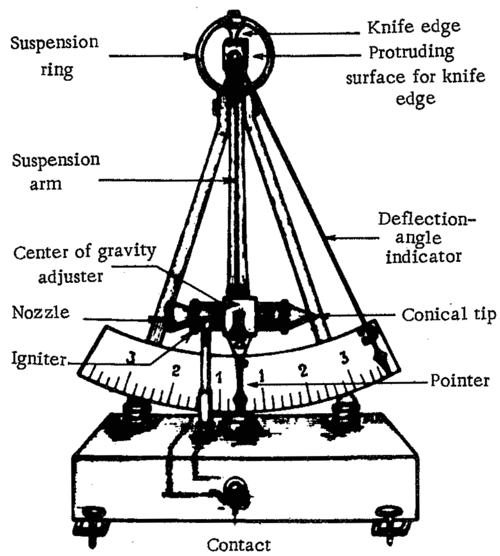


FIGURE 110. Ballistic pendulum

N. Zhirov's ballistic pendulum (project)

Figure 110 is a sketch of a ballistic pendulum for testing rockets, designed by N. Zhirov, laboratory assistant at the School of Artillery in Kiev. The rocket is secured to an arm suspended from a knife edge by a ring. The rocket is fired electrically. Nozzles of different shapes can be screwed to the rocket tail. The inclination of the arm indicates the magnitude of the reaction. The velocity can be determined with the aid of a chronograph.

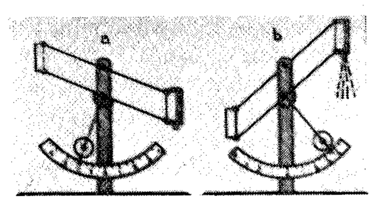


FIGURE 111. Rocket test stand

Ballistic test stand

We shall now describe a simple device for determining the reaction due to the discharge of gases from a rocket (Figure 111a and b). The sketches show a balance whose right-hand arm carries the rocket. The left-hand sketch shows the rocket before ignition, while the right-hand
118 sketch shows it during operation. The counterweight is on the left-hand side before the test, and the pointer indicates the weight G_1, of the rocket. During operation the reaction lifts the rocket, so that the pointer moves to the right. The reaction is equal to the weight of the rocket when the pointer is at zero. If the pointer moves to the right of zero and indicates, e. g., a force G_2, this shows that a force of this magnitude tends to lift the rocket, i. e., an overbalance Δ exists. When $G_2 = G_1$ numerically, the rocket moves upward at an acceleration equal to that due to gravity; the overbalance Δ increases further when the pointer moves more to the right. Let b be the acceleration of the rocket, and g the gravitational acceleration, while a is the number indicated by the pointer at some overbalance Δ.

Thus,

$$a = \frac{G_1 + G_2}{G_1 + \Delta}$$

whence:

$$\Delta = \frac{1}{a} G_2 - \frac{a-1}{a} G_1'.$$

Let the rocket now be arranged on the same balance so that the gases are discharged upward. Knowing the corresponding magnitudes G_1' and G_2', we obtain

$$a = \frac{G_2' - G_1'}{\Delta + G_1}; \quad \Delta = \frac{1}{2} G_2' - \frac{a+1}{a} G_1'.$$

The acceleration due to the reaction is $b = ag$ so that the rocket has an upward acceleration:

$$b - g = (a - 1)\, g.$$

b) Rocket bombs

The descent of bombs dropped from airplanes can be accelerated, according to the German Captain Ritter, by fitting rocket engines to them. This increases the velocity of the descent, reduces its duration, and improves the chances of hitting the target. We shall briefly discuss the theory of this bombing, neglecting the resistance of the air.

Let M_1 be the weight of the bomb and P the weight of the rocket engine fitted to it. The initial weight of the complete bomb is thus $M_0 = M_1 + P$.

Let c be the velocity at which the gases are discharged from the rocket, and v the final flight speed of the rocket with the bomb. We thus obtain

$$v = c \ln \frac{M_0}{M_1},$$

where $\ln$ denotes the natural logarithm, whence:

$$\frac{M_0}{M_1} = e^{\frac{v}{c}}.$$

Assuming, according to Goddard, $c = 2{,}400$ m/sec, we obtain the following Table:

$v = 2{,}400$ m/sec	$P = 1.72\ M_1$
1,600	$0.65\ M_1$
600	$0.285\ M_1$
300	$0.133\ M_1$

Thus, if a bomb weighing 100 kg has to hit the target at a speed of 300 m/sec, its initial weight together with the rocket must be 113.3 kg, which
119 is an increase of only 13.3%, neglecting gravity.

Ignition of the rocket has to be delayed for 5 sec after the bomb has been released, so as to prevent damage to the airplane. The bomb falls 122.5 m during this time, its speed attaining 49 m/sec.

The rocket begins to operate at this instant. We assume that burning lasts for 5 sec, and that the final velocity [due only to the rocket action] is 300 m/sec. This corresponds to an acceleration of 60 m/sec^2. The height lost between the end of the fifth and the end of the tenth second is 750 m [neglecting gravity]. Under the influence of gravity only, the bomb would have fallen 367.5 m during this time. During the first 10 sec it thus falls through a total vertical distance of $122.5 + 367.5 + 750 = 1{,}240$ m. The final velocity will be 398 m/sec. This velocity would have been attained only after 40.6 sec without rocket assistance. The corresponding height of fall is 8,077 m. Rocket assistance thus is equivalent to a 6,837 m increase in the flight altitude.

Consider now the effect of a rocket when a bomb is dropped from a height of 6,000 m, which is a case encountered in practice. The fall duration will be 34.6 sec if only gravity acts on the bomb. Rocket assistance makes the fall take place as if the bomb had been dropped from a height of $6{,}000 + 6{,}837 = 12{,}837$ m. The duration of free fall is 51.19 sec in this case, of which 40.6 sec are spent in descending a vertical distance of 6,837 m [should read 8,077 m]. Under the action of gravity and the rocket, the bomb will fall through a distance of 6,000 m in only 10.6 instead of 34.6 sec. [This is obviously wrong since during the first 10 sec the bomb falls through a vertical distance of 1,240 m only.] This represents a 60% reduction of the fall duration, while the total weight of the bomb is increased by only 13.3%. The resistance of the air has been neglected in these computations, so that the duration of the fall is slightly increased; it may nevertheless be assumed that a rocket-assisted bomb falls twice as fast as an ordinary bomb.

c) Increasing the ballistic effect of a gun

We shall investigate three examples in which the ballistic effect of a gun is successively increased. The work of the gas will be described only in an approximation, since the actual computations have to be performed differently, allowing for the inertia of the gases. This has a braking effect which we shall neglect, in accordance with Max Valier ("Flight in Interplanetary Space").

Consider a cylindrical gun barrel, closed at one end and open at the other, having any desired length and bore, its diameter being taken as unit of measurement.

Let a small but sufficient quantity of propellant be placed at the bottom of the barrel. The projectile is placed at a distance of 1 caliber from the bottom of the barrel (Figure 112, top).

We assume that combustion of the charge is instantaneous. Gas of high pressure and temperature is immediately formed between the bottom of the barrel and the projectile. Let the charge consist of gunpowder releasing an energy of 1,000 kcal or 427,000 kg · m per kg. Assuming all this energy to be used for propelling the projectile, we find that 15 kg of gunpowder will be sufficient to impart a velocity of 11,200 m/sec to a 1 kg projectile.*

After combustion of the charge the projectile moves toward the muzzle, the gas expands, and the pressure and temperature decrease.

Computations show that if the inertia of the gas is neglected, the pressure decreases according to a so-called adiabatic curve which is
120 similar to the curve representing the variation of the gravitational force with the distance from the earth. However, the slope of the adiabatic curve is less steep, since the pressure varies inversely as the 1.4-th power and not the second power of the distance. The area bounded by this curve, the vertical ***AB*** on the left, and the horizontal axis represent the work done by the gas during its expansion. Let the unit of work be equal to the area of the rectangle A'B'BA, whose height ***AB*** represents the initial pressure of the gas, and whose base ***AA*** is equal to the bore of the gun (unit length). Computation shows that the total work done
121 by the expanding gas is 2.5 · ***ABB' A'***.

The right-hand part of the diagram gives a comparatively small work, so that the barrel length may be limited, e. g., to 16 calibers ***(A'E)***, in which case the work done by the gas is represented by the area ***A BEF*** = 1.67 units.

In this way we cannot achieve a high projectile velocity. To increase the latter we would have to raise the initial pressure excessively, which would necessitate thickening the barrel walls.

Consider now a second example (Figure 112, center). Let all conditions be the same as before, except that the space between the bottom of the gun barrel and the projectile is filled with a slowly burning propellant. The latter burns in a finite time interval during which the projectile moves a distance of, for example, two calibers (2 units) while the gas pressure remains constant. Thereafter, the process takes place as before, the only difference being that the area ***A'CDB' = 3AA'B'B.*** is now taken as a unit of work.

* Since a 1 kg projectile requires for this $6378 \cdot 10^6$ kg · m ($6.378/0.427 = 15$).

(120)

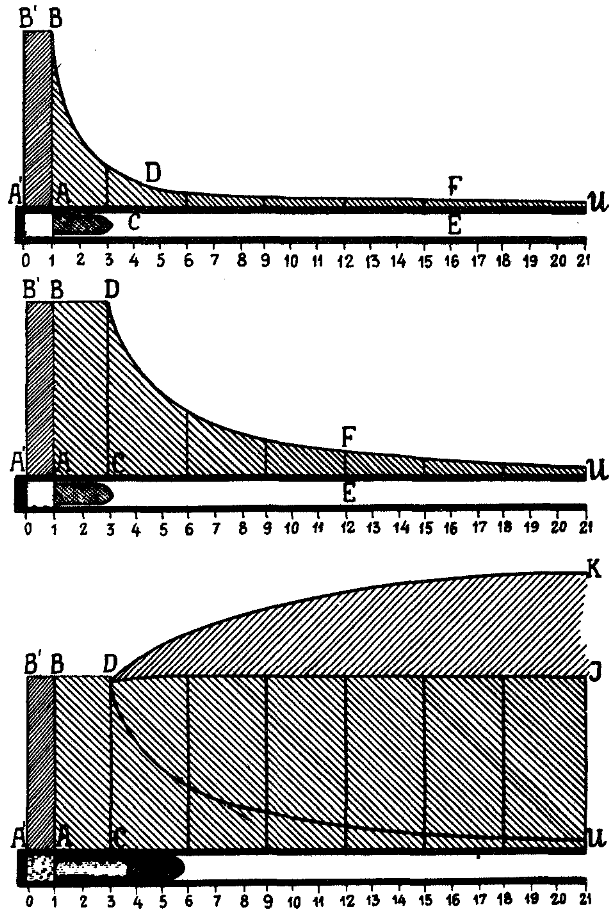


FIGURE 112. The theory of rocket-assisted artillery projectile

The pressure now attains the same value (EF) when the projectile end has moved to a distance of 12 calibers from the barrel bottom, as when it had moved to a distance of 3 calibers in the first example (DC Figure 112, top). The area of the expansion diagram will be three times larger, the total work done being 9.5 units ($3\times 2.5+2$). The muzzle velocity will be $\sqrt{9.5}=3.1$ units instead of 1.29 units in the first example. A large increase in the barrel length thus gives better results than in the first example. An "ideal state" still exists when the projectile end has moved to a distance of 3 calibers from the barrel bottom, i. e., the pressure has not yet decreased. Another 3.2 units of work are added while the projectile end moves from a distance of 3 calibers from the barrel bottom to one of 12 calibers. Further 1.84 units of work are added up to a distance of 48 calibers. Altogether 7.25 units of work are obtained when the barrel length is 60 calibers. Further increase of the work obtained is very slow and small; naval guns nowadays have barrel lengths of not more than 50–60 calibers. This gives a barrel length of 18 m for a 30 cm gun.

Guns firing as in the second example impart to the projectile a velocity which is 2.66 times higher than that in the first example (1,323 m/sec instead of 500 m/sec) at the same bore.

These considerations show that it is advisable to maintain a constant barrel pressure; a solution of this problem is given in the third example. (Figure 112, bottom).

Consider again the same barrel, but with a larger case containing slowly burning gunpowder, as in the second example. The projectile contains additional gunpowder at its rear in a case which is initially closed. The amount of gunpowder at the bottom of the barrel should be sufficient to impart a certain velocity to the projectile together with the cartridge as in the second example, so that a constant pressure is maintained until the projectile end has moved to a distance of 3 calibers from the barrel bottom.

Thereafter the pressure curve would drop (*DU*). At this instant the case should open, the gunpowder ignite, and the gas escape to the rear as in a rocket. We assume for the sake of simplicity that the amount of gunpowder in the case is sufficient to maintain a constant pressure in the barrel until the projectile leaves the muzzle (line ***DJ***). The contents of the case should be used up at this instant. We thus have a simple rocket.
122 The work performed by the gas at constant pressure is equal to the product of this pressure by the barrel length. The muzzle velocity will in this case be equal to seven units, against 1.29 units in the first example. The length considered is 49 units in a barrel of 50 caliber length, since one unit is taken up by the charge. Note that the mass of the projectile decreases as the propellant is burnt, so that the velocity will increase. This corresponds to an increase in the thrust if the mass of the projectile is assumed to be constant (curve *DU*).

We can attain any desired velocity if the mass of the gunpowder in the
123 case is large in comparison with the mass of the projectile.

Unfortunately, technical possibilities are limited in this respect. All types of gunpowder have a small specific weight, approximately equal to unity. Assuming the projectile to have a specific weight of 15, the gunpowder case would have to be 15 times longer than the projectile in order to have the same mass; this is difficult to achieve.

Nevertheless, the muzzle velocity of a gun having a barrel length of 60 calibers can be considerably increased in this way. It is possible to imagine a 40 cm gun with a barrel length of 24 m and a muzzle velocity of 2,000 m/sec.

Larger vertical and horizontal ranges can be obtained with multistage rocket shells. A large projectile is fired from the gun; it explodes at a certain height, part of its shell remains intact, and a second, smaller rocket is ejected from the latter. This second stage has a higher velocity, due to its smaller mass and the energy imparted to it by the explosion. This rocket can have a greater range, due to the lower resistance of the air at great altitudes. A further increase in speed and range is possible if more stages are provided.

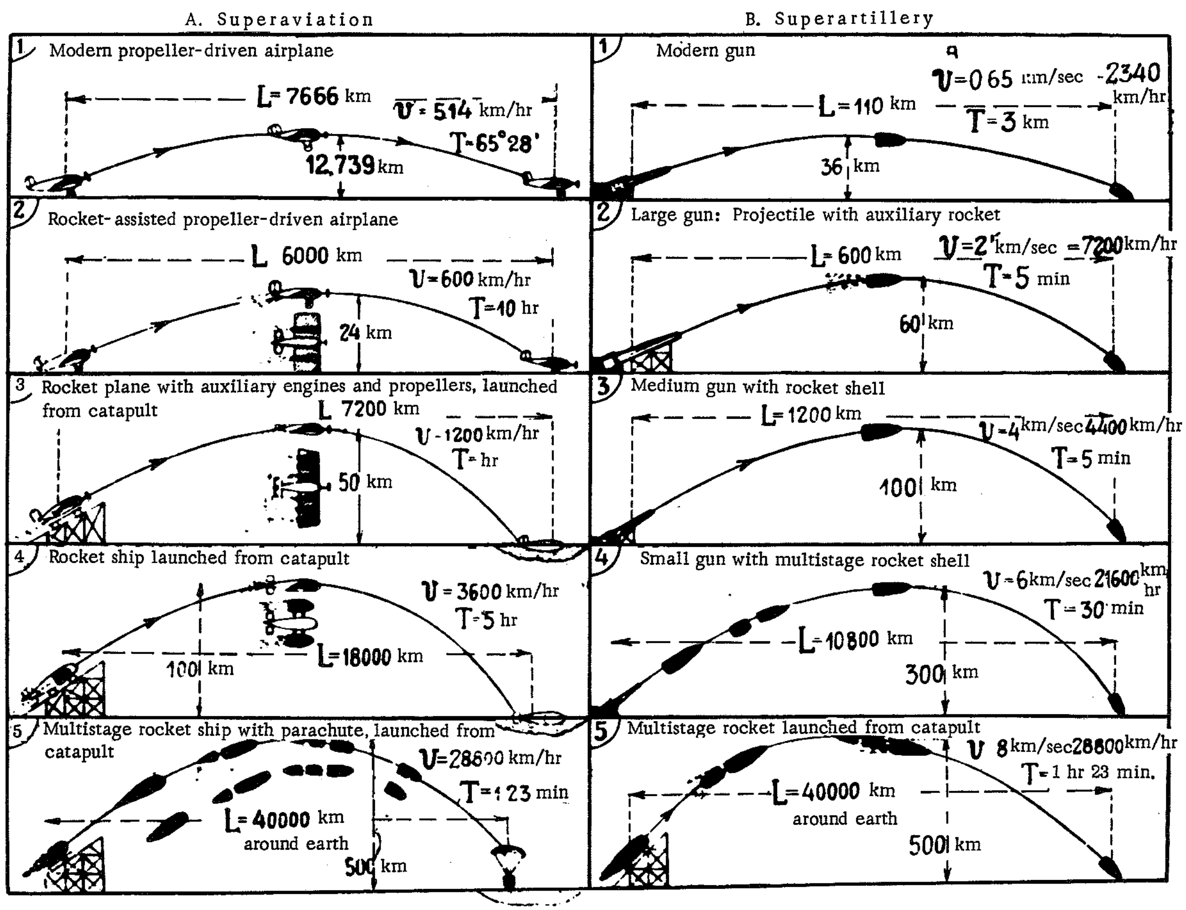


FIGURE 113. Flight of airplanes in superaviation and of projectiles in superartillery (schematic)

Some military authorities assume that the Germans may have used such projectiles when shelling Paris.

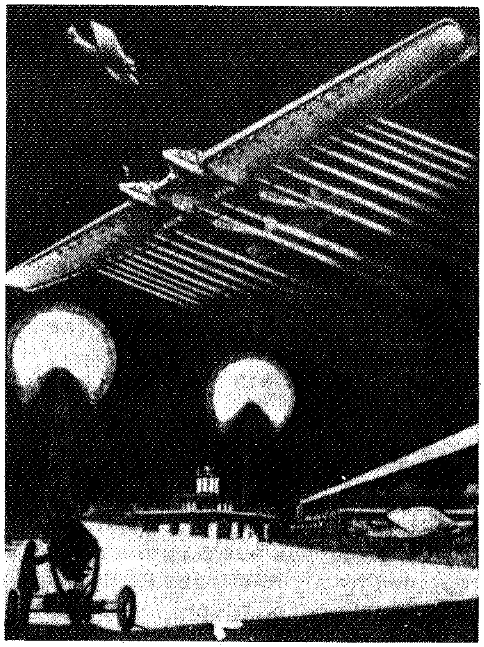


FIGURE 114. Future rocket plane

CONCLUSION

Summarizing the work done in superaviation and superartillery, we come to the conclusion that both are similar as regards the problem posed: how to transport a given load over a given distance in a minimum time. The solution consists in the use of reaction engines.

Figure 113 shows the similarity of the proposed development of aviation and artillery. The left-hand side of the diagram shows five stages referring to superaviation, namely:

124 1) A modern propeller-driven airplane together with all records attained.*

2) A future propeller-driven airplane with two auxiliary rockets.

3) A rocket plane with auxiliary engines driving propellers. The plane is launched from a catapult and lands on water. A rocket airplane without engines and propellers may form the transition to the next stage. Such a plane would represent a flying wing with retractable parts, would be launched from a catapult, and would land on water.

Figure 114 is an overall view of such a future plane, while Figure 115 is a cross section through the wings.

* At the time of printing this book the velocity record stood at 575 km/hr and the maximum range at 7,840 km.

4) A rocket ship launched from a catapult. Rocket engines are secured to the sides of the fuselage. The rocket ship is launched from a catapult and ordinarily lands on water, using retro-rockets. Emergency landings are possible with the aid of a parachute.

FIGURE 115. Cross section of rocket plane

5) A multistage rocket ship consisting of a spindle-shaped fuselage containing the rocket engines.

The rocket ship is launched from a catapult. The rocket engines operate during the flight. Their casings (stages) are discarded as the fuel is burnt up, in order to reduce the weight. The last stage lands on water by means of a parachute.

The right-hand side of Figure 113 shows the suggested stages of the development of superartillery:

1) A German gun which shelled Paris.

2) A future large gun. The projectile contains an auxiliary rocket.

3) A medium gun. The rocket in the projectile is enlarged.

4) A small gun. The projectile is of the multistage rocket type. Its parts are discarded as the fuel is burnt up during the flight.

5) The gun is replaced by a guiding and launching catapult. The missile consists of several rocket stages which are discarded during the flight as the fuel is burnt up.

125 *Chapter 3*

THE ATMOSPHERE

The atmosphere surrounding the earth supports airplane wings, ensures propeller thrust, and thus permits motion in the air. At the same time it gives rise to a resistance, causing the so-called drag. The drag increases with the air density and the flight speed.

Computation of the motion of airplanes in the terrestrial atmosphere necessitates a knowledge of:

1) its boundaries,
2) its composition, and
3) its properties.

Study of the atmosphere is the subject of meteorology; of the upper layers in particular, of aerology. The latter employs recording instruments carried by kites, sondes, airplanes, airships, and fixed balloons.

However, these methods permit only relatively low altitudes to be reached. Higher altitudes might be attained through the use of rockets which could be used for the following investigations:

1) Daily determination of the vertical pressure and temperature gradients; 2) determination of the wind velocities; 3) determination of the density and chemical composition of the air; 4) studies of the aurora borealis; 5) investigations with the aid of gyroscopes, of the origin of α, β and γ rays and their variation with the position of the sun, as well as of the ultraviolet part of the spectrum; and 6) investigations of the region of geocoronium, a gas which is not found on the surface of the earth. Detection of this still hypothetical gas will be of great importance for the development of astronomy, physics, and meteorology.

8. THE PROPERTIES OF THE ATMOSPHERE

Figure 116 shows the heights reached by man himself and by the instruments and missiles he has launched. The altitude record is held by a balloon carrying an automatically recording instrument, and probably also by projectiles fired from superlong-range guns. Figure 117 is a schematic section through the terrestrial atmosphere. The altitudes of clouds, meteorites, auroras, etc., are given.

(126)

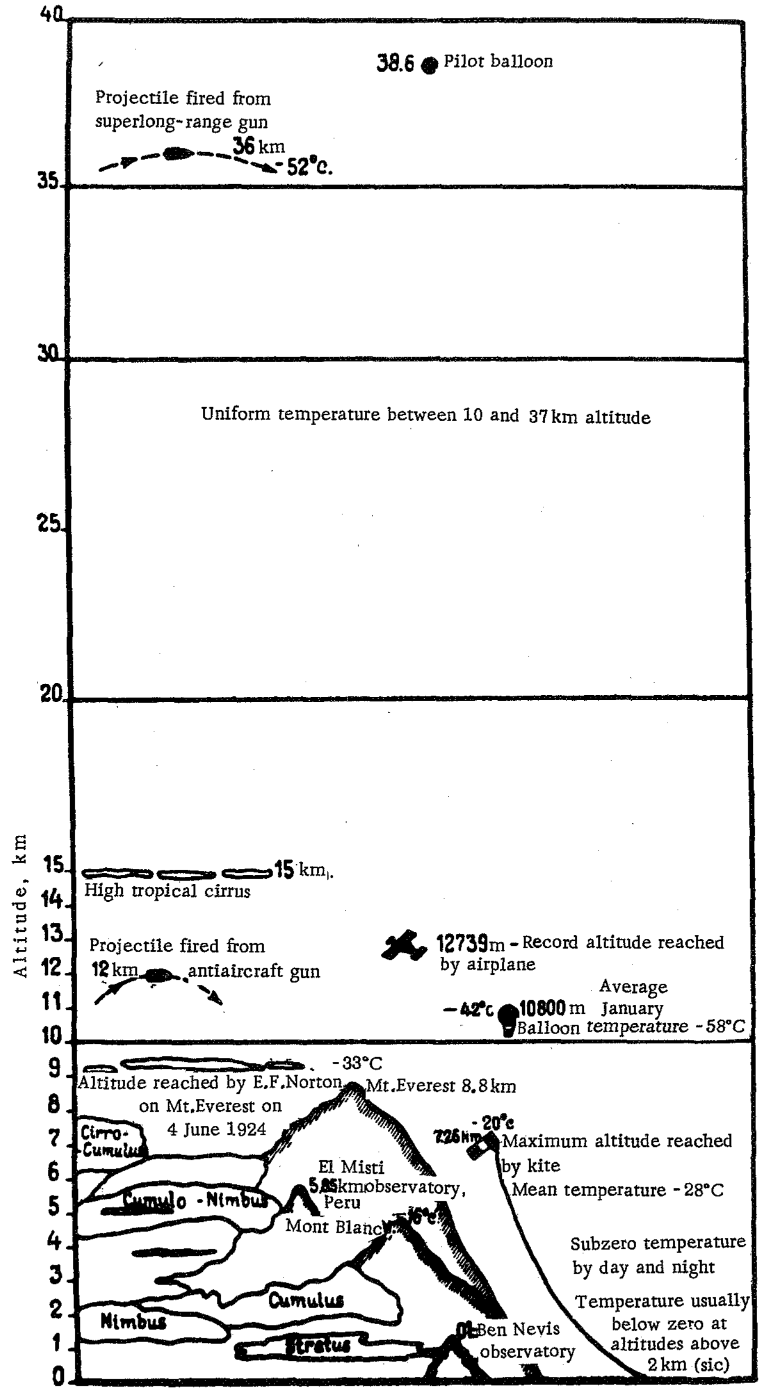


FIGURE 116. Heights attained by man and his missiles

The height of the atmosphere

The air consists of gas molecules having different velocities. However, their velocities are not great enough for escape from the gravitational
127 field of the earth into space. The average velocity is approximately $v = 500$ m/sec. A small percentage has velocities of up to 3,500 m/sec. From these values we can find the height to which these molecules rise, using the formula of mechanics:

$$h = \frac{v^2}{2g},$$

where g is the gravitational acceleration, taken as 10 m/sec^2. We thus obtain $h = 12.5$ km for $v = 500$ m/sec and $h = 602.5$ km for $v = 3{,}500$ m/sec.

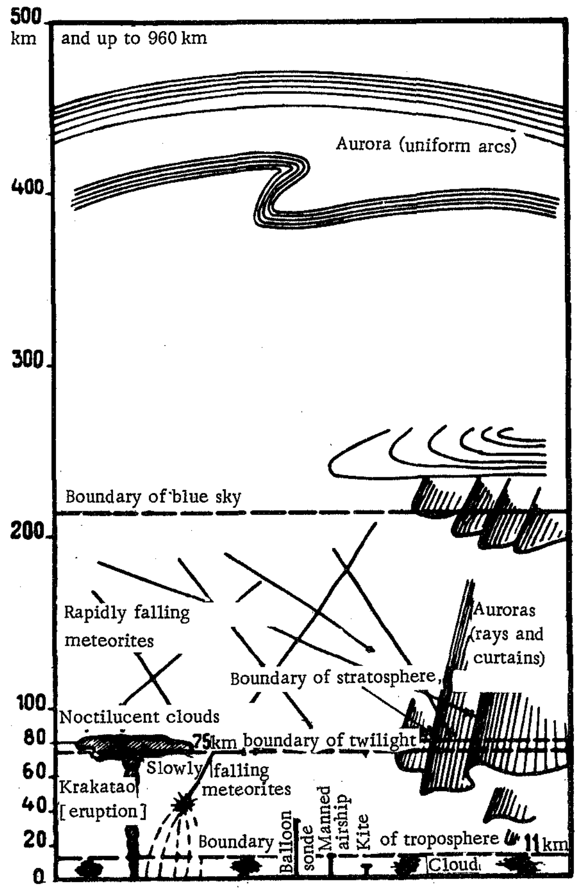


FIGURE 117. Section through atmosphere

If the particles of the air moved at the escape speed ($v = 11{,}800$ m/sec), the limit of the atmosphere would be at an altitude of

$$h \cong 7{,}000 \text{ km},$$

128 with $g = 10$ m/sec^2. This is approximately equal to the radius of the earth.
The composition of the atmosphere in % by volume is given in Table 27.

TABLE 27. Composition of atmosphere (in % by volume)

Altitude, km	Barometric pressure, mm	Argon	Oxygen	Nitrogen	Helium	Hydrogen	Geocoronium	Water vapor	CO_2
								in place of other constituents	
0	760	0.937	20.9	78.1	0.0005	0.0033	0.00058	1.20	0.03
20	41.7	0.00	15	85	0	0	0	0.02	0.01
40	1.92	–	10	88	0	1	0	0.06	–
60	0.106	–	6	77	1	12	5	0.15	–
80	0.0192	–	1	21	4	55	19	0.17	–
100	0.0128		0	1	4	67	29	0.05	–
120	0.0106	–	–	0	3	65	32	–	–
140	0.00900	–	–	–	2	62	36	–	–
200	0.00581	–	–	–	1	50	50	–	–
300	0.0329	–	–	–	–	29	71	–	–
400	0.00220	–	–	–	–	15	85	–	–
500	0.0162	–	–	–	–	7	93	–	–

This distribution is represented graphically in Figure 118, as given by A. Wegener in 1916. It is possible that actual observations will indicate a different composition of the upper layers of the atmosphere.* Thus, Lindeman and Dobson, who investigated the burning and extinction of shooting stars, as well as Vegard and Stormer, who studied the spectrum of the aurora borealis and compared it with the spectrum of nitrogen at low temperatures, concluded that solid nitrogen might exist as spray at high altitudes. They also found oxygen in the form of ozone, and determined the general composition of the atmosphere as shown in Figure 119.

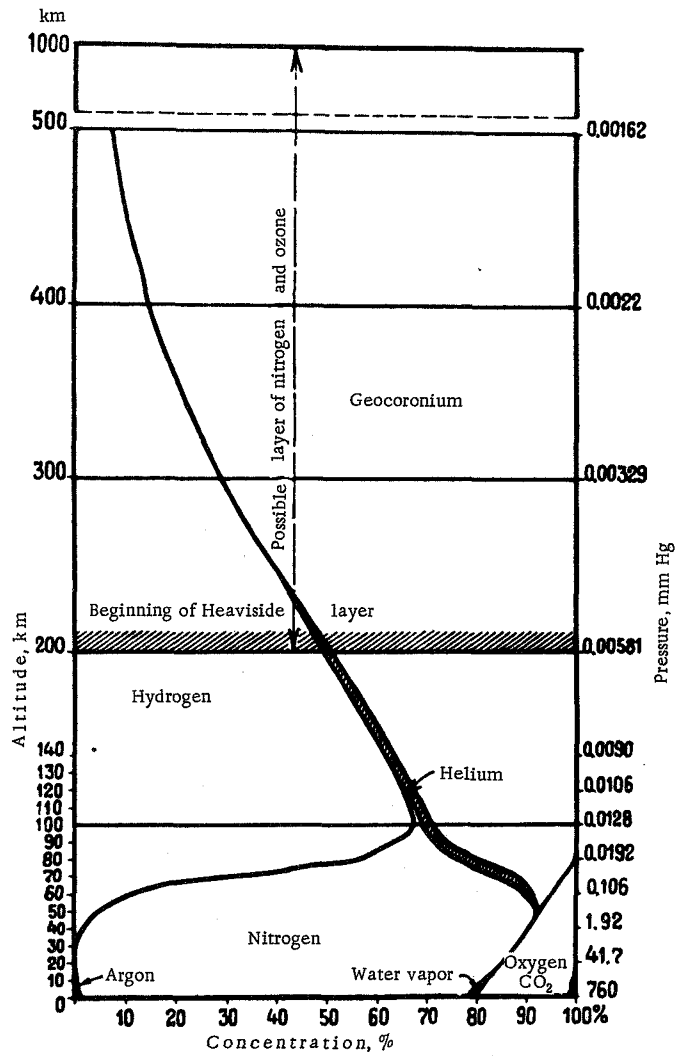
However, later investigations by MacLennan have shown that the same spectral lines are obtained by electrical discharges in helium containing admixtures of oxygen. There is thus no need to postulate the existence of solid particles of nitrogen in the upper layers of the atmosphere.

The latest data obtained by investigating the spectrum of the aurora borealis, occurring at altitudes of 300–1,000 km, indicate that oxygen and

* The lower and denser part of the atmosphere (up to a height of 11 km) is called troposphere, while the upper part (up to a height of 80 km) is called stratosphere.
The stratosphere is located higher at the equator than at the poles, and in general the level of the stratosphere shows yearly fluctuations. It is possible that a layer of ozone, heated by ultraviolet light to a temperature of +15°C, exists at large altitudes.

nitrogen are present at these heights. Oxygen is present in the form of ozone absorbing the short ultraviolet waves (less than 200 mμ long) which are harmful to organisms. This layer may also cause the phenomena ascribed to the Heaviside layer.

129



Composition of atmosphere, % by volume

FIGURE 118. Composition of atmosphere

We also mention the experiment carried out by the Norwegian scientists Stormer and Hansen to measure the altitude of the aurora borealis by photographing it simultaneously from two separate points in Norway at

a distance of approximately 258 km from each other. They found an average height of 100–150 km, and a maximum of 800 km. Birkeland and Stormer explain the origin of the aurora borealis by electronic eddies originating in sunspots. These eddies reach the earth and are deflected
130 toward the poles by the terrestrial magnetic field.

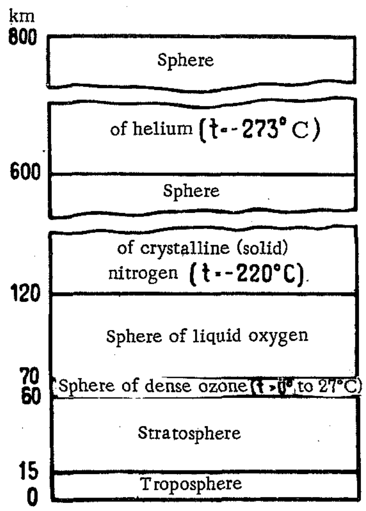


FIGURE 119. Composition of atmosphere

The temperature of the air decreases with increasing altitude. Observations indicate, however, that this decrease continues only up to approximately 12 km, beyond which (up to an altitude of 30 km) the air temperature is apparently constant (-54°C).

Figure 120 shows in solid lines the variation of the temperature in summer, winter, and averaged over the whole year, according to Hann's "Jahrbuch der Meteorologie" (Yearbook of Meteorology), 3rd edition, p. 126.

The broken line indicates the temperature variation according to the so-called "German Standard Atmosphere." A "Standard Atmosphere" in general is arbitrary, its properties being averages determined on the basis of many experiments carried out both in summer and in winter. It is used for the sake of convenience in comparing the results of airplane and
131 artillery-projectile flight tests at different altitudes.

Figure 121 represents the variation of the temperature with the altitude, in accordance with the U. S. Standard Atmosphere, corresponding to the latitude of 40°N. The curves are given for summer, winter and the yearly average. The diagram also shows, for comparison, the straight line representing the Toussaint formula $t = 15 - 0.0065Z$, where t is the temperature, °C, and Z is the altitude, m. The temperature is assumed to be constant (-55°C) at altitudes above 12 km.

(130)

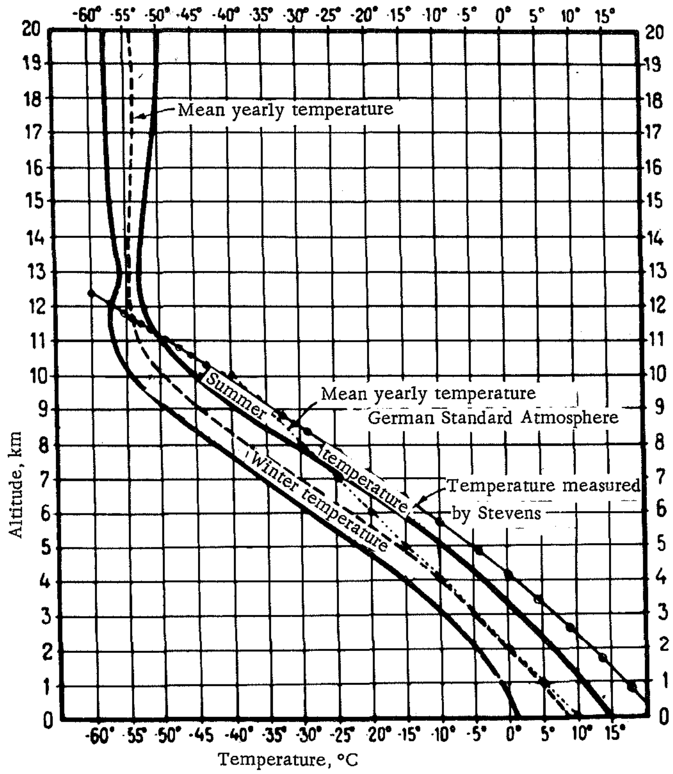


FIGURE 120

Note: As an historical reference we present in Figure 122 the temperature variation with altitude, suggested in 1903 by Tsiolkovskii in his paper "Issledovanie mirovykh prostranstv reaktivnymi priborami" (Investigations of Space with Rocket Instruments), published in Nauchnoe Obozrenie, 1903, No. 5.

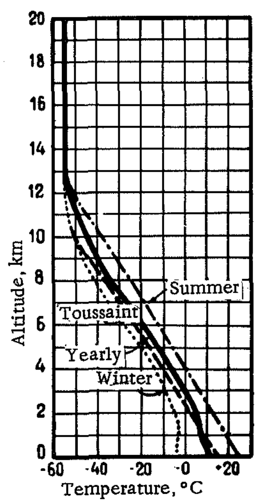


FIGURE 121

The air pressure decreases with increasing altitude. This variation, in accordance with the U. S. Standard Atmosphere (latitude 40°N), is represented in Figure 123 (up to an altitude of 20 km) (curves ab) for winter and summer. The diagram also shows the continuation of these curves up to an altitude of 40 km. For altitudes above 40 km see Figure 118 (right-hand scale).

Figure 124 shows the variation of the pressure of water vapor (summer, winter, and yearly average) in accordance with the U. S. Standard Atmosphere.

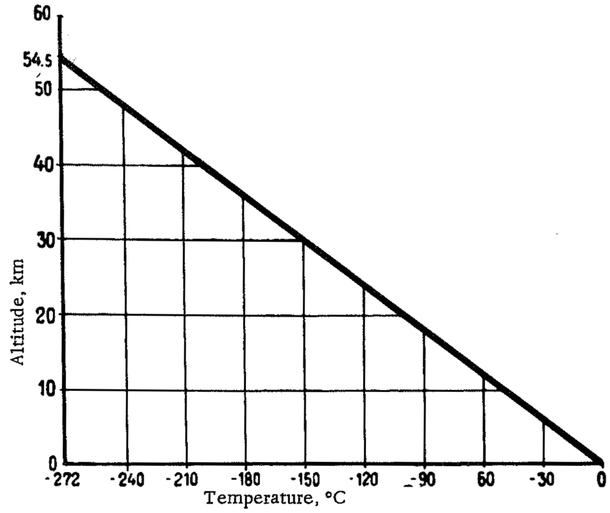


FIGURE 122

132

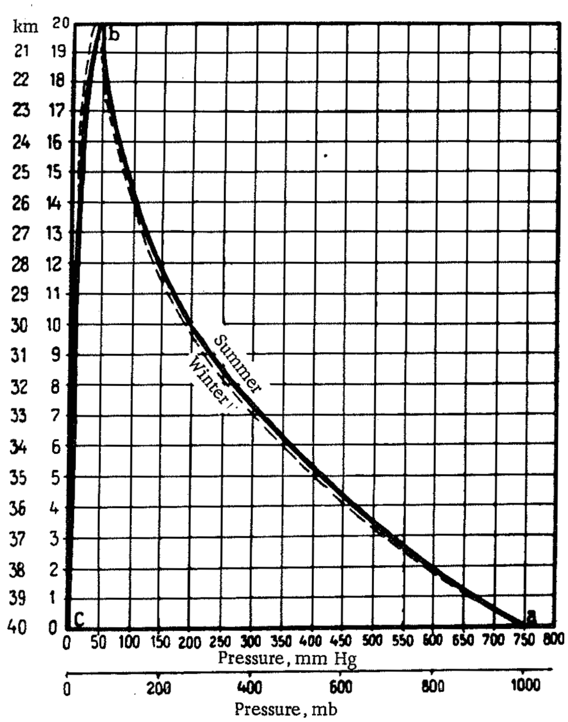


FIGURE 123

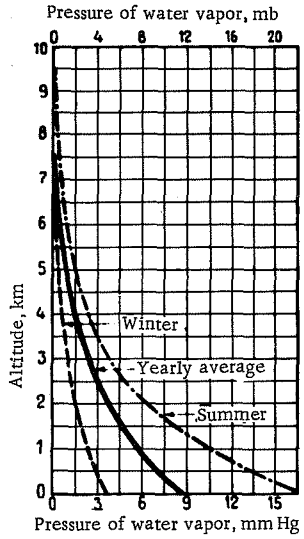


FIGURE 124

133 Table 28 gives the numerical values of these magnitudes.

TABLE 28. U.S.Standard Atmosphere (yearly averages for latitude of 40°N)

Altitude, m	Pressure		Temperature		Pressure of water vapor		Density	
	mb	mm	°C	°K	mb	mm	% of standard	kg/m³
0	1,017.0	763.0	11.5	284.5	11.5	8.5	95.9	1.240
500	957.5	718.0	9.5	282.5	9.5	7.0	91.0	1.176
1,000	901.5	676.0	8.0	281.0	7.5	5.5	86.2	1.114
1,500	848.5	636.5	6.0	279.0	6.0	4.5	81.7	1.056
2,000	798.0	598.5	4.0	277.0	5.0	4.0	77.4	1.001
2,500	750.5	563.0	1.5	274.5	4.0	3.0	73.5	0.951
3,000	705.0	529.0	-1.0	272.0	3.0	2.5	69.7	0.902
4,000	621.0	466.0	-7.0	266.0	2.0	1.5	62.8	0.813
5,000	546.0	409.5	-13.0	260.0	1.0	1.0	56.5	0.731
6.000	478.0	358.5	-20.0	253.0	0.5	0.5	50.8	0.657
7,000	417.0	313.0	-27.0	246.0	—	—	45.7	0.591
8,000	362.0	271.5	-33.5	239.5	—	—	40.7	0.527
9,000	313.5	235.0	-39.5	233.5	—	—	36.2	0.468
10,000	270.5	203.0	-44.5	228.5	—	—	31.9	0.412
11,000	232.5	174.5	-49.0	224.0	—	—	28.0	0.362
12,000	199.5	149.5	-53.0	220.0	—	—	24.4	0.316
13,000	171.0	128.5			—	—	21.1	0.273
14,000	146.0	109.5			—	—	18.0	0.233
15,000	125.0	94.0			—	—	15.4	0.200
16,000	107.0	80.5	-55.0	218.0	—	—	13.2	0.171
17,000	91.5	68.5			—	—	11.3	0.146
18,000	78.0	58.5			—	—	9.6	0.125
19,000	67.0	50.0			—	—	8.3	0.107
20,000	57.0	43.0			—	—	7.0	0.091

The density of air decreases with increasing altitude; this variation is represented in Figure 125 for altitudes up to 40 km. The mean density is referred to that of water, and to the latitude of 40°N. For instance, the density at sea level is $1{,}224:10^6=0.001224$ in summer, or the weight of one m^3 is 1.22442 kg (in the U. S. Standard Atmosphere).

The relative density (Δ) is the ratio of the air density at the altitude considered to the density at sea level.

Several formulas have been proposed for its determination:

1) The U. S. formula

$$\Delta = P_H - \frac{0.378e}{T_A} \cdot K,$$

134 where Δ is the density in % of the standard density; P_H is the barometric pressure, mb; e is the pressure of the water vapor, mb; T_A is the absolute temperature; $K = 0.26942$ is a constant.

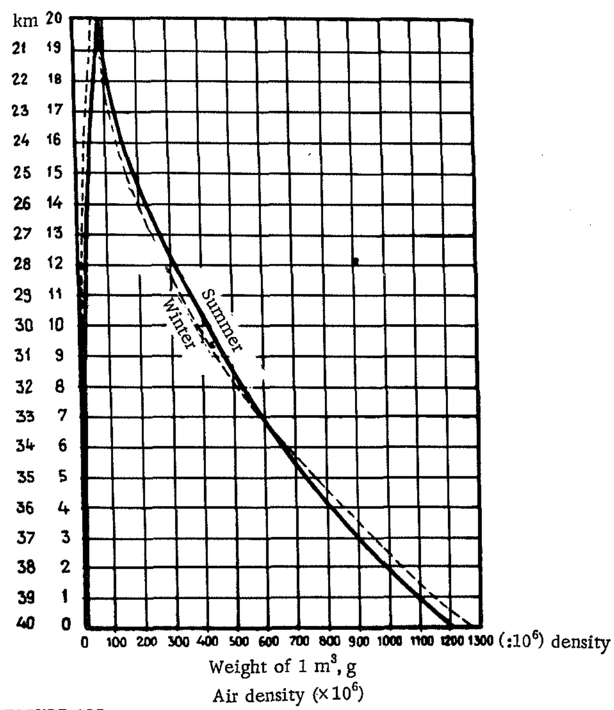


FIGURE 125

2) The German formula:

$$H_{met} = 21\,700 \ \lg \frac{1}{\Delta_1}$$

3) The German Standard formula:

$$\Delta=\frac{\rho_H}{\rho_0{}^2}=\left(1-\frac{0.005H}{T_0}\right)^{5.83}\quad \frac{P_H}{P_0}=\left(\frac{\rho_H}{\rho_0}\right)^{1.172}$$

4) $$H_{\text{met}}=21\,900\ \lg\frac{1}{\Delta_2}.$$

5) $$H_{\text{met}}=21\,850\ \lg\frac{1}{\Delta_2}.$$

6) $$\Delta=0.9^{\frac{H_{\text{met}}}{1\,000}}.$$

7) Knoller's formula:

$$\Delta=\frac{20-H_{\text{km}}}{20+H_{\text{km}}}.$$

8) The French formula: $\Delta=\left(\frac{228-0.0065\,H}{228}\right)^{4.256}$.

9) Tikhonravov's formula: $\Delta=\frac{T_0}{T_H}\times 0.1^{\frac{H}{A}}$.

10) According to Cranz the specific weight of air at an altitude H_{met} is:

$$\delta=\frac{1.294\cdot P_H}{760}\ \frac{273}{273+t_H}-\frac{0.174\ \frac{1}{2}\cdot E}{273+t_H}.$$

Here P_H is the barometric pressure at the altitude H and is determined from the formula $\lg P_H=\lg P_0-\frac{H}{18400\left(1-0.004\frac{t_H}{2}\right)}$,

where H is the altitude, m or km [sic]; T_0 is the absolute temperature at sea level; T_H is the absolute temperature at the altitude H; $A=18401.1+33.75\,(t_0+t_H)$, where t_0 and t_H are respectively the temperatures (°C) at sea level and at the altitude H; E is the vapor pressure of the water vapor saturating the air, at t_H; δ is the weight of 1 m^3 of air at the altitude H, kg.

135 11) According to Oberth the relative density of the air depends on the pressure, temperature, humidity, etc., and is a function of the altitude above sea level.

The value of Δ is precisely known for the upper layers of the atmosphere. For the lower layers it is given by the barometric formula. We can compute Δ in a first approximation from the formula:

$$\frac{1}{\Delta}=e^{\frac{H_{\text{met}}-H_{0\,\text{met}}}{h}}$$

Oberth takes h = 6,300 m and obtains $H-H_0=62\ 232$ m. Thus,

$$\frac{1}{\Delta}=e^{9.87881}=10^{4.2907}=19\,530.$$

In the final analysis this determination of $\frac{1}{\Delta}$ is doubtful. For the same altitude difference of 62,232 m, Oberth therefore assumes a 60-times larger value of the density, so that

$$\frac{1}{\Delta}=e^{\frac{62233}{h}}=10^{4.2907-\lg 60}=10^{2.5125},$$

whence $h=10{,}757$ m.

12) Hohmann's formula. The air pressure is assumed to vary with the altitude according to the formula (Figure 126):

$$P_H=P_0\left(\frac{y}{H'}\right)^n, \qquad (1)$$

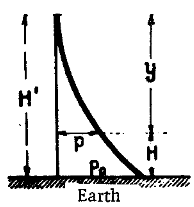


FIGURE 126

where $H'-y=H$ is the altitude above the sea level; H' is the altitude at which $P=0$; P_0 is the pressure at sea level.

Differentiating (1), we obtain:

$$\frac{dP_H}{dy}=\frac{nP_0}{H'^n}y^{n-1}.$$

However, on the other hand $dP_H=\delta\cdot dy$ or $\frac{dP_H}{dy}=\delta$, so that

$$\delta=\frac{nP_0}{H'^n}y^{n-1}.$$

At sea level $y=H'$; $P_H=P_0$ and $\delta_0=\frac{nP_0}{H'}$. Hence,

$$n=\frac{\delta_0}{P_0}\cdot H'.$$

Thus,

$$\delta=\frac{\delta_0}{P_0}\cdot H'\frac{P_0}{H'^n}\cdot y^{n-1}=\delta_0\left(\frac{y}{H'}\right)^{n-1}$$

$$\delta_0=1.293 \text{ kg/m}^3.$$

Setting $P_0=0.76 \text{ m}\cdot 13{,}600 \text{ kg/m}^3=10{,}330 \text{ kg/m}^2$, we obtain,

$$\frac{\delta_0}{P_0}=\frac{1.293 \text{ kg/m}^3}{10\,330 \text{ kg/m}^2}=\frac{1}{8000 \text{ m}}=\frac{1}{8 \text{ km}}.$$

Assuming the height of the atmosphere to be $H'=400$ km, we have

$$n=\frac{400}{8}=50;\ n-1=49;$$

$$\delta=1.293\cdot\left(\frac{y}{H'}\right)^{49}.$$

136 Table 29 gives δ for values of y between 400 km and 0 (Figure 127).

The broken line in Figure 127 shows the relative density of the air as a function of altitude. The full line represents the corresponding variation assumed by Arrhenius, and after him by Goddard and K. Tsiolkovskii in computing the drag of interplanetary spaceships.

(135)

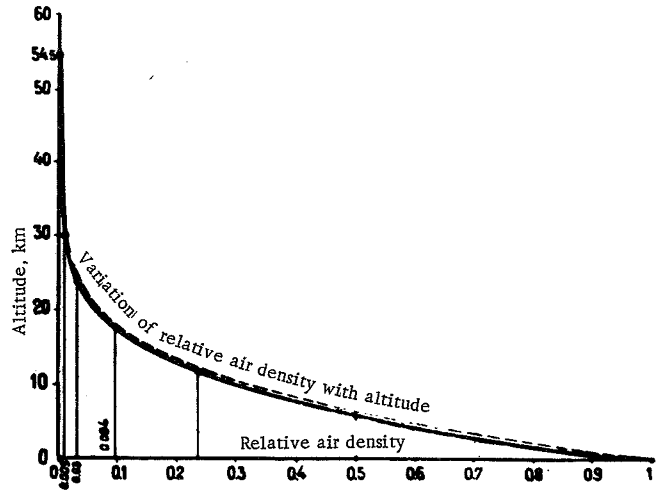


FIGURE 127

TABLE 29. Variation of specific weight of air with altitude

$H' - y$ km	Specific weight of air, $\delta = kg/m^3$	$H' - y$ km	Specific weight of air, $\delta = kg/m^3$	$H' - y$ km	Specific weight of air, $\delta = kg/m^3$
0	1.3	45	0.00376	110	0.000000185
1	1.15	50	0.00187	150	0.00000000013
2	1.00	55	0.000915	200	0.0000000000000023
3	0.90	60	0.000448	400	0.000000000000000000
4	0.80	65	0.000217		
5	0.70	70	0.0001025		
6	0.62				
8	0.48				
10	0.375	75	0.0000497		
15	0.215	80	0.0000230		
20	0.105	85	0.0000106		
25	0.055	90	0.0000049		
30	0.0283	95	0.0000022		
35	0.01464	100	0.00000098		
40	0,0074	105	0.000000423		

The density of the hypothetical ether filling space was assumed by K. Tsiolkovskii (cf. his book "Kineticheskaya teoriya sveta" (The Kinetic Theory of Light), Kaluga, 1910, p. 27) to be $1/(12 \cdot 10^{21})$ of the density of water, or $1/(16 \cdot 10^{18})$ of the density of air. Its vapor pressure was computed as 0.6 g/m^2, and the velocity of its atoms, $500 \cdot 16^6$ m/sec.

The French Standard Atmosphere is used in the USSR. This differs insignificantly from the U. S. Standard Atmosphere and is now considered as international. We shall now present formulae for the variations of the temperatures, relative pressure, and relative density of the air with the altitude H_{met} according to this international standard atmosphere.*

* Confirmed for the USSR by the Scientific and Technical Commission of the Armed Forces (cf. TsAGI Publications, No. 42, p. 161).

137 Let γ_0 and γ_H be the specific weights of air ($\gamma_0 = 1{,}2255\ \mathrm{kg/m^3}$).

We thus have $\Delta = \frac{\gamma_H}{\gamma_0}$ and $H = 44{,}300 - 42{,}230\,\gamma_H^{0.23^{r}}$

Altitudes between 0 and 11,000 m

Temperature:

$$t_H = 15 - 0.0065\,H.$$

Relative pressure:

$$\frac{P_H}{P_0} = \left(\frac{H}{44\,300}\right)^{5.256}.$$

Relative density:

$$\Delta = \frac{\delta_H}{\delta_0} = \left(1 - \frac{H}{44\,300}\right)^{4.256}.$$

Altitudes above 11,000 m

$$t_H = t_{11\,\mathrm{km}} = -56.5\ \mathrm{C}^\circ = \mathrm{const.}$$

$$\frac{P_H}{P_{11\,\mathrm{km}}} = \Delta = \frac{\delta_H}{\delta_0} = e^{-\frac{H - 11\,000}{6\,340}}$$

The values obtained from these formulae are presented in Table 30.

TABLE 30

Altitude H, m	Pressure P_H, mm Hg	Relative pressure $\frac{P_H}{P_0}$	Temperature t_H, °C	Specific weight γ_H of air, kg/m³	Relative air density $\frac{\delta_H}{\delta_0}$
0	760	1.000	+15	1.225	1.000
500	716	0.942	+ 11.75	1.1667	0.9528
1,000	674.1	0.887	+ 8.5	1.1120	0.9074
1,500	634.2	0.834	+ 5.25	1.0584	0.8637
2,000	596.2	0.784	+ 2.0	1.0068	0.8216
2,500	560.1	0.737	- 1.25	0.9572	0.7811
3,000	525.8	0.6918	- 4.50	0.9094	0.7420
3,500	493.2	0.6490	- 7.75	0.8634	0.7046
4,000	462.3	0.6082	- 11	0.8193	0.6686
4,500	432.9	0.5696	- 14.25	0.7770	0.6340
5,000	405.1	0.533	- 17.5	0.7363	0.6008
5.500	378.7	0.4983	- 20.75	0.6972	0.5689
6,000	353.8	0.4655	- 24.0	0.6598	0.5384
6,500	350.2	0.4344	- 27.25	0.6240	0.5091
7,000	307.9	0.4051	- 30.5	0.5896	0.4810
7,500	286.8	0.3773	- 33.75	0.5567	0.4542
8,000	266.9	0.3512	- 37.0	0.5252	0.4285
8,500	248.1	0.3265	40.25	0.4952	0.4040
9,000	230.4	0.3032	- 43.5	0.4664	0.3806
9,500	213.8	0.2813	- 46.75	0.4388	0.3580
10,000	198.2	0.2606	- 50	0.4127	0.3367
10,500	183.4	0.2414	- 53.25	0.3876	0.3147
11,000	169.4	0.2299	- 56.5	0.3636	0.2967

Altitudes above 11,000 m

138

H, m	p_H, mm	$\frac{p_H}{p_{11}}$	t_H, °C	γ_H, $\frac{kg}{m^3}$	$\frac{\delta_H}{\delta_{11}}$	$\frac{p_H}{p_0}$	$\Delta = \frac{\delta_H}{\delta_0}$
11,000	169.4	1.000	-56.5	0.3636	1.000	0.2229	0.2967
11,500	156.6	0.9241	—	0.3360	0.9241	0.2060	0.2742
12,000	144.6	0.8541	—	0.3104	0.8541	0.1903	0.2533
12,500	133.7	0.7893	—	0.2869	0.7893	0.1759	0.2341
13,000	123.7	0.7299	—	0.2653	0.7299	0.1627	0.2165
13,500	114.2	0.6441	—	0.2452	0.6741	0.1503	0.2001
14,000	105.6	0.623	—	0.2266	0.623	0.1389	0.1849
14,500	97.5	0.5768	—	0.2093	0.5758	0.1283	0.1708
15,000	90.14	0.5321	—	0.1935	0.5321	0.1186	0.1579

As reference we also present data on the German Standard Atmosphere (Table 31).

TABLE 31

Altitude H, m	Absolute temperature T, °K	Pressure, kg/m²	Pressure, mm Hg	Density	
				kg/m³	$\frac{\rho_H}{\rho_0}$
-500	285.5	10,990	808	1.312	1.050
0	283.0	10,360	762	1.250	1.000
500	280.5	9,750	717	1.188	0.950
1,000	278.0	9,180	675	1.127	0.902
1,500	275.5	8,620	634	1.069	0.855
2,000	273.0	8,100	596	1.013	0.810
2,500	270.5	7,610	560	0.960	0.768
3,000	268.0	7,140	525	0.910	0.728
3,500	265.5	6,700	493	0.862	0.690
4,000	263.0	6,280	462	0.815	0.652
4,500	260.5	5,890	433	0.770	0.616
5,000	258.0	5,510	405	0.729	0.583
5,500	255.5	5,160	379	0.689	0.550
6,000	253.0	4,820	355	0.651	0.521
6,500	250.5	4,510	332	0.614	0.491
7,000	248.0	4,210	310	0.579	0.463
7,500	245.5	3,930	289	0.546	0.437
8,000	243.0	3,660	270	0.515	0.412
8,500	240.5	3,400	250	0.488	0.386
9,000	238.0	3,160	233	0.454	0.364
9,500	235.5	2,950	217	0.427	0.342
10,000	233.0	2,770	204	0.405	0.324

139 Figure 128 has been copied from "Die Rakete," Aug. 1927. It represents the air density as a function of the altitude. On the basis of most recent investigations it has been assumed that the air density at an altitude of 50 km is 1/10,000 of that at sea level.

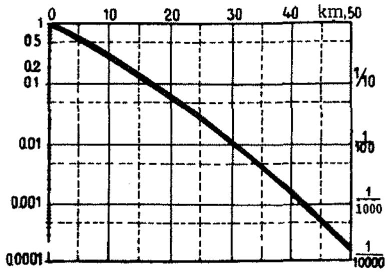


FIGURE 128

9. THE RESISTANCE OF THE AIR *

The influence of the air resistance during the ascent of a missile

Hohmann has determined the velocity (v) at which a missile rises to different altitudes. His results and the corresponding values $\delta \frac{v}{g}$ entering in his formula (p. 142) are given in Table 32 (cf. Figure 126).

The resistance of the air may be neglected at altitudes above 50 km for the velocities given in the Table. Hohmann assumed as a mean at altitudes between 0 and 50 km,

$$\delta \frac{v^2}{g} = 12\ 000\ \mathrm{kg/m^2}.$$

The deceleration due to the resistance of the air is:

$$\Delta\beta = \frac{W}{m} = \delta \frac{v^2}{g} \cdot \frac{F}{m} \cdot i.$$

For his missile (cone) Hohmann assumed

$$\frac{F}{m} = \mathrm{const} = \frac{1}{600}\mathrm{m^3/kg \cdot sec^2};\ i = 0.12,$$

where m is the mass of the missile.

Hence,

$$\Delta\beta = \frac{12\ 000 \cdot 0.12}{600} = 2.4\ \mathrm{m/sec^2}.$$

The influence of the air resistance during the descent of a missile

Hohmann assumed that a missile arriving from interplanetary space enters the terrestrial atmosphere at a speed $v' = 11.1$ km/sec.

* In this section we only give a brief computation of the resistance of the air, referring those interested in details to our book "Teoriya reaktivnogo dvizheniya" (Theory of Rocket Propulsion).

Table 33 gives the air resistance corresponding to this speed, per m^2 of the cross-sectional area perpendicular to the flight path.

It is seen from the table that the resistance of the air layers at altitudes above 100 km may be neglected. We assume the permissible drag to be

$$W=\frac{\delta v^2}{g}=\frac{1.3\ 50^2}{9.8}=330\ \mathrm{kg/m^2},$$

which is frequently used in airplane design. This value, according to the
140 table, corresponds to an altitude between 75 and 100 km. Hohmann therefore suggested that landing take place not radially but in a spiral around the earth, approaching the latter more closely after each turn and exploiting for braking the resistance of the atmospheric layers traversed, until the path becomes circular. Final landing is in a glide, so that the missile must have braking and gliding surfaces as well as a rudder.

TABLE 32

$H'-y$	v^2 km²/sec²	$\frac{\delta v^2}{g}$, kg/m²
0	0.00	0
1	0.04	4,600
2	0.08	8,000
3	0.122	11,000
4	0.162	13,000
5	0.202	14,200
6	0.243	15,100
8	0.323	15,500
10	0.404	15,200
15	0.606	13,000
20	0.810	8,500
30	0.214	3,440
40	1.620	1,200
50	2.028	370
60	2.434	110
80	3.250	7.5
100	4.070	0.4

The resistance of the air according to Oberth

The resistance of the air (drag) is:

$$L=F\beta\gamma v^2,$$

where F is the midsection area of the rocket; β is the density of the air; γ is the form factor: for speeds $v<300$ m/sec, $\gamma=0.025$; for $v>410$ m/sec, $\gamma=0.05$; for $v=340$ m/sec (velocity of sound, $\gamma=0.025$ (sic); $\beta=\beta_0 e^{-\frac{h}{H}}$; H=7,400 m; h is the flight altitude.

Thus,

$$L = F \, . \, \beta_0 e^{-\frac{h}{H}} . \gamma \, v^2.$$

The following method of determining the drag of a body flying at a
141 uniform speed is proposed in "Die Rakete" of 15 August, 1927:
The drag is

$$G = \frac{\psi \cdot \gamma}{2g} . \, Fv^2,$$

where F is the midsection area of the body considered; g is the gravitational acceleration; γ is the specific weight of air, equal to $1.293\,\mathrm{kg/m^3}$ at sea level; ψ is the form factor of the body (equal to 0.5 for a sphere); v is the flight speed of the body. The flight speed varies directly with the ratio $\frac{G}{F}$.

TABLE 33

(140)

$H'-y$	$W = \delta \frac{v'^2}{g}$, kg/m^2
400	0.000000000
200	0.00000003
150	0.0017
110	2.4
105	5.5
100	12.7
95	28.5
90	63.4
85	137
80	297
75	640
70	1,320
65	2,780
60	5,720
55	11,800
50	23,900

Gavre's ballistic formula

The drag of an ogival projectile is

$$R = p \, . \, \Gamma,$$

where the acceleration is

$$\Gamma = \Delta'_0 \frac{a'^2}{p'} \sin \gamma \, e^{-hy} F(v) \, . \, 100.$$

Here $\Delta_0' = 1.208\ \mathrm{kg/m^3}$ is the specific weight of the air at sea level; a' is the bore (projectile diameter), m; p' is the mass of the projectile, kg; $h = 10^{-4}$; βe^{-hy}, $F(v)$, y and v are expressed in m and m/sec; p is the mass of the projectile [g]; Γ is the acceleration in c. g. s. units.

Denoting by S the midsection area of the projectile, we obtain the specific drag:

$$\frac{R}{S} = \frac{400}{\pi}\left(\frac{p}{p'}\right)\left(\frac{a'}{a}\right)^2 \Delta_0' \sin\gamma e^{-hy} F(v);$$
$$S = \frac{\pi a^2}{4},$$

where S is given in cm^2 and a in cm, with

$$a' = \frac{a}{100};\quad p = 1000\,p'.$$

Hence,

$$\frac{R}{S} = \frac{40}{\pi} \Delta_0' \sin\gamma\, e^{-hy} F(v).$$

Esnault-Pelterie's formula (for analysis of rocket flight)

The drag is:

$$R = KaSw^2,$$

where a is the air density, $\mathrm{g/m^3}$; S is the midsection area of the rocket; w is the rocket speed; K is the form factor ($K=1$ for a standard shape, $K=0.70$ for a plane, and $K = 0.106$ for a sphere).

Alco's method

Alco gives the following approximate method of determining the resistance of the air (cf. "Voina i Tekhnika," 1925, Nos. 234–5, p. 49),
142 which we present in view of the originality of his approach in introducing an atmosphere of uniform density; we shall, however, also indicate the mistakes he made.

The atmosphere would extend to a height of approximately 8 km if the density of the air were uniform and equal to that at sea level.

We shall determine the energy lost by a projectile traveling over a distance of X km through air of this density.

These losses depend on the initial energy and on the distance X, and are proportional to the latter.

We introduce the following notation: A_0 is the initial energy of the projectile; A is the energy of the projectile after it has traveled a distance X; dA is the energy lost while traveling a distance dX; α is a proportionality factor.

We may then write*

$$\frac{dA}{A} = -\alpha\, dX. \qquad (1)$$

The minus sign has been used since the energy A decreases with increasing distance X.

Integrating (1), we obtain

$$\lg A + C = -\alpha X.$$

At the beginning of the motion $X=0$, $A=A_0$ and $C=-\lg A_0$, so that

$$\lg A - \lg A_0 = -\alpha X.$$

or,

$$\lg \frac{A}{A_0} = -\alpha X,$$

whence,

$$\frac{A}{A_0} = e^{-\alpha X}. \qquad (2)$$

However, the energy is approximately proportional to the velocity squared.

Let v_0 and v be the velocity at the beginning and end of the motion, respectively. We thus obtain**

$$\frac{v^2}{v_0^2} = e^{-\alpha X}$$

or

$$v = v_0 \sqrt{e^{-\alpha X}}. \qquad (3)$$

We may set $\alpha = 0.07$, according to experiments with rockets in the lower layers of the atmosphere, so that (2) and (3) respectively become

$$A = A_0 \cdot e^{-0.07X} \qquad (4)$$

$$v = v_0 \sqrt{e^{-0.07X}}. \qquad (5)$$

We shall use (4) and (5) to solve two numerical examples.

* Here Alco made a mistake; this should read $\frac{dA}{A_0 - A} = -\frac{dX}{X}$ so that Alco's conclusions are wrong.

** For instance, the velocity of a 12" projectile decreases from 762 m/sec (in the muzzle) to 586 m/sec after a horizontal flight of 8 km. According to (3) this corresponds to $\alpha = 0.07$.

143 **Example 1. Determine the velocity at which a projectile, fired by the Germans from a distance of 110 km, hits Paris**

The energy lost by the projectile while it passes through the air depends on the resistance of the latter. Assuming both the elevation and the angle of fall to be 54°, we obtain the thickness of the air layer traversed by the projectile as $\frac{8}{\sin 54^\circ} \cong 10$ km, whence $X = 20$ km. Assuming a muzzle velocity $v_0 = 1{,}600$ m/sec, we obtain the unknown velocity by (5)*

$$v = 1600 \sqrt{e^{-0.07 \cdot 20}} = 794 \text{ m/sec.}$$

Example 2. Determine the flight speed of an interplanetary missile fired from a horizontal gun

Let a long gun be arranged horizontally, the atmosphere of uniform density extending to a height of 8 km (Figure 129). The missile then has to travel a distance of 320 km through the air, since

$$\sqrt{8.(6400+6400+8)} = 320.$$

By (4) the energy at the end of this distance is

$$A = A_0 e^{-0.07 \cdot 330} = A_0 e^{-23.1}$$

while the velocity is

$$v = v_0 \sqrt{e^{-23.1}}.$$

The escape speed is approximately 8 km/sec [sic], so that the muzzle velocity should be

$$v_0 = \frac{8}{\sqrt{e^{-23.1}}} = 830{,}000 \text{ km/sec},$$

which is almost three times the speed of light, and thus clearly impossible.

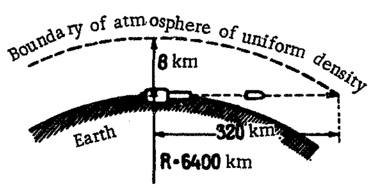


Figure 29.

* Cf. Figure 56 where $v = 707$ m/sec.

FEATURES OF SUPERAVIATION

The following problems arise when an airplane flies at high speed and high altitude where the air is rarefied and very low temperatures can be expected:

1. The human organism has to be protected against the effects of large accelerations during takeoff and large decelerations during landing.
2. Measures must be taken to ensure proper working conditions for human beings when the gravitational force is not felt, e. g., when the machine falls freely from a high altitude.
3. Correct breathing and temperature of human beings must be ensured at high altitudes.
4. Navigational methods have to be worked out.

Other problems may also arise, such as how to take off and land. These are considered in other books written by us, dealing in general with rocket flight.*

10. THE EFFECTS OF ACCELERATION

a) The influence of acceleration on human beings

Let an airplane pass from flight in a straight line into a curve. An inertia force will then appear, i. e., a centrifugal force equal to $p=\frac{Mv^2}{R}$, where M is the mass of the airplane, v is its speed, and R is the radius of the curve.

Let the airplane glide at a certain speed v, and then zoom rapidly, i. e., climb at a speed v_0, equal to the landing speed of the plane. The overload factor will then be $K=\left(\frac{v}{v_0}\right)^2$. Figure 130 gives the results of tests carried out in the U. S. A. in 1924–1925 in order to determine the overload factor of airplanes during the transition from diving to zooming. The full line indicates the test result, while the broken line represents the results obtained with the above-mentioned formula.

The experimental and theoretical results are in close agreement.

NACA Report No. 307, published in 1928, contains a description of the experiments carried out during the flight of a Curtiss-Hawk (F6C-4)
145 plane with wheeled undercarriage, in which the air pressure on the rudder
was determined during rapid turns. The acceleration attained a value of

* Cf. N.Rynin, "Rakety" (Rockets) and "Teoriya reaktivnogo dvizheniya" (Theory of Rocket Propulsion).

10.5 g during one experiment, causing serious sickness of the pilot. He had to stay in the hospital for a whole month after this flight, because of general conjunctivitis of both eyes and a nervous breakdown due to concussion of the brain and cerebral capillary hemorrhages. This may have been caused by the rapid variation of the centrifugal force when coming out of a dive (at a speed of 175 miles/hr) and zooming. No harmful effects on human beings were observed during these experiments when the acceleration attained 9 g for short periods.

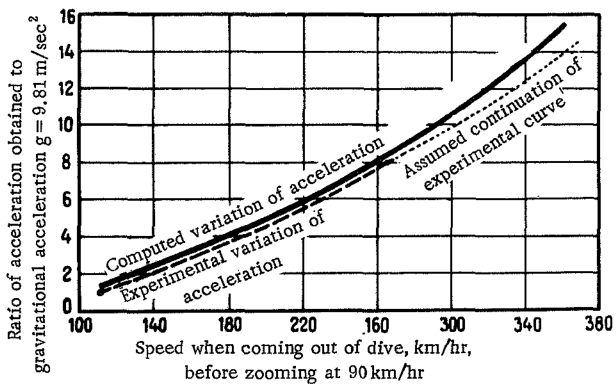


FIGURE 130

Interesting experiments to determine the influence of acceleration on the human organism were carried out in Italy with the aid of a catapult from which a hydroplane was launched. The plane had to acquire a speed of 100 km/hr over a distance of 15 m during less than 1 sec; this represents an acceleration which is more than three times larger than the gravitational acceleration. The experiments were initially (up to a speed of 32.1 m/sec) carried out with a catapult on the ground, having a length of approximately 80 m. A railroad track was laid along it, on which a carriage with the pilot was first accelerated and then decelerated. The pilot was examined before and after the experiments. Later experiments were carried out with a hydroplane launched from a catapult set up on a raft on water.

The test results are presented in Table 34.

The experiments showed that 1) launching an airplane from a catapult has no harmful effects on the pilot, 2) the pilot quickly returns to normal.*

Similar experiments were carried out near Brest in France by launching a hydroplane from a catapult. The catapult was 20.25 m long while the length of the launching path was 13 m. The final speed was 22 m/sec = 79.2 km/hr. The motor propelling the carriage with the hydroplane was driven by compressed air. The pilot seat could rotate
147 about a transverse horizontal axis and rested on rubber pads, so as to eliminate shocks. The acceleration at the end of the launching run was approximately 2 g. About 200 launchings were carried out altogether; all were successful.

* Cesare Talenti, Osservazzione sanitarie su alcuni piloti durante i lanci de velievoli a mezzo di catapulta (*Medical Observations of some Pilots during the Launching of Airplanes by means of Catapults*), Rivista Aeronautica, 1926, No.3, p.39.

TABLE 34

Date	Pilot	Operation of heart				Time of experiment	Acceleration, m/sec^2	Maximum speed, km/hr	Operation of heart				Analysis of urine
		hr	pulse/min	blood pressure					hr	pulse/min	blood pressure		
				systolic	diastolic						systolic	diastolic	
4–9–25	Carra	10.10	74	130	80	10.17	7.82	53.28	10.20	72	160	85	10 hr, sp.g. = 1023, no albumin or glucose
		11.35	78	140	80	11.40	15.85	67.32	11.42 11.47	72	150 150	90 80	12 hr, sp.g. = 1021, no albumin or glucose, traces of phosphoric-acid salts
5–9–25		10.50	75	130	80	11	23.2	82.08	11.3 11.13	70 72	190 150	85 80	10.15 hr, sp.g. = 1018, albumin and glucose +
		11.45	75	140	80	11.50	26.3	88.6	11.52 11.59	72	160 150	80 80	12 hr, sp.g. = 1021, albumin, glucose, and phosphoric-acid salts present
7–9–25		10.16	90	135	85	10.25	30.75	94.32	10.27 10.38 10.45	83	190 160 160	80 85 85	10.10 hr, sp.g. = 1023, albumin and glucose + 11.30 hr, sp.g. = 1021, albumin and glucose +, traces of phosphoric-acid salts
8–9–25		9.45	88	150	80	10	31.7	99.36	10.3 10.7	79	170 150	80 90	9.40 hr, sp.g. = 1021, albumin and glucose + 11 hr, ditto
7–9–25	Russo–Luigi	11.30	73	130	90	11.45	19.65	75.96	11.48	69	140	85	11.10 hr, sp.g. = 1014, no albumin 12 hr, sp.g. = 1009. Ditto
8–9–25		10.30	70	140	85	10.45	32.1	97.92	10.47 10.55	72	140 130	80 85	10.15 hr, sp.g. = 1016, albumin and glucose + 12 hr, Ditto
8–9–25	Missale	11	64	125	80	11.12	32	98.28	11.14 11.20	64	150 130	80 80	11 hr, sp.g. = 1021, albumin and glucose + 12 hr, Ditto + phosphoric-acid salts

A 20 m-long catapult was installed in 1929 on the German S. S. "Bremen" by Messrs. Heinkel. The acceleration attained 3 g during launching. The purpose of the installation was to speed up by almost one day the delivery of mail from Europe to America (Figures 131 and 132).

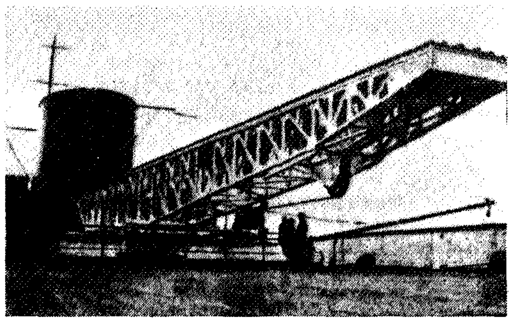


FIGURE 131. Heinkel catapult on S.S.Bremen (overall view)

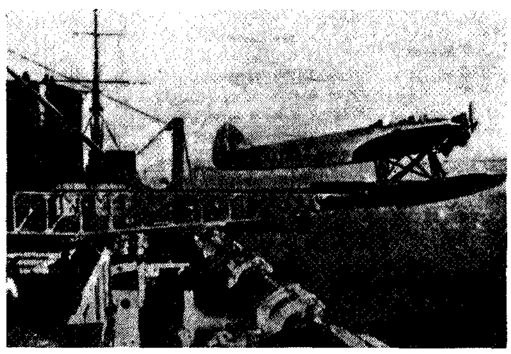


FIGURE 132. Heinkel catapult (launching of hydroplane)

The experiments described thus indicate that human beings are hardly able to withstand accelerations above 3—4 g for long durations without special protective devices. It takes approximately 12 seconds until a speed of 500 m/sec is attained at an acceleration of 4 g. Devices protecting human beings against the harmful effects of higher accelerations must be provided if this time is to be reduced.

K. Tsiolkovskii proposed the following experiment: A freshly laid chicken egg is placed in a strong vessel filled with salt water of such a
148 specific gravity that the egg floats in it. The egg is not damaged if the vessel is then thrown on the floor. (Trudy Obshchestva Lyubitelei Estestvoznaniya, Moskva, 1891).

The effects of large accelerations on dogs were investigated by Garsau. He placed dogs in a merry-go-round and rotated it at speeds of 4—6 turns/sec. Some dogs suffered brain damage as a result of the pressure on the skull. Some animals recovered, but some died. Autopsies showed that they had suffered from brain anemia and excessive filling of the blood vessels in the abdominal cavity. Flow of blood from part of the body to another, in particular from the brain, causes dizziness and fainting, and excessive filling of the blood vessels, in particular those in the interior, which extend easily. This eventually causes the blood vessels to burst.

b) Novel laboratories and new experiments

The effects of increased gravity, or the absence of it, may be investigated in new types of laboratories in which the following experiments are performed.

Laboratory rotating about a vertical axis. Such laboratories may have rectangular or oval cross sections (Figures 133 and 134). A person located in such a laboratory will experience different sensations at various speeds. Experiments with water, the growth of plants, etc., are of interest. The aim of the experiments is to investigate the phenomena occurring when the weight is increased.

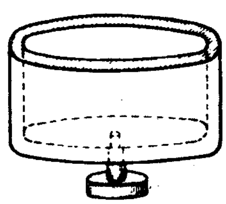


FIGURE 133

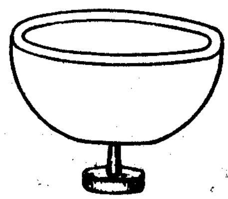


FIGURE 134

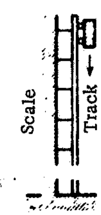


FIGURE 135

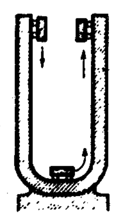


FIGURE 136

Laboratories for investigating effects of acceleration

In his book "Grezy o zemle i nebe" (Dreams of Earth and Heaven), K. Tsiolkovskii writes that he carried out experiments with cockroaches placed in a centrifuge. The weight of the cockroaches was increased 300-fold during the experiments, without the insects being hurt. Similar experiments were carried out on chickens which withstood a 500% increase in gravity without being harmed. Tests with human beings are better carried out at smaller angular velocities and larger radii, since the sensation of sickness is less in this case. For instance, we do not feel any sickness due to the rotation of the earth. A centrifuge having a radius of 100 m and a circumferential speed $v = 100$ m/sec makes one turn every 6.3 sec. The corresponding centripetal acceleration is 10 g.

Falling laboratory. Such a chamber may either fall along a wire or on a track (Figure 135), or be dropped from an airplane. A parachute may be used in the last case for braking. The experiments may first be carried out with animals and then with human beings. The aim of the experiments is to investigate the phenomena occurring at reduced gravity.

149

FIGURE 137. Homunculus' flight

Without experiments it is difficult to foresee the sensations experienced by human beings immersed in water or some other liquid. Even Goethe at the end of his "Dr. Faustus," Part II, Act II, describes some wonderful sensations and qualities of Homunculus, a person created artificially by Wagner in a phial. A condition of his existence was that he was kept constantly in a liquid filling the phial. The latter could fly with Homunculus who served as guide to Faustus and Mephistopheles in their wanderings. Homunculus himself is reborn in a different form and, at the advice of Proteus, breaks the phial and begins life in a different liquid, the sea.*

Homunculus:	Everything that I illuminate In this lovely liquid Is attractively beautiful.
Proteus:	Only in this liquid of life Does your light shine With beautiful ringing.
Thales:	It is Homunculus, seduced by Proteus. These are the symptoms of lordly desire. I sense the sound of fearful thunder, He will crash himself at the glistening throne. Now it burns, it flashes, it already pours.

Figure 137 shows Homunculus' flight in the phial filled with liquid, while he guides Faustus and Mephistopheles.

The sensations which a future space traveler enclosed in a vessel filled with water will experience in an interplanetary ship can be reproduced either in a laboratory or by placing such a vessel with a human being in a

* Collected Works of Goethe. English translation.

transparent cabin of an airplane. It is also possible for this person to control the flight of the airplane. Figure 139 shows the arrangement of such a vessel in an airplane, as suggested by C. Dienstbach in "The Flying Sensation," published in the Scientific American, 24 June 1916. Figure 138 shows such a person in a spacesuit.

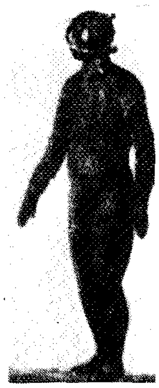


FIGURE 138. Pilot's impervious spacesuit

3. **Horseshoe-form laboratory** (Figure 136). The horseshoe-form laboratory consists of two vertical tracks connected by a band at the bottom. A carriage
150 which cannot be derailed travels on the track.

The carriage first descends on one track, is subjected to a large centrifugal force in the bend at the bottom, and finally ascends slowly on the other track. The purpose of the experiment is to investigate the phenomena occurring under increased and reduced gravity.

4. **Water laboratory.** A person, animal, or brittle object is immersed in water. The person is equipped with special eyeglasses and a tube for breathing. The experiment consists in letting the laboratory hit an obstacle; the person's perceptions and organs are studied during the impact and in the absence of gravity when the laboratory revolves. The bath itself may rotate.

Lastly, it is possible to build a chamber which rotates about two axes forming a universal joint and to investigate the effects on the organism of rotation about either axis.

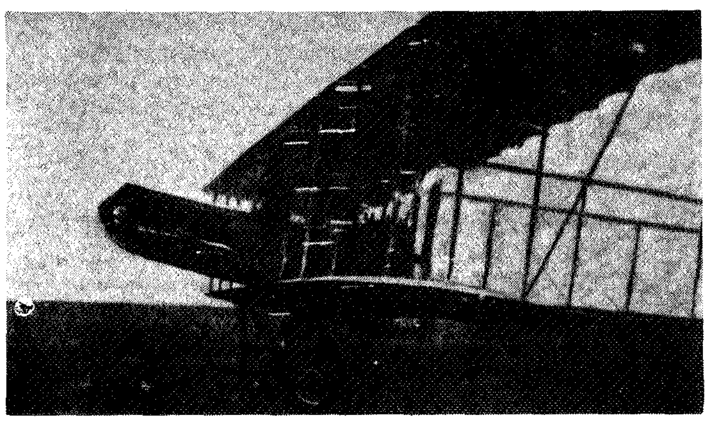


FIGURE 139. Flight of person in a water-filled vessel

151 It is in this case desirable that the mean density of the body be approximately equal to the density of the water. It is possible that the nonuniform density of a person's internal organs may unfavorably affect his health, but this must also be investigated.

(150) TABLE 35. Expected weight increases during experiments in horseshoe-form laboratory*

Height of horseshoe	300 m		75 m	
Duration of descent	15 sec		8 sec	
Radius of bend	15m	60 m	15 m	30 m
Weight increase	40 times	10 times	10 times	5 times
Weight increase at bottom of iceberg when sliding down with sledge			10–20 times	

* K. Tsiolkovskii, Grezy o zemle i nebe (Dreams of Earth and Heaven), p.47.

Nature itself places the delicate parts of living organisms in liquid, e. g., the human brain or the embryo in its mother's womb. Grapes and eggs are, for the same reason, placed in sawdust during transportation.

A rocket flying in space must rotate about a transverse diameter if the travelers want to sense gravity. The centrifugal force thus created gives the desired effect of gravity. Thus, an acceleration of 0.002 g is obtained when a 100 m-long rocket rotates so that the linear velocity of its end is 1 m/sec. An acceleration of 0.2 g is obtained when this velocity is 10 m/sec.

c) Instrument for determining the effects of braking

The following instrument may be used to determine the effects of braking on living organisms, similar in structure and weight to human beings, at different decelerations.

*

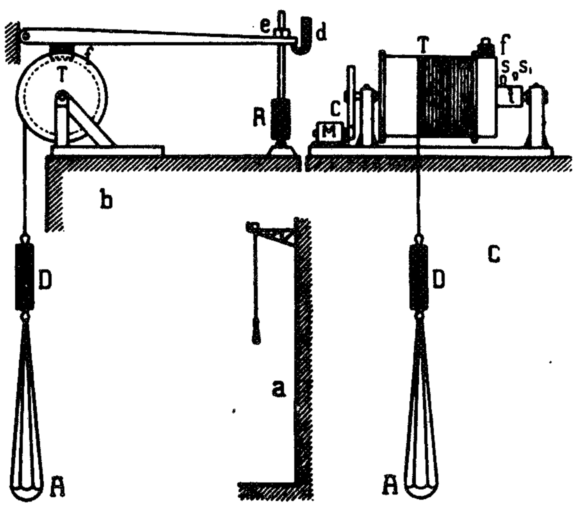


FIGURE 140

A cradle containing the object tested is lowered from a platform on a high tower by means of a steel cable (Figure 140a). The machine

TABLE 36.

Braking path l, m		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
v, m/sec	$\frac{v^2}{2g}$, kg																									
10	5	5																								
20	20	20	10	6	5																					
30	45	45	22	15	11	9	7	6	5	5																
40	80	80	40	26	20	16	13	11	10	9	8	7	6	6	5											
50	112	112	56	37	28	22	18	16	14	12	11	10	9	8	8	7	7	6	6	5	5					
60	180	180	90	60	45	36	30	25	22	20	18	16	15	13	12	12	11	10	10	9	9	8	8	7	7	6
70	245	245	122	81	61	49	40	35	30	27	24	22	20	18	17	16	15	14	13	13	12	11	11	10	10	9
80	320	320	160	106	80	64	53	45	40	35	32	29	26	24	22	21	20	18	17	16	16	15	14	13	13	12
90	405	405	202	135	101	81	67	57	50	46	40	36	33	31	28	27	25	24	22	21	20	18	18	17	16	16
100	500	800	250	166	125	100	83	71	62	55	50	45	41	38	35	33	31	29	27	26	25	23	22	21	20	20

located on the platform can lower the cradle at different decelerations, and is equipped with a recording instrument. The device is designed as follows (Figure 140b): A cable, wound on drum $\boldsymbol{T}$, supports cradle A via automatically recording dynamometer $\boldsymbol{D}$. The drum is equipped with brake $\boldsymbol{f}$, actuated by spring $\boldsymbol{R}$, adjustment nut $\boldsymbol{e}$, and hook $\boldsymbol{d}$. The rotational
153 speed of the drum is recorded on its shaft t by pen S, which slides at uniform speed parallel to the axis and thus describes a curve. Another pen S_1 is connected to the end of the braking lever and records the braking action on the same shaft.

d) The force acting during braking

We shall now determine what force acts on a body moving at a speed of v m/sec, when it is braked over a distance l. Let the mass of the body be m kg, and the unknown force, P kg. The gravitational acceleration is g m/sec^2.

The kinetic energy of the body must be equal to the work done during braking. Thus,

$$\frac{mv^2}{2} = P \cdot l,$$

whence

$$P = \frac{mv^2}{2l}.$$

Assuming the body to weigh 1 kg, and substituting for m its value $\frac{1}{g}$, we obtain

$$P = \frac{v^2}{2gl}. \tag{1}$$

Table 36 (p. 160) gives the forces acting on bodies moving at speeds between 10 and 100 m/sec, braked over distances varying between 1 and 25 m. The gravitational acceleration has been taken in the computations as $g = 10$ m/sec^2.

Forces less than 5 kg are not given in the table, since a five-fold weight increase, corresponding to a five-fold increase in the gravitational force, may be assumed to cause no harm to the human body.

Consider the different magnitudes of the force acting on a human body at various decelerations.

Let a train traveling at a speed $v = 40$ m/sec be stopped over a distance of 300m. The force acting on each kg weight of the human body is then $\frac{80}{0,1} = 0.26$ kg. This is hardly felt and is completely innocuous.

Assume that a body falls at a velocity of 100 m/sec on the earth, making a 0.1 m deep hole in the ground. The force acting per kg of the body weight is then

$$\frac{500}{0.1} = 5\,000 \text{ kg}.$$

A velocity of 100 m/sec corresponds approximately to a fall through a height

$$H=\frac{v^2}{2g}=\frac{100^2}{2.10}=500 \text{ m.}$$

Consider some intermediate cases of deceleration and the corresponding forces.

Let a train traveling at a speed of 40 m/sec be stopped suddenly. The passengers will be thrown forward at this velocity and hit the opposite wall of the carriage. We assume that the upholstery of the walls can yield to a depth of 10 cm. The force acting per kg of the body weight will then be $\frac{80}{0.1}$ =800 kg, and the results will be fatal.

Assume now that the passengers are secured to their seats by irreversibly elastic bonds (i.e., bonds which do not permit return motion)
154 which enable the passengers to move in the direction of the opposite wall for a distance of 2 m. The force acting per kg of the body weight will then be only $\frac{80}{2}$ =40 kg, which can still be withstood, in particular if it is uniformly distributed over the body. It is then important that each part of the body is decelerated to an extent permissible for it. Thus, let the body weight be 40 kg; each kilogram of the body weight then experiences a force of 40 kg. The head, which weighs 12 kg, is thus subjected to a force of 480 kg, which is excessive. The force must therefore be more suitably distributed over the body.

Consider some other cases.

1) An acrobat jumping from the ceiling of a circus tent into a net. The height of the fall is 20 m in this case, so that the speed at the net is $v=\sqrt{2gh}=\sqrt{2\cdot10\cdot20}=20$ m/sec. The braking path (sagging of the net) is 1 m, so that according to the table the force is 20 kg.

2) A jump onto the tilled earth from a height of 5 m. The velocity at the ground is 10 m/sec, the depth of penetration of the foot into the soil and the deflection of the knee are 0.5 m, so that the force is 10 kg.

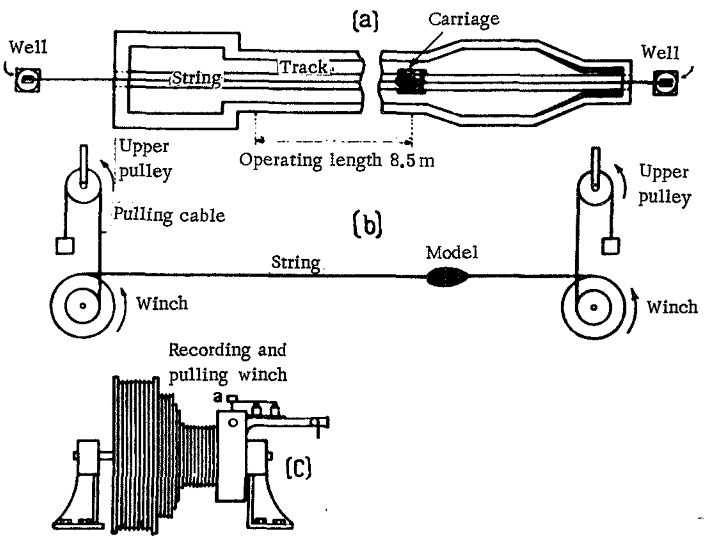


FIGURE 141

e) Acceleration of bodies moving in water

The British scientists R. Frazer and L. Simmons in 1919 carried out experiments in order to determine the resistance of a medium to the accelerated motion of a body in it. The model tested (Figure 141) was secured to a carriage running on a track laid on the bottom of a test tank. The carriage was pulled by a string wound on a winch. The latter was rotated by a cable passing over a pulley and ending in a weight lowered into a well. The model moved in the water with an acceleration since the string was successively wound on parts of the winch drum which had different diameters. The winch also had a sheave with a paper tape wound on it, on which pen *a* was moved electrically at uniform intervals so as to record time marks. Knowing the mass of the weight, the diameter of the drum, and the distance traversed by the model at various times, it is possible to determine the deceleration of the model and the resistance of the water.

155 f) Measuring the deceleration of bodies falling into water

W. Cowley and H. Lewy carried out experiments in the U.K. in 1918 in order to determine how the resistance of water varies when a body moves in it with an acceleration. The body tested *B* was for this purpose placed inside a high vessel *P*, filled with water (Figure 142). One wall of the vessel was made of glass so that the body could be photographed at various times during its fall. Light source *A* emitted a beam passing through lens *L*, which after reflection in mirror *M*, fell on the body. A disc with cutouts (shutter) *D* was located in the light path. Rotation of this disc interrupted the light beam falling on the body. A schematic representation of a photograph of the illuminated body is given on the left-hand side of the sketch. The acceleration varied between 0 and 0.2 g.

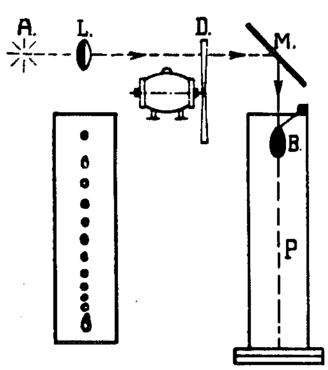


FIGURE 142

In 1918 the British scientists E. Relf and R. Jones carried out experiments in order to determine the effects of acceleration on the motion of an airship model. For this purpose they made the model swing in air and in water like a pendulum. They also tested it in the Froude tank of the National Physical Laboratory.

The sensations during high-speed flight, in particular during coasting, may vary greatly due to changes in or the absence of gravity and the possibility of different accelerations. In fact, the airplane may fly in a straight line or along a curve, it may turn about its axes of inertia, and it may accelerate or decelerate. All this can cause different sensations in the passengers. In particular, the most peculiar sensations are caused by the Coriolis acceleration which occurs when a body has simultaneously translational and rotational motion.

It is useful first to investigate these sensations in special moving laboratories. One was recently built in Göttingen; it consists of a cylindrical chamber of 3 m diameter, rotated by an electric motor at a speed which may attain 60 rpm.

The acceleration may attain 5 g at the circumference (Figure 143). This peculiar laboratory is used to investigate the effects of the rotation of solid bodies on the motion of liquids or gases contained in them. Two problems are considered in this case. Firstly, the influence of the rotation of the earth on air and ocean current is studied; secondly, the laws of motion of water or steam in a turbine are determined. These two problems, although apparently quite distinct, are closely related. The difficulty of investigating them directly is due to the fact that the earth rotates very slowly, while a turbine rotor revolves very rapidly. Being located in a laboratory which rotates at a few rpm, the experimenter is able to observe the phenomena in all details. It is not difficult to apply
156 the results obtained to meteorology and engineering.

The rotating laboratory was finished only recently. Before beginning with the experiments it was necessary to determine the effects of the rotation on the observers themselves.

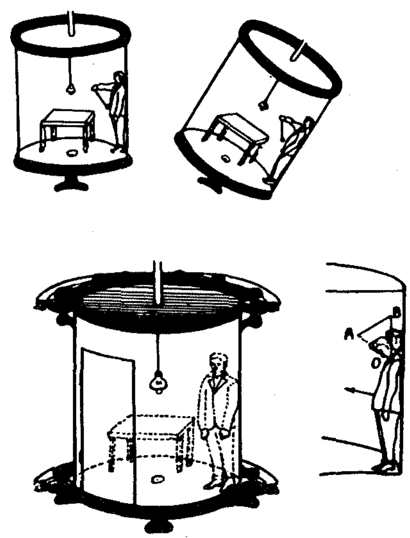


FIGURE 143

The influence of the rotation on vision was first investigated. No anomalies are observed at speeds of up to 10 rpm when complete immobility is preserved in relation to the rotating chamber; the chamber and all objects located in it appear to be stationary. However, the beginning and end of the rotation are noticed by the eye; the chamber appears to rotate in the opposite direction when the motion is stopped. Many readers will probably have experienced similar sensations during sharp turns of streetcars.

The observer located in the chamber notices when the chamber begins to rotate, and senses the acceleration as velocity. A similar impression is obtained during deceleration. However, during uniform rotation it appears to the observer that the chamber is stationary.

An extremely unexpected and uncomfortable visual sensation is obtained if the head is inclined in a uniformly rotating chamber. The chamber appears to turn upward or downward like the deck of a pitching ship if the head is inclined sideways.

On the other hand, the chamber appears to incline toward the left or right like a rolling ship when the head is raised or lowered. This causes the visual sensation to be accompanied by "sea sickness."

However, this feeling, like the visual sensation, disappears completely when the head is held stationary.

Interesting phenomena are also caused by gravity and centrifugal force. An observer located near the wall senses the resultant force like the ordinary gravitational force. It thus appears to him that the entire chamber is inclined backward, and that he himself rests semi-inclined on a sloping wall against which he is pressed by the centrifugal force, while the floor rises upward like a mountain slope.

This sensation becomes weaker as the observer approaches the center of the chamber; in the center itself, the chamber regains its usual look. Walking in a rotating chamber also gives rise to some strange sensations; one irrepressively tends to deviate from the goal to the right or left, depending on the sense of rotation of the chamber; this phenomenon is the more pronounced, the quicker one walks. Trying to walk toward the center, one must, so to speak, aim sideways.

We would experience this sensation in our daily lives if the earth were to
157 rotate faster. Such forces are called Coriolis forces, and cause deviation of rivers, ocean currents, and winds on the earth. To a slight extent they are experienced in a rapid train (the right-hand rail is worn more than the left one) or an airplane. An apparent deviation of the vertical from the direction toward the center of the earth occurs also under ordinary conditions, due to the centrifugal force. The centrifugal force acting on the walls is five times as large as the gravitational force when the laboratory described rotates at its maximum speed. Under these conditions the observer willy-nilly comes to rest against the wall, which to him appears more like a floor.

Prandtl explains these sensations by the fact that the human brain center which gives a person the sense of equilibrium "forgets" the rotation of a chamber in which the person is located if this rotation lasts for a long time. It thus gives the impression of a new rotation if there is any change in the motion. We can consider the rotation as a vector [in fact it is a "pseudovector"] and resolve it along the axes of rotation. The experimental results can then be easily explained by combining these rotations. Thus, an inclination of the head to the left recalls the "forgotten" rotation about axis $\boldsymbol{A}$ (Figure 144), which takes place when the head is in its normal position. However, when the head is inclined this rotation vector remains in the same position as before in relation to the head and its equilibrium organ. On the other hand, the actual [absolute] rotation takes place about axis $\boldsymbol{B}$. This creates the impression that a rotation about axis $\boldsymbol{C}$ takes place.

A similar impression is created when the head is inclined forward (Figure 143). The axis of rotation $\boldsymbol{OB}$ retains its absolute position in space, but its position in relation to the head (axis $\boldsymbol{OA}$) changes. This has the same effect as when the head retains its previous position while the axis of rotation is inclined toward the back of the head. The sensation felt in this case is the same as when the entire chamber is rotated rapidly about axis ***AB*** from right to left.*

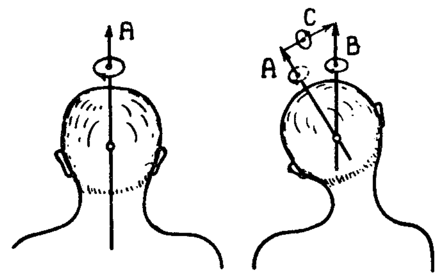


FIGURE 144

Mach used a chair which could be inclined and rotated in a vertical frame enclosed in a cardboard shell in order to eliminate the visual effects on the observer. Similar experiments were also carried out by Purkinje.

g) The Saint-Cyr merry-go-round for investigating the effects of dizziness

A merry-go-round with a vertical axis was built at Saint-Cyr. It was
158 used to study the psychological effects of rapid turns on fliers. One arm of the merry-go-round was 16 m long, and the other, 8 m. The entire merry-go-round turned in a circular hall of 40 m diameter and was driven by a d. c. motor with voltage regulation between 0 and 500 v. The motor developed 80 hp at 1,500 rpm. Box ABCD, made of steel pipes was fixed to the end of one arm (Figure 145). An armchair with floor was secured to the box at point O. The visual, auditory, and other reactions were recorded by means of electric chronographs located in the building on a table and connected to the person sitting in the armchair.

The person sitting in the armchair had the impression of taking off when the merry-go-round was started, and of landing when it was being stopped. No special sensations, depending on the speed or inclination of the body, were experienced during steady rotation. However, the test person felt a strong dizziness when turning his head during uniform rotation, tending

* Experiments in rotating laboratories were carried out earlier 1) in 1826 by Purkinje ("Physiologische Beobachtungen über den Schwindel" (Physiological Observations of Dizziness)), Bulletin No.10 of the Section of Natural Sciences of the Schlesische Gesellschaft (Silesian Society), 1825, p.35, and Bulletin No.12, 1826, p.1, Breslau, and 2) in 1875 by Ernst Mach ("Grundlinien der Lehre von den Bewegungsempfindungen" (Fundamentals of the Theory of the Sensations of Motion)), p.25–31, Leipzig, 1875.

to fall to the outside when the head was moved outward, and to the inside when the head was turned inward. This sensation did not depend on the initial position of the body, but only on the rotational speed of the merry-go-round and on the motion of the head.

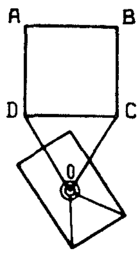


FIGURE 145

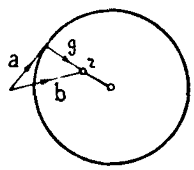


FIGURE 146

h) Compound acceleration

The centrifugal acceleration of a point moving on a circle of radius r is (Figure 146)

$$a = 4\pi^2 \frac{r}{t^2} = 40 \frac{r}{t^2},$$

where t is the duration of one turn.

Let the gravitational acceleration be g; the total acceleration is then

$$b = \sqrt{a^2 + g^2}.$$

The maximum velocity with which a person weighing 81.5 kg fell in air was 191 km/hr after 12 sec (height of fall = 45 m), and 137 km/hr for a person weighing 56.6 kg.

Experiments on the free fall of a person from an airplane were carried out in the U.K. This was done by delaying the opening of the parachute.

11. ABSENCE OF GRAVITY

Flight is possible by inertia under the action of gravity only. This may occur when an airplane falls from a high altitude or with a future spaceship while coasting. Any body inside such a spaceship will then lose its weight and
159 be able to float in the air.

It can be demonstrated by some simple experiments that a falling body becomes weightless.*

First experiment. An iron bar is placed in one of the balance pans (Figure 147a) so that it rests with one end on the pan. The other end of the bar is secured with a thread to the balance beam. The bar is then equilibrated by a weight. The bar will fall to the bottom of the pan if the thread is burnt through, while the other pan will descend.

Second experiment. (Due to Prof. de Metz of Kiev). A tight diametral partition is inserted into a vessel (Figure 147b). One compartment is then filled with water. If the vessel is now dropped while the partition is pulled out simultaneously, the water will not flow along the bottom of the vessel [to the other half] during the fall.

Third experiment. A cork is placed in a vessel full of water. According to Archimedes' Principle the stopper will float at a certain level if

* Cf. Ya. Perel'man, Mezhplanetnye puteshestviya (Interplanetary Travel).

it is pulled to the bottom of the vessel by a spring (Figure 147c). If the vessel is now dropped, Archimedes' Principle, which is valid only when the liquid has weight, will no longer apply, and the cork will be pulled to the bottom by the spring (Figure 147d).

We shall now present descriptions, according to some authors, of life in a future interplanetary rocket ship in the absence of acceleration.

The rocket continues to coast after the bursts; the passengers then lose their sensation of gravity and become "weightless;" the definitions "top" and "bottom" then lose their meaning. This sensation lasts as long as there is no "acceleration effect," and can be prevented by giving some acceleration to the rocket ship. In the absence of acceleration the travelers can, dressed in spacesuits, leave the rocket ship through airlocks and move fearlessly in space, obtaining air from knapsacks carried on the back while being linked to the rocket ship by a cable. They may thus investigate the surrounding space.

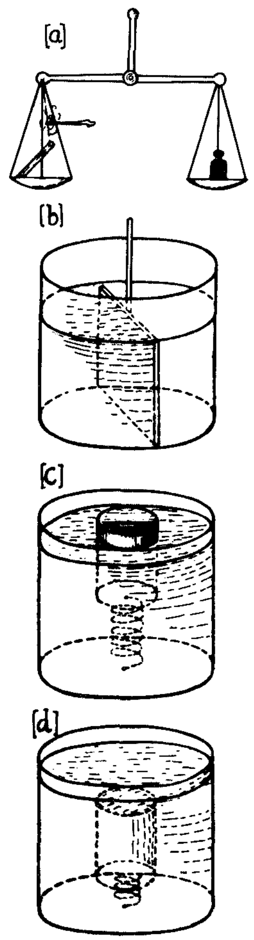


FIGURE 147

Figure 148 shows two space travelers inside their interplanetary rocket ship during unaccelerated flight (according to Wells). They and their belongings are weightless, and everything floats inside the rocket ship.

Water poured into the air assumes the shape of a sphere (Figure 149).

Artificial gravity may be created if desired, by rotating the rocket ship about some axis, or by rotating separate compartments inside the ship.

Stability of human beings inside or outside the rocket ship may be achieved in the absence of acceleration by means of portable gyroscopes secured, for example, to the back.

160

The sensations of human beings in fields of reduced gravity have been considered by many novelists and scientists. Below we present a description of the sensations which a person falling on the moon or an asteroid may experience.

Sensations on the moon. The gravitational acceleration on the moon is only one sixth of that on the earth, so that any body loses weight; a person thus may jump six times as far as on earth. The shadows are extremely black and sharp due to the clearness of the environment, while penumbras due to reflection of light are possible. It is very cold (-250°C) in the shade and very hot (100–150°C) in the sun. Talking is impossible since there is no atmosphere. Day and night are 30 times longer than on earth. The earth appears on the moon to be four times larger than does the moon on the earth.

Sensations on an asteroid. The smaller the asteroid, the less is its gravity and the lighter a person on it will feel himself. A person may jump to distances of hundreds of meters on a small asteroid. He may even fly off the asteroid with a strong kick.

FIGURE 148. Weightless passenger inside spaceship

Ray Kemmings, in his story "Man on a Meteor," describes the assumed sensations of a person falling on a meteor. He does not state how this person comes to be on the meteor. The story begins when the hero of the tale, Nemo, finds himself on the surface of the meteor, dressed in a rubber suit and with a transparent helmet of a hard material on his head. The helmet may be taken off since there is air on the meteor. The horizon is very near since the surface is extremely convex.

161

The meteor is one of those which in large numbers form the rings around Saturn. Its diameter is only about 10 km. The weight of any object on it is only 0.00039 times its weight on earth, so that a person will weigh a little more than one ounce.

Because of this small gravity Nemo can fly freely in the air, but has to take care not to jump forcefully, since he might fly off the meteor into space, or become its satellite.

He gradually becomes used to living and breathing under water since, on one hand, water on the meteor contains far more air than on earth, while on the other hand, the lungs can work in water with greater ease due to its much smaller specific weight.

(160)

FIGURE 149. Weightless water spilled in spaceship

12. PHYSIOLOGICAL CONDITIONS OF HUMAN FLIGHT AT HIGH ALTITUDES

The flight conditions at high altitudes force balloonists and fliers to employ special means for their protection against the effects of low temperatures and pressures, and lack of oxygen.

We shall now describe some of the sensations experienced by pilots during flights at high altitudes.

1. On 27 February 1920 Schroeder rose to an altitude of 10,093 m at Dayton, Ohio. His oxygen apparatus ceased to function at an altitude of 10,000 m, so that he almost lost consciousness. Fortunately, he could still control his plane and landed after 2 hrs 5 min. He was found completely numb and blind. An eye had become frozen. The temperature at the maximum height he attained was -50°C. Schroeder reported that at the instant when the oxygen supply failed he felt something like a terrific explosion in his head. His landing speed was 77 m/sec.

2. Another flyer named Weiss attempted unsuccessfully to beat Schroeder's record a few weeks later. His speed rose to 200 m/sec during the descent.

3. A new altitude record was established by Macready on 28 September 1921 at Dayton. He rose to an altitude of 10,518 m where the temperature was -50°C. The pilot seat was heated by means of hot water flowing in pipes from the radiator. Breathing was facilitated by oxygen flowing from a bottle through a tube to the mask worn by the pilot. In addition, a tube with a mouthpiece was connected to another bottle, and could be used by the pilot. The eyeglasses were covered on the inside with a layer of a gelatinous compound preventing the formation of ice at temperatures down to -50°C. The pilot was dressed very warmly. The weather was clear and warm. The pilot used oxygen above an altitude of 6,000 m. He experienced no sign of sickness up to a height of 9,000 m. Thereafter, he began to feel weak and his senses became blurred, this being most noticeable while bending toward the instruments. Special difficulties occurred when the ice in the mask caused by breathing clogged the oxygen tube, and the pilot had to take the emergency tube into his mouth.

The pilot remained for five minutes at the maximum altitude. No stars were seen despite the clear sky. The sky was very bright with almost no blue tint. The airplane tended to nosedive during the descent; the water in the radiator was thus cooled to such an extent that it ceased to heat the pilot. The eyeglass became covered with ice and the pilot was for some time unable to control the plane. This irregular descent continued down to an
162 altitude of 9,000 m where the pilot again gained control over the airplane and descended from 9,000 m to 6,000 m in 30 min in order to reduce the harmful effects of the pressure difference on the organism. The plane landed successfully.

4. The next two records were established by the French flier Sadi Lecoin who attained an altitude of 10,741 m on 5 October 1923, and of 11,115 m in November 1923.

5. The maximum altitude with a free balloon was reached by the Germans Berson and Süring who rose to a height of 10,800 m.

They prepared themselves very carefully for this flight and carried out a series of experiments on the effects of the rarefied air on blood circulation, respiration, strength and pulse rate. The flight was carried out on 31 June 1901 in good weather. The balloonists reached an altitude of 9,000 m after 4 hrs. The temperature at this height was -30°C.

Prof. Süring reported on this record flight as follows:

"We were proud to know that we had attained a greater altitude than anybody else before us. We made out all necessary records accurately and did not even feel the need for breathing oxygen. The only inconvenience was that because of the thick fur earflaps we could hardly hear each other and therefore could not talk. During further ascent no deterioration in our sensations occurred, except that a weakness seized us, which became more and more intensive, so that I involuntarily closed my eyes and had to make a great effort to open them again.

At an altitude of approximately 10,000 m we felt ourselves still able to carry out four observations at intervals of six minutes. The temperature at this altitude was between -30 and -40°C. It should be added that a purely incidental factor caused the reduction of our physical strength: our recording barometer became frozen, the clockwork mechanism stopped, and the ink congealed. Berson used all his strength in order to repair the instrument, but without success. During this time I remained without any work to do and felt a constantly increasing weakness. I carried out another observation at an altitude of 10,230 m when repair of the instrument was found to be impossible. It is striking that despite the great height we were able to make the observations with hardly any difficulty. The mercury barometer was set up and we read it very easily in a very uncomfortable position of the body; I also read the thermometer very easily and entered all observations very clearly and accurately in the logbook. This good feeling can be explained only by the fact that during this time we breathed oxygen uninterruptedly and were well protected against the cold. It is therefore not surprising that we considered ourselves able to rise even higher, but our bodies now required their normal equilibrium. We were quickly reminded of this when we suddenly lost the freshness of our memory at an altitude of 10,250 m, so that our recollections of this time are very inaccurate and inconsistent. Only one thing is certain, namely that Berson pulled the valve lanyard so that the balloon started to descend slowly. Shortly before that he glanced at the barometer and noticed that the air pressure was 202 mm hg, corresponding to an altitude of 10,500 m. Berson hurried to open the valve since he had called and shaken me several times and had found that I was in a deep faint; this scared him greatly.
163 However, the small physical effort which he had to make for this purpose affected him to such a degree that he fell down and for some time lay in a deep faint.

I dimly remember that after recovering consciousness I beheld my comrade as if quietly asleep, but as much as I tried to awaken him, I failed. I then threw myself on the valve in order to open it still more, but at this instant I felt such an attack of weakness that I had to hurry and grasp the oxygen tube. Here my recollections are again interrupted, and it is clear that I again became unconscious.

It must be assumed that just then the balloon began to descend. It is difficult to establish whether this lasted for half or three quarters of an hour, but when we recovered consciousness we found ourselves at an altitude of 6,000 m.

It is interesting that we now felt much worse, without a trace of our previous freshness; breathing was very difficult — although this quickly improved due to increased use of oxygen—and a leaden tiredness remained accompanied by headaches and terrible weakness, greatly resembling an attack of seasickness; in fact, this may really be called airsickness.

With great effort we forced ourselves to perform the necessary work, first of all by jettisoning ballast to reduce the rate of descent of our balloon. We then had to free ourselves a little from our furs and pack our instruments, which we managed to do, although with a great effort. In full control of the balloon we succeeded in regulating the flight in such a way that two hours later we landed slowly and smoothly on a small field."

6. The following is a description of the author's own flight with a balloon, carried out on 21 and 22 September 1910, for a prize offered by the All-Russian Aero Club for an altitude record:

"F. I. Odintsov flew with me as passenger.

We provided ourselves with warm clothes — sweaters, trousers, several pairs of socks and gloves, felt boots, and fur caps — assuming that the temperature at high altitudes would be low.

We ascended rapidly at 1705 hrs.

We intended to fly during the night at a low altitude, jettisoning as little ballast as possible. This ballast consisted of 21 bags (21 pud). We assumed that the sun would heat the gas in the morning, so that the balloon would rise to a certain altitude. We then intended to gain altitude by jettisoning ballast until only the quantity necessary for a safe landing remained. We assumed that we would need one bag for every 100 [1,000?] m descent and that 2 bags would be necessary for the landing itself. This was supposed to suffice for landing under favorable circumstances. However, if the wind was to drive us out over the sea or a lake, we would have to ascend again by jettisoning more ballast, retaining only as much as was necessary for preventing a landing on water.

We had to preserve our strength since we intended flying all night, so we arranged watches.

At sunrise we began to prepare ourselves for gaining altitude.

We connected the rubber tubes to the oxygen bags. We had initially
164 decided to use only one bag. A tube led from it and then divided into two branches, each of which ended in a mouthpiece convenient for inhaling oxygen. The branch tubes were closed with a clamp whose removal permitted the oxygen to flow to the mouthpieces. We then transferred the bags with the ballast toward the sides of the basket in order to have them at hand. The sun began to heat the gas at 7 a.m. and we gradually began to rise from an altitude of 725 m. Unexpectedly, the temperature was quite high at -5°C.

We were already at an altitude of 3,300 m by 8 a.m. and attained a height of 5,000 at 0835 hrs.

We jettisoned ballast in amounts of $\frac{1}{4}$—1 bag from time to time, in order to speed up the ascent.

We knew from oral accounts and from the literature that we might rapidly become exhausted at a high altitude, due to the low oxygen content of the air at a height of 5,000 m. Therefore, we began to take a few gulps of oxygen every five to ten minutes.

At 0854 hrs, at an altitude of 5,700 m, a few drops of blood appeared unexpectedly at the tips of my fingers and toes, due to the rarefied air, so that I unintentionally soiled my logbook. I did not feel sick and observed with interest how the blood gradually appeared out of the pores, as if extracted by some invisible instrument.

We continued to gain altitude by gradually jettisoning ballast. At 0910 hrs we were at a height of 5,900 m, and attained our maximum altitude of 6,400 m at 0930 hrs. The balloon did not rise higher since we could not jettison more ballast; only eight bags remained, the quantity needed for descent from this altitude.

We took a few deep breaths of oxygen every 3—5 min at the altitude of 6,400 m. We heard noises in our ears; our hearts beat at an increased rate; we felt fatigue and a desire not to move, only to rest. When I had moved the one-pud bags from the outside of the basket to its inside I felt such an intense fatigue that I had to sit down and rest for 3 min.

We began gradually to descend from the altitude of 6,400 m. However, the gas again became hot when the sun reappeared from behind a cloud and we reascended to the previous altitude. We saw cirrus clouds above us at an enormous height; cumulus clouds extended below us in a continuous ring on the horizon.

In view of the impending landing, we now had to clear the rupture band and the valve lanyard. I climbed onto the side of the basket, stood on it and gripped the collar with my hands, while I untangled the band and the lanyard. I felt my comrade S. I. Odintsov holding me by the edge of my overcoat to prevent me from falling overboard.

After freeing the lanyard I descended from the basket side and rested for 10 min from this gymnastic exercise.

To characterize the air currents at the various altitudes it should be added that up to a height of 4,500 m the wind carried us to the north. The wind direction changed when we rose higher, and we were carried to the south, toward Lake Ladoga. However, when we descended to the former altitude we again encountered a south wind.

The temperature at 6,400 m was only -6°C; this was due to the strong south wind below and the light north wind above. The temperature at
165 St. Petersburg was approximately +4°C at takeoff.

During this time our balloon was gradually descending. At an altitude of 3,000 m we jettisoned ballast in order to slow down the descent. I forgot to mention the effects of breathing oxygen on the body; with every gulp we felt an influx of strength and energy, and perhaps enjoyed ourselves. The effects of oxygen may be compared with those of champagne. After descending to below 5,000 m and finding that there was still sufficient oxygen in the bag we decided to "relish" it, i. e., not to waste it by throwing it into the air, and gradually inhaled it all. One bag appeared to suffice us for the flight. The other bag could be left unused, especially since an unnoticed hole had caused all the oxygen in it to escape during the night. After descending to 1,500 m we were again carried toward the north. The wind was quite strong, and we inspected the location attentively, selecting the safest place for landing.

A field attracted us first, but upon inspection we found huge boulders were strewn on it; landing in their midst with a strong wind might have caused our death. We therefore decided to land directly in a wood. Having noticed on our flight a place covered with dense wood, not far from some sawmill, we opened the valve for some seconds. The balloon quickly approached the ground. We jettisoned ballast in order to reduce the speed of descent. The guiderope was finally caught by the treetops. The balloon became lightened and we stayed for some time on the guiderope whose end had become entangled in the treetops, tearing off branches due to the speed. Soon a river appeared in front of us. We reached it with the guide rope. A few minutes later we dropped the anchor which caught hold; we were thrown into the basket and clutched the ropes in order to reduce the initial impact. To stop the balloon we tried to tear off the rupture band, but to our misfortune it was so strongly secured that we were twice unable to remove it. I then grasped the valve lanyard in order to open the valve.

Meanwhile the balloon, held by the anchor, began to make huge jumps. Sometimes we were lifted to a height of 100 fathoms, at other times we were again thrown into the wood. After tearing off the rupture band we both grasped the valve lanyard and held the valve open, assuming that the gas would gradually escape from the balloon. From time to time the wind gusts tore the lanyard out of our hands and carried it out of the basket.

The rapid descent from a high altitude had affected our strength. Our bodies, deprived for some time of regular respiration, had obviously become weak; our fingers could no longer hold the lanyard. It became necessary to compress the valve lanyard into a ball and clutch it with both hands. After this struggle with the wind the balloon net finally got caught in the branches of the trees. It was now 1205 hrs. We left the valve open, however, fearing a new gust of wind.

Having ascertained that the balloon was firmly settled in the branches we let go of the valve lanyard and began to give signals by blowing into a horn. Some 20—30 min passed. We then heard people running and very soon we were surrounded by a crowd of Finns from the neighboring woods. They had seen us when we were carried past them in the wood, and they had run after us for some time. It appeared that we had landed not far from the Ellola wood near the Värtsila station of the Finnish railroads to the north of Lake Ladoga.

After together folding and packing the balloon we returned to St. Peterburg by rail."

166 AT THE "CEILING" OF THE ATMOSPHERE*

Article written by Hawthorne C.Grey (USA) who reached an altitude of 42,470 ft.with a balloon

What are the sensations of a person who has reached an altitude of 13 km, higher than any living being had been before, where the air is too thin to sustain life, and where the thermometer indicates a temperature of about -60°C?

* Vechernaya Moskva, 1927, No.204.

A few weeks ago I experienced these sensations when I attained an altitude of 42,470 ft above sea level. The balloon, whose capacity was 80,000 ft^3, lifted me from the Belleville landing ground, and, when I had attained this "ceiling," lowered me again at such a speed that I was forced to abandon it and jump into space with a parachute.

I freed the balloon from the remaining ballast (47 lbs of sand) at an altitude of 40,000 ft, but nevertheless the balloon stopped climbing. I was prepared for this, since I had special parachutes for every item of the equipment. I attached one of these parachutes to the oxygen bottle which had already been emptied on the way. When I threw this 25-lb steel bottle overboard it appeared to me that it weighed at least 150 lbs; I was weakened to such a degree by my long stay in the rarefied atmosphere, breathing oxygen. Release from the weight of the oxygen bottle was sufficient for the balloon to rise another 2,000 feet.

The clouds were far below me, and a deep blue, almost cobalt, sky extended above me. Particles of dust, which cause the sunlight to appear white, remain below, and the pure and thin air gives rise to this magnificent color. I clearly heard the radio which brought to my ear the loud sounds of an orchestra; this was the only reminder of the earth.

However, the balloon had now stopped. The gas remaining in it at this altitude was less than one eighth of the amount present at takeoff. The gas expanded and gradually escaped from the balloon as I rose to lighter layers of the atmosphere and the external pressure decreased. The balloon was full (gas had already been discharged) at the maximum altitude, but as soon as I began to descend, the balloon became flat since the gas in it was again being compressed in correspondence with the outside pressure. Even if I had been able to retain all the remaining gas in the balloon there would not have been more than 10,000 ft^3 left upon landing.

All this was known to me, but in some way or another I had to begin to descend. For this it was necessary to touch the valve lanyard slightly. After beginning to descend the balloon gathered speed from minute to minute, as the statoscope indicated. In order to slow down the descent I began to attach parachutes to all items in the basket, and threw them overboard.

The parachutes were supposed to descend at a speed of 16 m/sec, but the balloon dropped so quickly that the items thrown overboard appeared not to fall down but to fly above me. It was strange to see a 25 lb steel bottle rising into the air.

Another two oxygen bottles, the battery for the electric heater, the
radio and loudspeaker, the wooden frame supporting the bags with the ballast,
167 and all empty bags were thrown overboard in order to lighten the balloon.
But all this was too little. At an altitude of 8,000 ft I found that the balloon was going to inevitably fall into a swamp which I could see below me. The speed of my descent was then 1,800 ft/min, which was twice the safe speed of a parachute descent. I therefore decided to abandon the balloon and descend by parachute.

After being freed of a load of 240 lbs, 183 lbs of which represented my own weight, the balloon began to rise slowly and landed after traveling another 10 miles. The point where I landed was located at a distance of 110 miles from the starting point.

This was my 107th balloon flight. I had enough experience to start in the best possible way. All the ballast bags were arranged on a wooden frame outside the basket; they could all be easily released together when it was necessary to jettison them during the flight. The oxygen apparatus was improved by covering it with a tubular asbestos cylinder containing an electric heater supplied from a 2-v battery. The heater provided a hot current of oxygen to the lungs, which was very easy to inhale. My oxygen helmet externally greatly resembled a gas mask used during the war, but in fact it had very little in common with it.

The oxygen current flowed to the mask in such a way that the eyeglasses did not become covered with ice.

To protect myself against the unbearable cold I wore, in addition to warm woolen underclothes, two woollen shirts, a sweater, and a winter overcoat on top. Above all this I wore a flying suit with fur on the outside, deer skin beneath, and two layers of heavy cloth in between. My boots were designed in the same way.

During my next flight I jettisoned superfluous equipment at a high altitude and not during the descent, as I had done until then, and thus managed to attain a height of 43,000 ft. A prize had been set out at that time for attaining an altitude of 50,000 ft, but later investigations showed that this height could not be reached with the equipment available at that time. This was established by physicians who carried out experiments in a chamber where the air density was reduced in correspondence with the different altitudes. It was found that there is an altitude between 40,000 and 50,000 ft, at which the air pressure is so low that a person's muscles are no longer able to contract the lungs; a person breathing oxygen thus cannot actuate the muscles sufficiently in order to empty the lungs again and prepare them for taking a new breath. A person thus perishes at this altitude.

Sensation of sickness during flight at high altitudes was to be prevented by designing airplanes with airtight [pressurized] cabins for crew and passengers. These cabins were to be heated by the exhaust gas, while normal air pressure was to be maintained in them with the aid of turbochargers. Figure 150 is a section of such a future airplane designed
168 by Bréguet.*

The airplane has four engines with a total power of 950 hp. The letters in the drawing have the following meaning: *A* — propeller shaft, *B* — engines (4), *C* — gear box, *D* — radiator, *E* — oil tank, *F* — gasoline tank, *G* — turbocharger, *H* — lower wing, *I* — passenger cabin, *J* — pilot seat, *K* — radio, *L* — bulkhead, *M, N, O, P, Q, R, S* — stabilizers and rudder, *T* — skid.

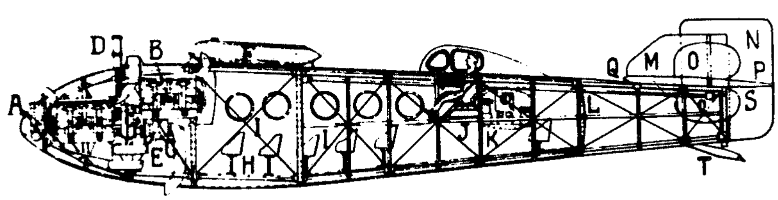


FIGURE 150. Bréguet airplane with pressurized cabin

* Cf. Proceedings of French Academy of Sciences, Vol.170, p.782 (paper by Rateau).

Figure 151 shows schematically a section of the basket of an air balloon, specially designed by the Frenchman Tridon for flight at large altitudes. The basket is pressurized. The letters in the drawing have the following meaning: B — barometer, C — collar, N — upper open basket, C_1 — valve lanyard, T — hatch, T_1 — fabric soaked in lime water, H — illuminators, S — bench, S_1 — safety valve, E — lime water, E_1 — funnel for ballast, LL_1 — ballast, P — air pump, OX — compressed oxygen.

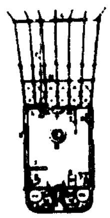


FIGURE 151. Pressurized basket of Tridon's balloon

A pressurized cabin has to be provided in an airplane flying at high altitudes. Heating the clothing is difficult, as is motion at excess pressures of even 0.5 atm.

Such a cabin is not difficult to construct. Its walls will be subjected only to tension if it forms a body of revolution. A higher tightness can be ensured by providing double walls with a pressure difference. The space between the double walls should be heated in order to prevent the formation of deposits. The air supply should be independent of the engine, blowers and pumps being used.

Let the pressure in the cabin be 14.7 lb/in^2, while the outside pressure is zero. We assume that the cabin has a circular cross section, so that its walls are subjected only to tension. The wall thickness required is

$$t = K\frac{pd}{2kf}.$$

169 where t is the wall thickness, in; p is the pressure in the cabin, lb/in^2; d is the outside cabin diameter, in; f is the permissible tensile stress, lb/in^2; K is the safety factor; k is the efficiency of the riveted joints.

Consider a five-seater airplane with a 425 hp engine and a fuselage diameter of 72 in. The fuselage is built of dural sheets for which the permissible stress is $f = 55{,}000$ lb/in^2. The efficiency of a joint consisting of three rows of rivets is 80%. The safety factor is taken as 4.

Inserting these values in the above formula, we obtain

$$t = 4 \cdot \frac{14.7 \cdot 72}{2 \cdot 0.80 \cdot 55\,000} = 0.049 \text{ in.}$$

Orientation with respect to the ground is possible with the aid of periscopes. It thus remains, generally speaking, only to construct and flight-test such a cabin.

The American writer Edgar Allen Poe, in his story "Unparalleled Adventures of One Hans Pfaal" published in 1835, was apparently the first to propose a blower to compress the rarefied air at high altitudes in order to permit breathing and thus the existence of human beings at such heights. This machine was, according to Pfaall, invented by a certain Grimm and perfected by Pfaall himself. The design of the basket was as follows (Figure 152):

Basket (A) of the balloon contains flexible rubber membrane (m) which is impervious to air. This membrane covers tightly the bottom and walls of the basket and is brought out at the sides, cables (f) being inserted through hermetically sealed openings. The membrane then envelopes collar C and runs beneath cables (*e*) connecting the basket to the balloon. Cables (e) are connected to the membrane by means of studs. Wire netting (d) is inserted into the membrane on top in order to prevent its crumpling through the pull of cables (*e*). Windows (*o*) of thick glass are arranged in the membrane and in the bottom of the basket. Pipe (*a*) leads through one of the windows to blower (*b*) located inside the basket. Rarefied air is aspirated through this pipe, compressed to the normal pressure, and discharged into the airtight space inside the basket and membrane (*m*),, where the balloonist is located. The spent air is discharged through flap (*g*) in the bottom of the basket. The blower had to be operated each hour. Pfaall therefore built water clock (*h*) which every hour automatically poured water over the sleeping balloonist in order to awaken him; he then situated the blower and let it run for 5 min, after which the water clock began to fill again.

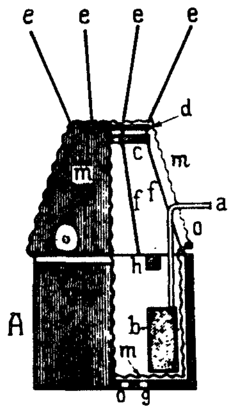


FIGURE 152. Pressurized basket according to E.A.Poe

13. BREATHING OF HUMAN BEINGS AT HIGH ALTITUDES

The lack of oxygen in the air has a harmful effect on the human organism beginning at an altitude of 4.5 km. The first symptoms are headaches followed by palpitations, fatigue, numbness of limbs, pains in one or both ears, which may last for several hours, weakening of the memory, reduction of the sense of equilibrium, dizziness,
170 loss of strength, and finally, loss of consciousness. Long flights at altitudes of 6,000 m or more may cause disorders of the central nervous system. However, all these physiological symptoms can be prevented by the inhalation of oxygen.

A normal person inhales about 16 times per minute at normal atmospheric pressure. The volume of air inhaled at each breath is approximately half a liter, so that about 8 l/min of air, containing 1.6 l oxygen, are aspirated. This is the minimum required by a person. A human being can rarely pass through his lungs more than 28 l/min even while doing very heavy work.

During flight a person has an increased activity and thus requires more oxygen. The amount of oxygen may be taken as 4 l/min, allowing for leakages through the mask. The air pressure at sea level is approximately 1 kg/cm^2; it decreases by about 0.1 kg/cm^2 for every 1,000 m altitude for the first three km, and more slowly at high altitudes.

The air pressure and density at an altitude of 6,000 m are slightly more than one half of the corresponding values at sea level. A person thus obtains only half the required amount of oxygen with each breath. Table 37 indicates the amounts of additional oxygen required by a person at various altitudes (at a constant temperature of 10°C). It is assumed that the normal consumption of oxygen is 4 l/min. This table corresponds to British and U.S. standards.

TABLE 37.

Altitude, m	Amount of oxygen inhaled with air, l/min	Oxygen deficiency, l/min	Additional oxygen required, l/min
0	4	0	0
1,500	3.3	0.7	0.7
3,000	2.8	1.2	1.2
4,500	2.3	1.7	1.7
6,000	1.9	2.1	2.1
7,500	1.6	2.4	2.4
9,500	1.3	2.7	2.7

The U. S. Air Force assumes that the [additional] oxygen requirement varies with the barometric pressure P, mm Hg, according to the formula

$$R=4-\frac{4P}{760}.$$

In general the oxygen content of the inhaled air must correspond to a partial pressure of not less than 160 mm Hg this being the value at sea level. It is shown in Table 38 how the oxygen content (in %) of air inhaled at different altitudes (or different pressures) must be increased, so that this condition is satisfied. These values were obtained from the formula

$$p=100\%\,\frac{160}{P}.$$

171

TABLE 38.

Barometric pressure P, mm Hg, corresponding to altitude H	Necessary oxygen content of air, %
760	21.0
530	30.2
440	36.4
375	42.7
315	50.8
260	61.6
220	72.8

Addition of 0.5 l/min at an altitude of 3,000 m and of 2.5 l/min at an altitude of 8,000 m are considered sufficient in France.

A person thus requires about 150 l/hr additional oxygen during high-altitude flights. A supply for 2.5 hrs is considered necessary. A space of 350 l or more must therefore be provided in the basket for oxygen. Bags were used in France. They weighed 1.2 kg and contained 360 l oxygen at atmospheric pressure, but could expand during the ascent to a volume of 720 l without bursting. Methods of maintaining a constant pressure in them were also envisaged. This was to be achieved by means of an auxiliary bag in which constant pressure was to be maintained by filling it with air during the flight.

Two methods of storing the oxygen during the flight are employed at present. These are 1) use of compressed oxygen, 2) use of liquid oxygen.

Compressed oxygen is used in France. It is kept in steel cylinders having 3 mm thick walls, each holding 362 l [at NTP] and weighing 4—4.5 kg. The volume of such a cylinder is 2,000 cm^3, while the inside diameter is 6.5 cm; the pressure in it may reach 175 atm. Chrome-steel cylinders are used in the UK. Such a cylinder holds 500 l [at NTP] and weighs 3 kg; the pressure in it may reach 150 atm. Steel cylinders having a capacity of 500 l are used in the USA; the pressure in them may reach 150 atm at 16°C. The outer diameter of such a cylinder is 10 cm, and it weighs 5 kg.

Only liquid oxygen is used in Germany. It is kept in vessels from which the air has first been evacuated, and which are much lighter than the steel cylinders required for compressed oxygen.

Special vessels for storing and transporting liquid oxygen have been developed in the UK. Such a vessel consists of two flasks, one inside the other, rolled or forged metal, having diameters of 35.5 and 38 cm, respectively. The vessel can hold 4.5 gallons (=20 l) liquid oxygen, corresponding to 15,000 l gaseous oxygen. The neck of the inner flask consists of a long metal tube of 9.5 mm diameter, which is surrounded by the neck of the outer flask. An airtight ring is located between the necks. The air in the space between the two flasks is evacuated to a pressure of approximately 0.001 mm Hg; the surfaces of this space are brightly polished in order to prevent radiation of heat. About 300 g charcoal are placed in a small tray under the bottom of the inner flask in order to absorb the remainder of the gases in the evacuated space. A lead pipe with removable cap is used to exhaust the air from this space. The loss through evaporation in 24 hrs does not exceed 7% of the capacity of the vessel.

An essential part of any vessel containing oxygen is control equipment recording the gas used by the flier. Equipment of the most recent type is provided with a device ensuring an oxygen supply at the pressure and quantity required at the given altitude. Various types of such control equipment have been designed: **A. Equipment for compressed oxygen:** Dreyer, Clark-Dreyer, King-Dreyer, Munerelle, Clark, Van Sickler, Garsaux, Gourdou, Gibbs. **B. Equipment for liquid oxygen:** German, British, French.
172 The operation of control equipment was thoroughly tested in the USA by the Bureau of Standards at pressures corresponding to a flight altitude of 9 km and at temperatures between +29 and -40°C. The quantities of oxygen required by one and two persons were determined, it being assumed that two airmen need only 60% more than one, since they do not use oxygen simultaneously.

Historical notes. The problem of air and food for persons flying at high altitudes, in particular in space, has for a long time engaged scientists and novelists.

Jules Verne in his book "From the Earth to the Moon" indicates the need for regeneration of the air in the spaceship, and proposes the following method for the voyagers with two dogs who have to fly in a missile for 4 days:

Their daily air consumption is 2,400 l, i. e., about 2.9 kg. The problem is formulated in the following way: while the nitrogen which a person inhales and exhales remains unchanged, it is necessary to regenerate the consumed oxygen and eliminate the exhaled carbon dioxide. This is done very easily with the aid of potassium hypochlorite and potassium hydroxide.

Potassium hypochlorite heated to 400°C is transformed into potassium chloride and oxygen which are easily separated. An amount of 7.37 kg potassium hypochlorite yields 2.9 kg oxygen, i.e., exactly the amount required each day by the space travelers.

Potassium hydroxide is a substance which readily absorbs carbon dioxide from the air. It is sufficient to shake a solution of it in order to make it absorb CO_2 and form K_2O.

K. Tsiolkovskii went even further and proposed the following measures for ensuring the life of human beings in future rocket ships:

Oxygen and nitrogen are necessary for breathing. The products exhaled must be absorbed by special substances (alkalis), or better still by plants which transform them and other matter excreted by human bodies, into food while the oxygen required for breathing is again formed under the effects of sunlight. Enough plants for a complete food and oxygen cycle can be taken, so that the requirements of the space travellers are fulfilled. Tsiolkovskii thought that 1 m^2 of hothouse area, turned toward the sun, would be sufficient to supply food for one person (Vestnik Vozdukhoplavaniya, 1912, No. 2, p. 2–4).

In "Vne zemli" (Outside the Earth), p. 40, he assumes that this area must be 16 m^2. He also suggests in this book that pure oxygen at a pressure of 0.1 atm should be breathed, assuming that 10 kg of various substances are necessary per person per day for the purification of the air (ibid., p. 18).

FIGURE 153. Bastian oxygen apparatus

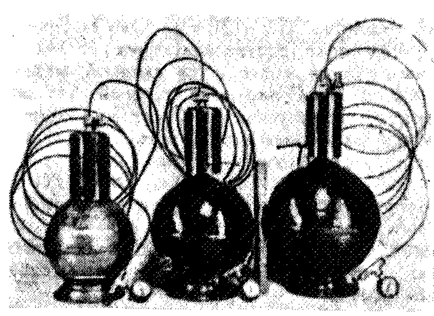


FIGURE 154. Blessing oxygen apparatus

Oxygen apparatus for breathing

High-altitude flights have for a long time been performed in the USA. The fliers are provided with warm clothing and oxygen equipment for breathing. Such equipment is produced by Messrs. Bastian, the Blessing Co., 173 and the Purox Co. (Figures 153 and 154). Liquid oxygen is contained in a double-walled metal vessel, similar to a thermos flask. Vaporizers are connected to the mouth of the vessel, which automatically, according to the atmospheric pressure, discharges gaseous oxygen into the airman's mouth.

The temperature of the liquefied gas is -295°F. Figure 154 shows three models of this apparatus, having capacities of 5, 10 and 15 l, respectively.

FIGURE 155. Airmen with Blessing apparatus

Figure 155 shows airmen in their flying suits, together with oxygen equipment for high-altitude photographic flights.

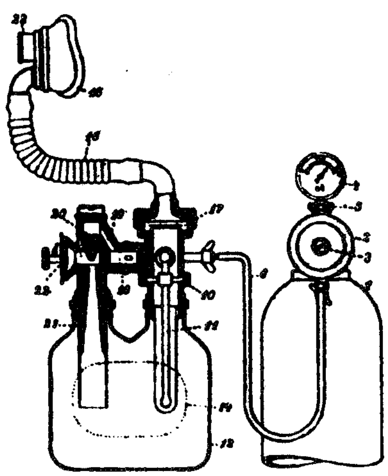


FIGURE 156. Dreger oxygen apparatus

Dreger oxygen apparatus

The apparatus developed by Messrs. Dreger (Lübeck, Germany) is used to produce the oxygen needed for breathing during high-altitude
174 flights. This apparatus consists of three principal parts (Figure 156): 1) a steel bottle containing oxygen at a pressure of 150—250 atm, equipped with reducing valves lowering this pressure to 3 atm at the outlet; 2) a chamber in which the oxygen is mixed with the outside air, equipped with further reducing valves; 3) a mask with an inlet tube and an outlet valve.*

Apparatus for altitudes below 10,000 m, supplying oxygen at variable density

An oxygen apparatus of the old type (Figure 157) consists of a steel bottle containing oxygen, a device for discharging the gas, and a mask. The gas-discharge device indicates the quantity of gas remaining in the bottle and has a pressure gauge showing the oxygen consumption in liters. The airman obtains 3.5 l oxygen per min. The total capacity of the apparatus is 1,000 l [at atmospheric pressure], which is sufficient for one person for five hours. A shortcoming of this apparatus is the fact that the reduced pressure of the outside air at high altitudes also affects the oxygen whose density decreases with increasing altitude, so that it does not supply the airman's needs.

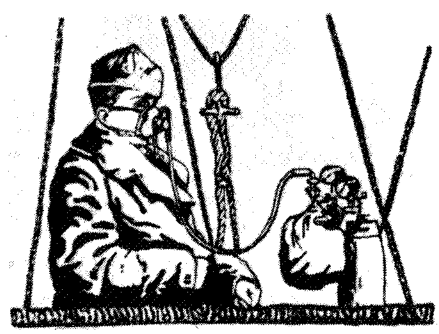


FIGURE 157. Oxygen apparatus of old type

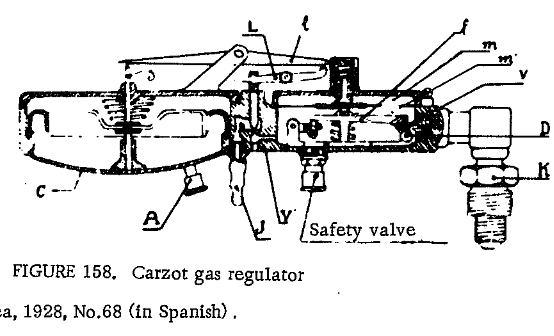


FIGURE 158. Carzot gas regulator

* For details cf. Aerea, 1928, No.68 (in Spanish).

Carzot apparatus. This apparatus ensures automatic maintenance of the density of the oxygen supplied to the airman. The apparatus consists of the following parts: 1) a bottle for compressed oxygen (150–175 kg), 2) a gas regulator, 3) a gas meter, and 4) a mask. The regulator is set in such a manner that 30 l oxygen are delivered per hr at an altitude of 350 m, 270 l/hr at an altitude of 8,000 m, and so forth. The gas regulator is shown in Figure 158.

The operating principle of the gas regulator is as follows: Aneroid box C opens needle valve v to a varying degree, depending on the outside pressure, by means of a lever system; the valve permits oxygen to flow from the right-hand part of the regulator into tube J leading to the mask.
175 The right-hand part of the regulator contains oxygen flowing from the bottle through tube K and an orifice closed by needle valve v whose position is regulated by lever l and a spring adjusted by means of the screw on top. The pressure in the regulator is less than that in the bottle.

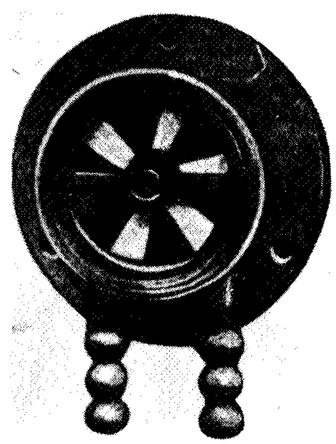


FIGURE 159. Carzot gas meter

The gas meter (Figure 159) consists of a small turbine between the bottle and the regulator, which measures the oxygen consumption.

The mask (Figure 160a) is an aluminum dish with a 2 cm-diameter opening at the bottom for the discharge of the used air.

A rubber band encircling the mask ensures a tight fit to the face. Oxygen is supplied to the mask through two copper tubes with small apertures directing the gas into the nostrils. The mask is heated electrically.

The **Panhard-Levassor** apparatus (Figure 161) is a more recent and lighter design, but operates according to the same principle as the Carzot apparatus.

176 The Goudrou apparatus (Figure 162) weighs still less than the preceding types. Its aneroid box is even more sensitive (four boxes C). Its operating principle is the same as that of the preceding designs.

(175)

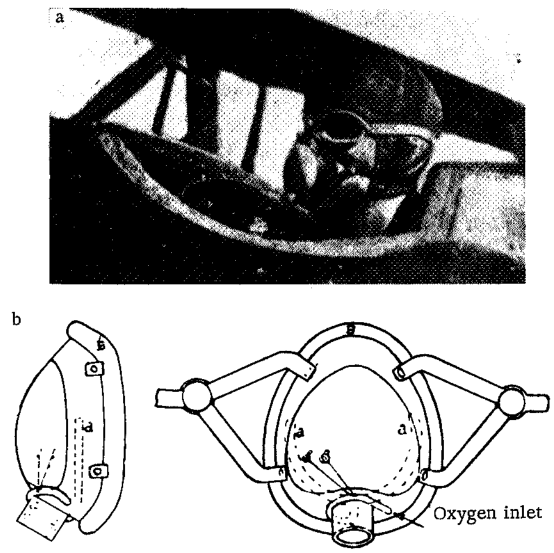


FIGURE 160a, b. Carzot mask

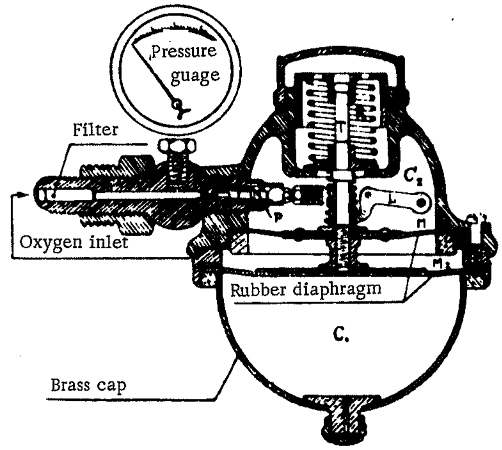


FIGURE 161. Panhard-Levassor apparatus

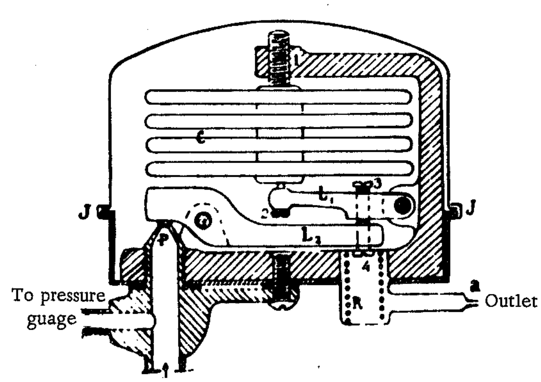


FIGURE 162. Goudrou apparatus

177 ## 14. HEATING AND FLYING SUITS

Very low temperatures, which according to many scientists may approach absolute zero (-273°C), will be encountered during flight at high altitudes and outside the terrestrial atmosphere. K. Tsiolkovskii in his book "Vne zemli" (Outside the Earth), p. 108–110, gives the following values for the temperature of the sunlight at various distances from the sun (Table 39).

TABLE 39.

Distance from sun in earth radii	0	1/36	1/25	1/16	1/9	1/4	1/3	1/2	1	2	3	4	5	9	16	25	36	infinite
Temperature, C°	+6,427	+2,277	+1,852	+1,427	+1,002	+577	+450	+322	+152	+27	-27	-61	-83	-131	-167	-188	-202	-273

The mean temperature of a planet is less than that given in Table 39 for the corresponding distance from the sun; since a planet is spherical, the sunlight falls on it at an angle, while part of it is reflected by the atmosphere, where one exists.

K. Tsiolkovskii gives the following values for the temperatures of the planets:

Table 40

Planet	Mercury	Venus	Earth	Mars	Asteroids	Jupiter	Saturn	Uranus	Neptune
Temperature if no atmosphere present	+200	+90	+27	-23	—	-138	-174	-204	-218
Temperature if atmosphere present......	+176	+72	+14	-35	-35 to -145	-145	-179	-207	-221

It is possible during flight to concentrate the sun rays with a mirror and thus raise the temperature inside the spaceship. The same purpose may be achieved by coating the side of the spaceship facing the sun with a dark metal,
178 while keeping the side in the shade brightly polished. It is also possible to cool the spaceship by interchanging these sides. Furthermore, the walls of the spaceship must be of a material which is a poor heat conductor. In the fantasy of novelists it is already assumed that sunlight can be stopped from reaching the earth.

Thus, a fantastic novel by C. Brisbane, translated from English, appeared in "Priroda i Lyudi," 1917, p. 77, under the title "Absolyutnyi nul'" (Absolute Zero). The novel describes how a medium which is impermeable to thermal radiation penetrates the solar system from outer space. It stops the heat flow from the sun to the earth, so that the latter freezes up. This, fortunately, does not last long: the curtain moves on, and the earth again receives heat.

Lastly, we end this section by presenting descriptions and drawings, given by various writers and scientists, of spacesuits worn by persons leaving interplanetary ships moving freely in space.

A spacesuit must cover the whole body, be impervious to heat, gases, and vapors, and be pliable and strong in order to withstand the internal gas pressure. The spacesuit should have plates opposite the eyes, to allow vision and at the same time protect the eyes against the harmful effects of strong sunlight. The inside of the spacesuit should be covered by a thick heated lining, impervious to gases and vapors and containing tanks for the urine, perspiration, etc. The spacesuit is connected to a box which continuously releases oxygen, at a rate of about 1 kg per person per day. The carbon dioxide, water vapor, and the other products released by the body are absorbed by other boxes. The gases and vapors are circulated beneath the clothing and in a permeable lining by automatic pumps. All stores are calculated for 8 hrs; their weight, including the clothes, does not exceed 10 kg.

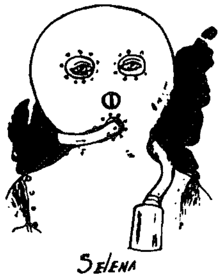


FIGURE 163. Spacesuit according to LeFaure and Graffigny

A long time ago the French writers LeFaure and Graffigny presented some drawings and descriptions of such spacesuits. Figures 163 and 164 show spacesuits for ladies. Fresh air is supplied from a bottle. The used air is discharged to the outside. Each liquid-oxygen bottle can supply 3,000 l gaseous oxygen, which, according to the authors of the novel, should be sufficient for three days.

FIGURE 164. Spacesuit according to LeFaure and Graffigny

An analysis of the phenomena occurring in human beings ascending to high altitudes without protective devices is given by Gillert in the Yearbook of the Scientific Association for Aeronautics (Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt), 1928, p. 78, under the title "Neue Medizinische Ergebnisse über Flug und Höhenflug" (New Medical Results on Flight and High-altitude Flight).

Figure 165 shows the pilot of an airplane, dressed and equipped for
179 high-altitude flight. The upper picture shows (from left to right) the
engine turbocharger, the interior of the fuselage with the covering removed, the connection box, the steering wheel, the instrument panel, the oxygen mask and microphone, the electrically heated eyeglasses, the straps holding the pilot to his seat, the tube from the oxygen generator, the oxygen regulator, the parachute harness, the parachute which forms a pillow on the sea, the radio equipment, the barograph, the distance meter, the liquid oxygen for breathing and the generator. A windmill generator supplying current for heating the pilot's clothes is arranged between the wheels.

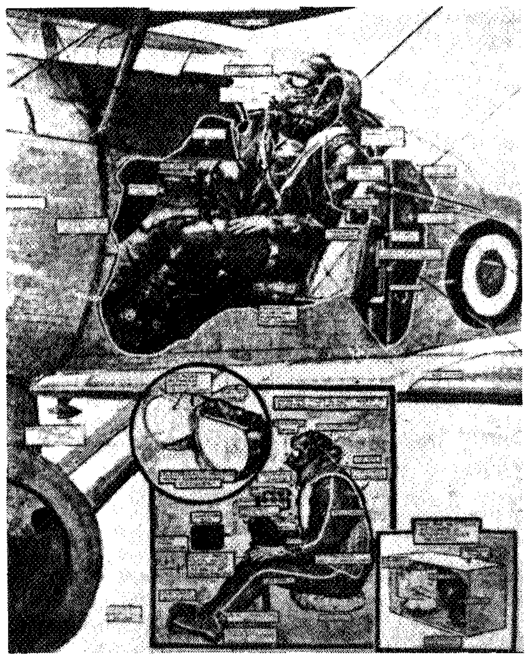


FIGURE 165. Pilot dress for high-altitude flight

The center picture shows the eyeglasses which are heated electrically by thin wires between triple layers of glass (triplex), in order to prevent moisture condensation, and the electric cable.

The picture on the bottom shows how the pilot keeps warm with the aid of electricity; we see the oxygen mask, the helmet, the telephone wires, the contacts for the plugs, the outer fur gloves, the connection box, the leads from the generator, the electrically heated inner cotton gloves, the outer fur boots, the inner boots with electrically heated soles, cable connections, the foot drive, the parachute, the electrically heated waistcoat, cable connections, the heating cable, the manual controls, the fur coat, and the breathing tube.

The picture on the bottom right shows how the altitude is recorded automatically by a barograph provided with window, pen, drum, and casing.

180 15. ACCELEROMETERS

a. General notes

The acceleration must be determined during superaviation flights and also during future interplanetary voyages; instruments called accelerometers are used for this purpose. Most accelerometers presently available measure only the accelerations in directions lying in the plane of symmetry of the airplane and perpendicular to the longitudinal axis of the airplane. The motion of an airplane is usually determined with the aid of three orthogonal coordinate systems having a common origin at the center of gravity of the airplane* (Figure 166). These systems are denoted as follows:

1. The coordinate system (ξ, η, ζ) fixed to the airplane (body axes);
2. " " " (x, y, z) " " " surroundings (wind axes);
3. " " " (X, Y, Z) " " " earth (earth axes)

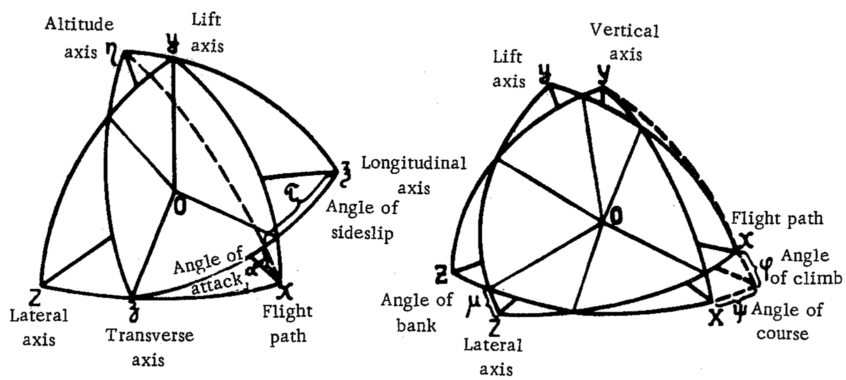


FIGURE 166

The directions of the axes of these coordinate systems are defined in the following tables.

1. Body axes

Designation of axis	Position of axis			Positive direction	Definition
ξ	principal axes of inertia	in plane of symmetry	in direction of fuselage center line	forward	longitudinal axis
η			perpendicular to fuselage center line	upward	altitude axis
ζ		perpendicular to plane of symmetry		to right	transverse axis

* According to Fuchs and Baranoff, ZFM 1927, p.32

181 The relative positions of the coordinate systems are determined by angles. The y-axis (lift axis) of the wind system lies in the $\xi\eta$ plane, i.e., in the plane of symmetry of the airplane so that the position of the airplane in relation to the surroundings is determined by two angles, namely the angle of attack α and the angle of sideslip τ.

2. Wind axes

Designation of axis	Position of axis	Positive direction	Definition
x	direction of airplane motion	opposed to drag	flight-path axis
y	direction of lift	direction of lift	lift axis
z	direction perpendicular to preceding two axes	in direction of lateral force	lateral axis

The position of the wind axes in relation to the coordinate system fixed to the earth are determined by three angles: the angle of bank μ, the angle of climb φ, and the angle of course ψ.

3. Earth axes

Designation of axis	Position of axis	Positive direction	Definition
X	horizontal component of steady longitudinal motion	in direction of longitudinal velocity	–
Y	vertical	upward	vertical axis
Z	horizontal and perpendicular to X	to right	–

The five angles $\alpha, \tau, \mu, \varphi$, and ψ may be used to determine the angular velocities of the airplane about any three axes. We know all six variables necessary for determining the motion of the airplane in space if the flight speed v is also given.

Accelerometers must measure the accelerations in relation to the body axes. Six cases are possible, namely accelerations along any of the three axes ξ, η, z and about these axes. These accelerations affect both the
182 strength of the airplane and the endurance of persons located in it.*

* The motion of an airplane in relation to the earth is determined with the aid of instruments located in the airplane (barographs, navigraphs, etc), as well as with special theodolites used to observe the motion of the airplane from the earth (at least from two points). A so-called "Kymograph" located in the airplane itself is now ordinarily used for this purpose. It reflects the sunlight onto a moving film and thus records the angles which the rays make with the body axes. If the position of the sun in relation to the earth at the instant considered is known, the angles between the body-axes and the earth axes can be determined from the film.

The operating principle of the accelerometer is as follows: the acceleration in any direction can be determined by measuring the force which imparts to a small mass, located in the airplane, the same acceleration that acts on the airplane itself. The deviations of this mass from its zero position may be used to measure the accelerations of the airplane if the mass is linked to it elastically. This method permits quick, and even almost instantaneous determination of the acceleration if the oscillations of the mass are damped rapidly. The mass is therefore connected to the airplane, e.g., by a stiff spring having a high natural frequency, provided in addition with a device causing rapid damping of these oscillations. Flat springs are used in most accelerometers. The deformations of the spring are recorded on a film; the record after calibration serves as a measure of the acceleration.

The simplest method of calibrating an accelerometer is to turn it slowly about a horizontal axis at a constant angular velocity. The spring is then extended in the direction of the radius of rotation. The difference of the forces, acting on the mass m at its top and bottom positions, does not depend on the distance of the mass from the axis of rotation or on the angular velocity, and is equal to 2mg, where g is the gravitational acceleration. The wavy line recorded indicates the distance between the highest and lowest positions of the mass, which is proportional to 2g. Accelerometer springs are made of glass, steel, or quartz. Mercury is sometimes used instead of a spring. However, instruments with springs are obviously preferred. Accelerometers may either indicate the magnitude of the acceleration at a given instant, in which case a pointer moves along a scale, or may trace a curve on a drum rotated by a clockwork mechanism. The ordinates of this curve then indicate the accelerations. The curve is recorded either by a pen or photographically by a reflected light beam.

Accelerometers may be classified in the following way:

A. Linear accelerometers record the accelerations along one coordinate axis, preferably perpendicular to the longitudinal and transverse axes of the airplane.

B. Three-dimensional accelerometers record the accelerations along all three coordinate axes.

C. Angular accelerometers record the angular accelerations of the airplane.

D. Light accelerometers are used at very large flight speeds.

Accelerometers are classified as follows according to the method of suspending the "test mass":

a) torqued pendulums;

b) accelerometers with flat horizontal springs;

c) accelerometers with coil springs, preferably vertical;

d) accelerometers employing liquids or gases;

e) accelerometers with moving masses.

183 Description of the design and operation of accelerometers will be preceded by a short outline of the theory of spring-mass accelerometers. We shall then present a detailed paper by K. Lürenbaum, dealing with the general principles of these instruments, which appeared in the Z. F. M., 1929, No. 2, p. 38.

b) Short outline of theory of spring-mass accelerometers

The following instrument may be used to determine the accelerations when a spaceship coasts only under the action of gravity with its propulsion engines not working (Figure 167). Cylinder ***aa*** contains weight (test mass) ***M***, carrying bar ***b***, ending in pen ***d***, sliding on a paper tape wound on a cylinder rotated by a clockwork mechanism in the direction indicated by the arrow. Spring ***e*** is secured to weight ***M*** and to the bottom of the cylinder. Let the spaceship fly to the left without acceleration. The weight will then be in its zero position in which spring ***e*** is not loaded (mass ***M*** is weightless during coasting).

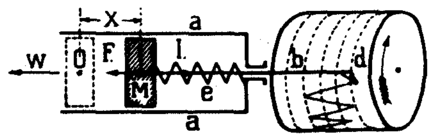


FIGURE 167

Let the spaceship now acquire an acceleration w to the left. Because of its inertia weight M will then move to the right over some distance x, so that the spring is compressed. The magnitude of x depends on the acceleration w, and is recorded or measured on the drum where pen d traces some curve.

Let the mass of weight M be m.

Two forces then act on M. One is the spring force which is proportional to the deformation of the spring: $F = c \cdot x$, where c is constant and equal to the force needed to compress the spring by 1 cm. The other force is due to the inertia of the spring: $J = m \cdot w$.

The differential equation of the motion of weight M is

$$mx'' = J - F = mw - cx,$$

whence

$$mx'' + cx = mw \quad \text{or} \quad w = \frac{c}{m}x + x''.$$

The magnitudes c and m are known while x and x'' are determined from the record on the drum. The above formula thus yields the unknown acceleration w at any instant t.

Let us set up three such instruments so that their axes are directed toward three fixed points in space, e.g., three stars lying in the continuations of the edges of a solid right angle. We can then determine the accelerations of the spaceship, referred to these edges taken as coordinate axes.

The accelerometer springs may be replaced by elastic plates.

184 c) Measurement of accelerations by means of vibrating mass-spring systems

(Translation of Karl Lürenbaum's paper)

α) Introduction

1. Fundamentals

An airplane moving in space may undergo translational accelerations along and angular accelerations about the three coordinate axes.

Many devices have been proposed for the determination of these accelerations. Such devices may be divided into two main groups. The first group comprises devices whose purpose is only to indicate certain limiting accelerations which it is dangerous to exceed. Devices of the second group indicate the accelerations at any instant and thus give their complete range. Accelerometers with oscillating masses belong to this group. Such an instrument consists of a test mass m and a spring restraining it, which is tensioned by some force C. This instrument has one degree of freedom (Figure 168). The symbol m may denote either a mass or a moment of inertia. C is a spring device permitting translational accelerations to be determined. Let such an instrument be suspended from a body undergoing translational accelerations. The indications of the pen connected to the spring will then be a measure of the accelerations of the body. The natural frequency of the instrument is very important with this arrangement; this applies especially to the ratio of the natural frequency to the frequency of the oscillations of the test body.

FIGURE 168

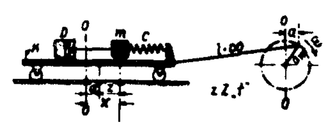


FIGURE 169

The paper reproduced deals only with instruments belonging to the second group. The equations of motion of a mass-spring system are introduced in connection with the measurement of the accelerations of bodies. This work was stimulated by the appearance of many instruments which are not free from shortcomings, known under the name of "accelerometers." It is shown in this paper that satisfactory results are obtained only with accelerometers whose natural frequencies considerably exceed the oscillation frequencies of the test bodies. The limits of applicability of any accelerometric mass-spring system depend on the frequency of its oscillations.

In this case we encounter a phenomenon which is the opposite of that observed when the oscillations of bodies are measured with a stationary instrument: in this latter case the oscillation frequency of the body must be above the natural frequencies of the instrument parts.

A high natural frequency necessitates a stiff spring C and very small displacements of mass m. However, practical considerations require that these displacements be increased; this introduces difficulties in the design.

185
2. Notation.

t [sec] — time;
m [kg sec^2 cm^{-1}] — mass of mass-spring system;
c [kg cm^{-1}] — spring constant of mass-spring system;
$\omega_e = \sqrt{\frac{c}{m}}$ [sec^{-1}] — undampened natural frequency of mass-spring system;
ω_e [sec^{-1}] — damped natural frequency of mass-spring system;
k [kg sec cm^{-1}] — damping constant referred to first power of velocity;
Δ — logarithmic decrement referred to half-period of vibrations;
ε — phase shift of forced vibration with damping;
e — base of natural logarithms;
0—0 — part of system (instrument), which is either fixed or moves with uniform velocity;
K — body whose acceleration in relation to 0—0 has to be measured;
a_m [cm] — maximum oscillation amplitude of body K;
ω [sec^{-1}] — angular frequency of oscillations of body K;
a [cm] — displacement of K in relation to 0—0 at instant t;
x [cm] — displacement of m in relation to **0—0** at instant t;
z [cm] — displacement of m in relation to K at instant t;
$\dot{x}, \dot{a}, \dot{z}, \ldots$ [cm sec^{-1}] — first derivatives of x, a, z, with respect to time $t: \frac{dx}{dt}, \frac{da}{dt}, \frac{dz}{dt}, \ldots$;
$\ddot{x}, \ddot{a}, \ddot{z} \ldots$ [cm sec^{-2}] — second derivatives of x, a, z with respect to time $t: \frac{d^2x}{dt^2}, \frac{d^2a}{dt^2}, \frac{d^2z}{dt^2} \ldots$;
A, B, C — integration constants.

β) Theory of mass-spring system used as accelerometer

1. Setting up the equations of motion

Figure 169 shows a body K (land vehicle, boat, airplane, machine foundation, etc.) performing an accelerated motion (a) in relation to some other system **0—0**. This motion is represented as a displacement in the accelerometer (mass-spring system) secured to body K.

The natural oscillations of the measuring part, whose relative velocity is $\dot{z}$, are eliminated by applying to it a braking force $D = k \cdot \dot{z}$ whose magnitude will be determined later.

D'Alembert's equation of motion is

$$m\ddot{x}+kz+cz=0.$$

However,

$$x=a+z \text{ and } \ddot{x}=\ddot{a}+\ddot{z};$$

hence

$$m(\ddot{a}+\ddot{z})+k\dot{z}+cz=0,$$

$$\ddot{z}+\frac{k}{m}\dot{z}+\omega_e^2 z=-\ddot{a} \qquad (1)$$

186 This is the equation of motion of m relative to K. Its solution is $z=f(t)$. In other words, we must first know the displacement of m along a line rigidly fixed to K, in order to determine the displacement $a=f(t)$ and the acceleration $\ddot{a}=f(t)$.

2. Solution of the equation of motion

The accelerated motion a of body K is often of the form

$$a=a_m \sin \omega t,$$

or in the most general case

$$a=a_{m_0}\sin\omega_0 t+a_{m_1}\sin\omega_1 t+\ldots+a_{m_n}\sin\omega_n t=\sum_0^n a_m \sin\omega t.$$

This motion is represented schematically in Figure 169 as the rotation of a crank and the movement of a connecting rod.

The solution of (1) is

$$z=e^{-\frac{kt}{2m}}.(A\sin\omega_e' t+B\cos\omega_e' t)+C\sin(\omega t-\varepsilon), \qquad (2)$$

which corresponds to the case of natural [damped] and forced vibrations.

Consider first the case when the motion has already taken place for some time at $t=0$ so that the first term in (2), which represents the natural vibrations of the system, has vanished; only the particular solution

$$z=C\sin(\omega t-\varepsilon) \qquad (3)$$

then remains, which represents forced vibrations.

The integration constant C and ε are found by inserting (3) into (1):

$$C=\frac{a_m\,\omega^2}{\sqrt{(\omega_e^2-\omega^2)^2+\left(\frac{k}{m}\omega\right)^2}},$$

$$\varepsilon=\operatorname{arctg}\frac{\frac{k}{m}\omega}{\omega_e^2-\omega^2}. \qquad (4)$$

The solution of (3) is therefore

$$z = \frac{a_m\,\omega^2}{\sqrt{(\omega_e^2-\omega^2)^2+\left(\frac{k}{m}\omega\right)^2}}\sin\left(\omega t-\operatorname{arctg}\frac{\frac{k}{m}\omega}{\omega_e^2-\omega^2}\right). \tag{5}$$

3. Investigation of the equation of motion

For further investigation it is advisable to rewrite (5) in the form

$$z = \underbrace{\frac{1}{\sqrt{\left(\frac{\omega_e^2}{\omega^2}-1\right)^2+\frac{\frac{k^2}{m^2}}{\omega^2}}}}_{v}\underbrace{a_m\sin(\omega t-\varepsilon)}_{\text{distance}} = v\times\text{distance}, \tag{5a}$$

$$z = \underbrace{\frac{1}{\sqrt{\left[\omega_e^2\left(1-\frac{\omega^2}{\omega_e^2}\right)^2+\frac{k^2}{m^2}\frac{\omega^2}{\omega_e^2}\right]\omega_e^2}}}_{v'}\underbrace{a_m\,\omega^2\sin(\omega t-\varepsilon)}_{\text{acceleration}} = v'\times\text{acceleration}. \tag{5b}$$

187 Equation (5a) states that the displacement z of the mass-spring system is equal to the displacement a multiplied by v at an instant at which the phase difference is ε.

Equation (5b) states that under the same conditions the displacement of the mass-spring system is equal to the acceleration $\ddot{a}$ multiplied by v'. The unknown displacements and accelerations can thus be determined if we know v and v' which are functions of the ratios $\frac{\omega_e}{\omega}$ and $\frac{k}{m}$. These magnitudes are found easily if $a = a_m \sin\omega t$ in the case considered.

We can then compute ω; the values of ω_c and $\frac{k}{m}$ are given constants of the mass-spring system. From this we obtain v and v'.

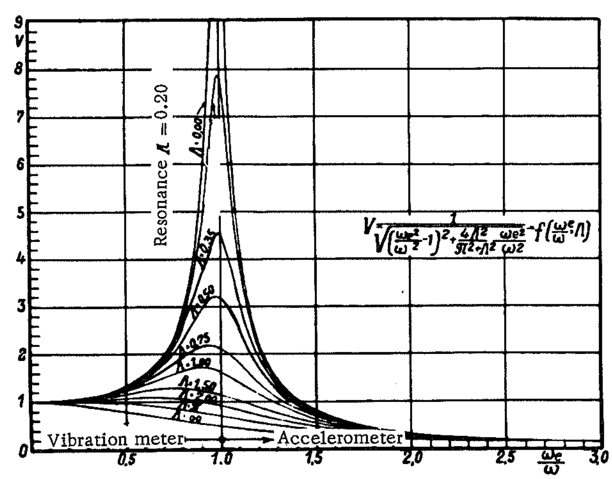


FIGURE 170

An example is Figure 170 which represents the values of

$$v = f\left(\frac{\omega_e}{\omega}, \frac{k}{m}\right) \text{ in the range of } \frac{\omega_e}{\omega} = 0.0 \text{ to } \frac{\omega_e}{\omega} = 3.0$$

at different values of $\frac{h}{m}$.* All data on the two independent variables $\frac{\omega_e}{\omega}$ and $\frac{k}{m}$ (or Λ), and on their influence on v, can be obtained from this diagram.

188 Consider the most frequently encountered case when the acceleration to be measured is the sum of several components of different frequencies, the motion being represented in the form

$$a = a_{m_0} \sin \omega_0 t + a_{m_1} \sin \omega_1 t \ldots + a_{m_n} \sin \omega_n t = \sum_0^n a_m \sin \omega t,$$

the terms being arranged in ascending order of ω. The particular solution in the case of forced vibrations is then, in analogy to (5a) and (5b),

$$z = v_0 [a_{m_0} \sin(\omega_0 t - \varepsilon_0)] + v_1 [a_{m_1} \sin(\omega_1 t - \varepsilon_1)] + \ldots = \sum_0^n v(a_n \sin(\omega t - \varepsilon)) \,. \quad (5c)$$

or

$$z = v_0' [a_{m_0} \omega_e^2 \sin(\omega_0 t - \varepsilon_0)] + v_1' [a_{m_1} \omega_1^2 \sin(\omega_1 t - \varepsilon_1)] + \ldots =$$

$$= \sum_0^n v' [a_m \omega^2 \sin(\omega t - \varepsilon)], \quad (5d)$$

where

$$v_0 = \frac{1}{\sqrt{\left(\frac{\omega_e^2}{\omega_0^2} - 1\right)^2 + \frac{\frac{k^2}{m^2}}{\omega_0^2}}}; \quad v_1 = \frac{1}{\sqrt{\left(\frac{\omega_e^2}{\omega_1^2} - 1\right)^2 + \frac{\frac{k^2}{m^2}}{\omega_1^2}}}; \; v_2 = \ldots\ldots\ldots$$

and

$$v_0' = \frac{1}{\sqrt{\left[\omega_e^2\left(1 - \frac{\omega_0^2}{\omega_e^2}\right)^2 + \frac{k^2}{m^2} \cdot \frac{\omega_0^2}{\omega_e^2}\right]\omega_e^2}};$$

$$v_1' = \frac{1}{\sqrt{\left[\omega_e^2\left(1 - \frac{\omega_1^2}{\omega_e^2}\right)^2 + \frac{k^2}{m^2} \cdot \frac{\omega_1^2}{\omega_e^2}\right]\omega_e^2}}; \ldots v_n' = \ldots\ldots\ldots$$

In practice it is hardly possible to determine the various frequencies between ω_0 and ω_n and the corresponding factors v_0 to v_n or v_0' to v_n' from

* The influence of damping is in Figure 170 expressed in terms of the logarithmic decrement Λ referred to a half-period. This is more convenient in practice, since Λ is determined easily.

this expression for $z=f(t)$, especially since the motions are harmonic. Measurements enable us simply to determine the unknown quantities if we assume that the values v_0 to v_n or v_0' to v_n' do not depend on the frequencies and are respectively equal. This is approximately true for (5c) if

$$\frac{\omega_e}{\omega_0}=\frac{\omega_e}{\omega_1}= \;\ldots\; \frac{\omega_e}{\omega_n}=0$$

or

$$\omega_e<\omega_0,\ \omega_1,\ldots\ldots\omega_n.$$

In this case

$$v_0=v_1=\ldots.\ v_n=(1)$$

and (5c) assumes the simple form

$$z=1\ .\ \sum_0^n a_m \sin(\omega t-\varepsilon) \tag{5e}$$

Writing

$$\frac{\omega_0}{\omega_e}=\frac{\omega_1}{\omega_e}=\ldots\ .\ \frac{\omega_n}{\omega_e}=0,$$

or

$$\omega_e>\omega_0,\ \omega_1,\ldots\ldots\omega_n$$

189 in (5d) we obtain

$$v_0'=v_1'=\ \ldots\ldots v_n'=\frac{1}{\omega_e^2},$$

so that (5d) assumes the form

$$z=\frac{1}{\omega_e^2}\sum_0^n a_m\,\omega^2 \sin(\omega t-\varepsilon)\cdot \tag{5f}$$

The formulas obtained lead to the following conclusions:

1. Let an accelerated motion, expressed in the form

$$a=a_{m_0}\sin\omega_0 t+a_{m_1}\sin\omega_1 t+\ldots.+a_{m_n}\sin\omega_n t=\sum_0^n a_m \sin\omega t,$$

be measured with a mass-spring system whose natural frequency is considerably less than the lowest frequency ω_0. The expression $z = f(t)$, obtained with the aid of the instrument, will then directly yield the amplitude or the displacement $a = f(t)$ of the test body.

All mass-spring systems operating according to this principle are called "vibration meters."

2. Let an accelerated motion, expressed in the form

$$a = a_m \sin \omega_0 t + a_{m_1} \sin \omega_1 t + \ldots + a_{m_n} \sin \omega_n t = \sum_0^n a_m \sin \omega t,$$

be measured with a mass-spring system whose natural frequency is considerably greater than the highest frequency ω_n. The expression $z = f(t)$, obtained with the aid of the instrument, will then yield the acceleration $\ddot{a} = f(t)$ if the multiplier $\frac{1}{\omega_e^2}$, which depends on the instrument, is known.

All mass-spring systems operating according to this principle are called "accelerometers."*

Only instruments of the latter type yield reliable acceleration values. However, it is also possible to measure accelerations with the aid of vibration meters if the results obtained are differentiated twice; this method is, however, laborious and inaccurate.

4. Investigations of impacts

The above considerations refer to the case when the accelerated motion lasts for a certain time, while the natural vibrations of the mass-spring system have already ceased. This steady state is in practice attained very quickly by means of artificial damping; it is, however, of interest to measure sudden accelerations, e.g., impacts, with the aid of the instruments described.

We make, as before, the sufficiently accurate assumption that the path traveled by the body after the impact is part or the whole of a sinusoid; in this case (1) is valid. We may therefore proceed from it.

190 Ordinarily we have the following initial conditions (at $t = 0$):

$$z = 0, \quad \dot{z} = 0.$$

The constants A and B are determined from (2):

$$A = \frac{C}{\omega_e'}\left(\frac{k}{2m}\sin\varepsilon - \omega\cos\varepsilon\right);$$
$$B = C\sin\varepsilon.$$

The distance traveled since instant $t = 0$ is thus

$$z = e^{-\frac{k}{2m}t}\left[\frac{C}{\omega_e'}\left(\frac{k}{2m}\sin\varepsilon - \omega\cos\varepsilon\right)\sin\omega_e' t + C\sin\varepsilon\cos\omega_e' t\right] + C\sin(\omega t - \varepsilon). \qquad (6)$$

* Seismographs, shock meters, vibration meters, etc., employ this principle.

The constants C and ε are given by (4).

We shall now consider two numerical examples in order to determine what influence the use of "vibration meters" ($\omega_e < \omega$) or "accelerometers" ($\omega_e > \omega$) has on measurements of accelerations during impacts. Assuming the mass-spring system to perform a dead-beat motion, we have

$$\frac{k}{2m} = \omega_e; \quad \omega_e' = 0.$$

Equation (6) cannot in this case be used as given, since the first term in square brackets then assumes the form $\frac{0}{0}$.

The limit is found through differentiation with respect to ω_e'. In the case of dead-beat motion (6) then becomes

$$z = e^{-\frac{k}{2m}t}\left[C\left(\frac{k}{2m}\sin\varepsilon - \omega\cos\varepsilon\right)t + C\sin\varepsilon\right] + C\sin(\omega t - \varepsilon). \tag{6a}$$

1st numerical example: $\omega_e < \omega$ (vibration meter),

$$\omega_e = \frac{k}{2m} = 5\,\mathrm{sec}^{-1}; \quad \omega = 20\,\mathrm{sec}^{-1}; \quad a_m = 1\ cm.$$

We then obtain from (4): $C = 0.942$ cm; $\varepsilon = 151°55'$.
Equation (6a) becomes

$$z = e^{-5t}[0.942(5\cdot 0.471 + 20\cdot 0.882)t + 0.942\,.\,0.471] + 0.942\,.\,\sin(20t - 151°55').$$
$$z = e^{-5t}(18.8t + 0.442) + 0.942\sin(20t - 151°55'). \tag{7}$$

2nd numerical example: $\omega_e > \omega$ (accelerometer),

$$\omega_e = \frac{k}{2m} = 20\,\mathrm{sec}^{-1} \quad \omega = 5\,\mathrm{sec}^{-1} \quad a_m = 1\ \mathrm{cm}.$$

By (4) $C = 0.0588$ cm; $\varepsilon = 28°05'$;
and (6a) yields

$$z = e^{-20t}[0.0588(20\,.\,0.471 - 5\,.\,0.882)t + 0.0588\,.\,0.471] + 0.0588\sin(5t - 28°05');$$
$$z = e^{-20t}(0.296 + 0.0277) + 0.0588\sin(5t - 28°05'). \tag{8}$$

Equations (7) and (8) are represented graphically in Figures 171 and 172. It is seen from Figure 171 that the influence of the natural vibrations is strong at the very beginning of the motion, despite its dead-beat nature. The z-curve has ordinates which are 150% larger than those of the impact curve (1.0 sin $20t$). "Vibration meters" are therefore not suitable in the case of impacts.

On the other hand, the natural vibrations play no important role in the case of accelerometers (Figure 172). Their influence has almost vanished after $1/6$ of the duration T of the impact; there remain only the forced vibrations which are a measure of the effects of the impact.

191 An accelerometer gives even more accurate results if artificial damping is not applied to the limit of dead-beat motion.

Damping is generally adequate if it corresponds to a logarithmic decrement $\Lambda = 1.2$ to 3.0, while dead-beat motion is undoubtedly important in a "vibration meter."

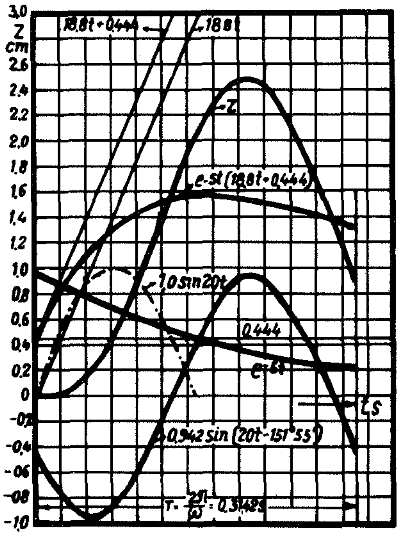


FIGURE 171

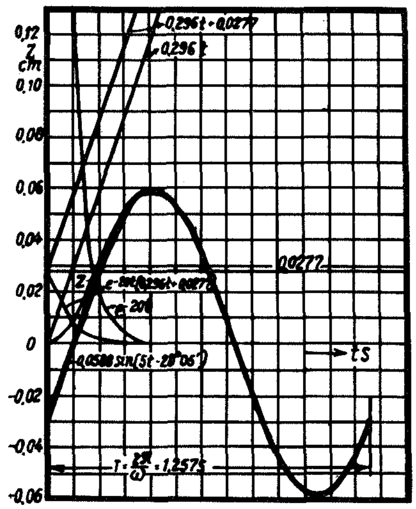


FIGURE 172

γ) Artificial damping

1. The purpose and meaning of damping

It was shown above that artificial damping should rapidly eliminate the natural vibrations of the mass-spring system. We are here considering only damping which depends on the first power of the velocity. Friction which is independent of the velocity (i.e., friction between solid bodies) is not applicable to our case, since no amplitudes which are less than the so-called "constant friction travel" will then be registered by the instrument. Friction depending on the square of the velocity is also inapplicable, since it does not satisfy the condition of isochronism of the natural vibrations.

The natural vibrations are a function of e when damping depends on the velocity [viscous damping]. The larger the exponent $\frac{k}{2m}$, the more rapidly will the amplitude decrease with increasing time, and the more quickly will the natural vibrations become damped. It can be shown that the least time will be required for the natural vibrations to vanish if $\frac{k}{2m} = \omega_e =$ natural

frequency of mass-spring system. This case corresponds to dead-beat motion. There will be no natural vibrations, and the mass will only return from the end of its path to the position of rest without moving beyond. More intense damping is useless and even harmful, since in the case of "super dead-beat motion" the time required for the mass to return to its position of rest will be very long. Damping at the dead-beat limit is most
192 advantageous. It is necessary whenever a "vibration meter" is used.

It is, however, sufficient for accelerometers if $\Lambda' = 1.2$ to 3.6 due to the small influence of the natural vibrations on the forced vibrations.

2. Possibilities of practical realization

Viscous damping requires the use of gases or liquids (air, oil, glycerin, syrup, etc.). A piston connected to the mass moves with small play in a cylinder filled with such a substance. Damping takes place due to friction of the fluid in the narrow clearance between the piston and the cylinder, and is almost proportional to the velocity if the latter is not excessively great. However, the fluid flow pattern in the clearance is altered at high velocities, so that the damping effect is reduced. The temperature also affects the damping.

Magnetic damping is free of these effects, and is frequently used in engineering. A short-circuited conductor (aluminum or copper disc) is secured to the mass, and moves in an electrically created magnetic field. A current, whose intensity is proportional to the velocity, is then generated (Lenz's law). The damping intensity can be varied by altering the magnetic-field strength (e.g., by altering the distance between the poles). Furthermore, the damping effect is practically independent of the temperature.

δ) Enlarging and tracing the readings

The indications of existing mass-spring acceleration meters are so small that they cannot be seen with the unaided eye. They must therefore be enlarged. This is particularly the case with "accelerometers" ($\omega_e > \omega$) where the multiplier v entering in the expression for z is very small (see Figure 170). The simplest means of magnifying the indications is to use a lever system. However, this method has drawbacks, the most important of which is the influence of the mass of the recording device on the accuracy of the readings. The test mass must in this case be increased in proportion to the square of the transmission ratio. This, however, increases the instrument weight. In addition, the use of levers involves friction at the hinges, which is difficult to take into account.

Let us, however, assume that a lever system is used in order to magnify the vibration amplitudes, and that a pen is fixed to the end of the last lever. This pen draws a trace on a tape moving beneath it. The pen must press tightly against the tape in order to leave a trace, but this gives rise to friction which was to be prevented.

However, all these drawbacks can be eliminated if the levers have no mass. This may be achieved by replacing the levers with a light beam whose deflections are magnified by a system of mirrors or microscopes. The use of a light beam and mirrors necessitates the provision of light-weight mechanical transmissions, but this can be achieved in practice. Use of microscopes requires a strong light source but no transmissions.

The tape must be photosensitive so that the deflection of the light beam can be traced on it.

193 Another method is to magnify the motion of the mass-spring system electrically. Small oscillations of the mass can be greatly magnified by means of suitable electric contacts. A shortcoming of this method is the need for mechanical measuring instruments, such as galvanometers or oscillographs, parts of which vibrate themselves. An instrument operating according to this principle is described below.

Magnification by means of magnetic induction is successfully used in seismology and in practical engineering measurements. The principle considered is as follows:

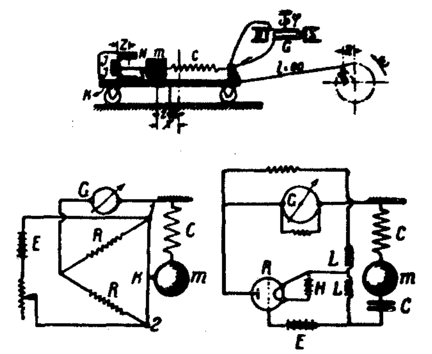


FIGURE 173

Two induction coils (J, J) are fixed to the test mass of the mass-spring system (Figure 173, top). These coils move between two magnetic poles in a field which is as uniform as possible. The coils intersect the lines of force during their motion; currents are thus induced in coils, which according to Lenz's law depend on the velocity $\dot{z}$ of the test mass in relation to the poles. The coefficient of this relationship can be easily determined and given any magnitude.*

This method is convenient when vibration meters ($\omega_e < \omega$) are used for measuring accelerations. It is then only necessary to differentiate $\dot{z}$, which is always possible.

Figure 173, bottom left, illustrates another method of electrical magnification. In this instrument the test mass of the mass-spring system moves contact K along one branch of a Wheatstone bridge. The bridge is adjusted in such a way that no current flows when test mass m is at rest,

* The theory and practice of magnification by magnetic induction is explained in "Lectures on Seismometry" by B. Golitzin, published in German by Teubner, Leipzig, 1914.

so that galvanometer G gives a null reading. Displacement of the left mass from its position of rest induces a proportional reading of the galvanometer, provided the vibration amplitude of m is not excessive.

The instrument shown in Figure 173, bottom right, is far more sensitive. It is based on variations of the electrical capacity. One plate of capacitor C is secured to test mass m of the mass-spring system, while the other plate (on the other side of the air gap) is connected to an ammeter. The capacity varies when the former plate oscillates; this causes changes in the intensity of the current in the anode loop of electronic tube R. These changes are within certain limits proportional to displacements of test mass m, and are indicated by galvanometer G. The sensitivity of the instrument is such that the galvanometer already gives an indication when the capacitor plate is displaced to a distance of 10^{-7} cm.

A galvanometer indicates the displacements of the test mass in all these instruments; the readings are therefore best recorded in the known way, on photosensitive paper by means of mirrors and a light beam. This permits further magnification.

194 ε) Application of theoretical results

1. Recommendations for the design of accelerometers ($\omega_e > \omega$)

a) The natural frequency ω_e of the instrument should be at least 4 to 5 times higher than the vibration frequency ω of the test body. This is achieved by a suitable choice of the ratio $\frac{C}{m} = \omega_e^2$ and the corresponding quantities C and m of the instrument.

b) The equation $m.\ddot{a}_{max} = P_{max}$ yields the maximum force P_{max} acting on the test mass m (the inertia force). From this, knowing m and the maximum acceleration $\ddot{a}_{max}$ we find the maximum amplitude z_{max}.

c) Knowing z_{max}, which ordinarily is very small, we establish the desirable magnification by means of devices which should, if possible, have no mass.

d) It is necessary to provide for damping devices which should ensure approximately dead-beat motion.

e) The accelerometer can be calibrated statically by loading it with weights equivalent to the inertia force acting on m, since the multiplier determining z ($\frac{1}{\omega_e^2} = \frac{1}{C/m}$ (cf. (5 f)) is then obtained from the statical measurements.

2. Example from aviation

The principal design data must be established for an accelerometer to be used for measuring the vertical accelerations of an ordinary airplane during flight. The minimum duration of acceleration during flight is $T = 0.1$ sec, while the maximum vertical acceleration is $\ddot{a}_{max} = 6$ g.

Solution: From $T = 0.1$ sec we obtain

$$\omega = \frac{2\pi}{1/10} = 60 \text{ sec}^{-1}$$

The natural frequency of the accelerometer is

$$\omega_e = \sqrt{\frac{C}{m}} = 5.60 \quad = 300 \text{ sec}^{-1}$$

The test mass of the mass-spring system is

$$m = 0.0002 \ kg \text{ sec}^2/\text{m}.$$

The spring constant is then obtained from the equation $\omega_e^2 = \frac{C}{m}$

as

$$C = 0.0002 \cdot 300^2 = 18 \text{ kg cm}.$$

The force acting on m at the maximum acceleration $\ddot{a}_{max} = 6{,}000 \text{ m/sec}^2$

$$P_{max} = m\ddot{a}_{max} = 0{,}0002 \cdot 6\,000 = 1.2 \text{ kg}.$$

The displacement of m from its position of rest is

$$z_{max} = \frac{1.2}{18} = 0{,}0667 \text{ cm}.$$

195 Optical magnification by 100 yields the following amplitude of the readings:

$$z = 6.67 \text{ cm}.$$

It is of interest to check the value $z_{max} = 0.0667$ cm theoretically. We have

$$\frac{\omega_e}{\omega} = 5/1 \text{ and } \ddot{a}_{max} = 6\,000 \text{ cm/sec}^2,$$
$$\ddot{a}_{max} = a_m \omega^2; \text{ i.e. } 6\,000 = a_m 60^2,$$

whence

$$a_m = \frac{6\,000}{60^2} = 1.667 \text{ cm}.$$

With $\frac{k}{m} = 400$ sec^{-1} we obtain from (5a) the corresponding dynamic amplitude z_{max} of the test mass:

$$z_{max} = \frac{1}{\sqrt{\left(\frac{300^2}{60^2} - 1\right)^2 + \frac{400^2}{60^2}}} \cdot 1.667 = 0.0669 \text{ cm.}$$

This is in satisfactory agreement with the value $z_{max} = 0.0667$ cm, obtained through static calibration.

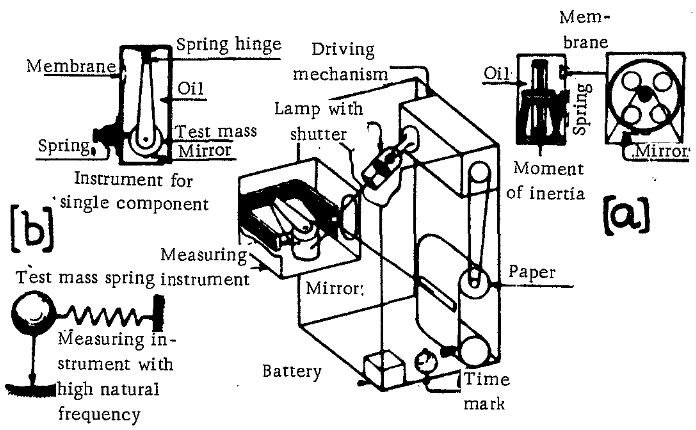


FIGURE 174. Lürenbaum accelerometer

Ing. Lürenbaum proposed an accelerometer based on the mass-spring principle. The vibrations are damped by immersing the test mass in oil. The vibrations are recorded by means of a light beam reflected by a mirror (Figure 174).

Figure 175 represents:

1) The natural vibrations of the test mass and the damping effect of the syrup (top). The vibration period is $T = 0.025$ sec, while $v = 2.6$.

2) The drag of a Junkers A 35 airplane (measured along its longitudinal axis) during uniform straight flight at various rpm ($n = 650$, 1200, and 1400) (three diagrams in center).

3) As above, for a three-engine Junkers G 24 airplane (bottom).

Figure 176, top, represents the natural vibrations of a similar mass-spring system with a low natural frequency and damping by means of oil. The natural-vibration period is $T = 0.38$ sec, while $v = 7$.

196 Figure 176, bottom, represents the longitudinal vibrations of the fuselage of a Junkers A 35 airplane during take-off with vibrating engine. The mean amplitude is $a = 0.5$ mm, while the angular velocity is $\omega = 330\ \text{sec}^{-1}$. The acceleration caused by the engine vibrations is $b = a \cdot \omega^2 = 5.4\ g$.

The piezo effect (use of a quartz block) is employed for measuring small and rapidly varying forces.

Figure 177 shows schematically instruments and the recordings made with them in a large airplane. A Siemens oscillograph was used.

Below we shall describe existing and planned accelerometers.

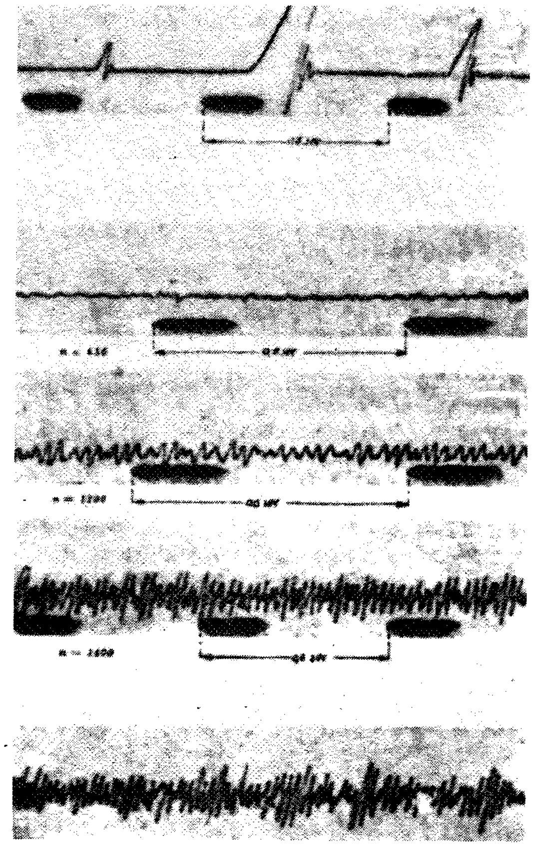


FIGURE 175

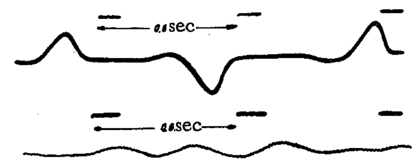


FIGURE 176. Vibration diagram

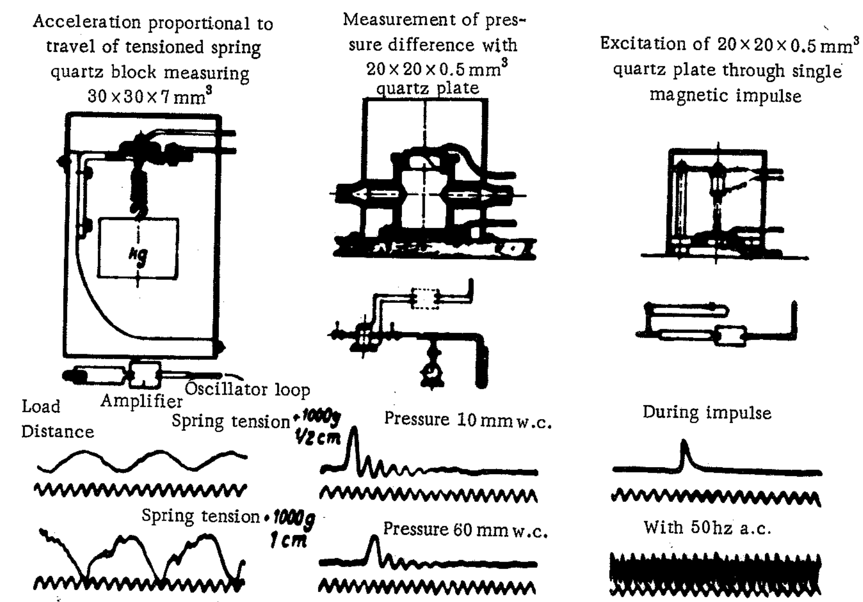


FIGURE 177

197d) Linear accelerometers

α) De Doune´torqued pendulum

The French engineer De Doune proposed the use of a torqued pendulum for determining the accelerations during the nonuniform motion of bodies. This was to be done by measuring the angle of inclination. His method is suitable both within the gravitational field of the earth and for interplanetary flight along a circular path (g = const).

We shall consider two cases: a) constant acceleration; b) variable acceleration.

a) Constant acceleration

Let a spaceship move within the gravitational field of the earth in the direction of arrow a (Figure 178) at a constant acceleration w, which has to be determined. This acceleration causes a pendulum, suspended from the ceiling of the spaceship, to be deflected by a constant angle (α). Three forces, which are in equilibrium, then act on the pendulum. These are the inertia force $J = m.w$; the weight $P = mg$, where m is the mass of the pendulum and g is the gravitational acceleration, and the string tension N.

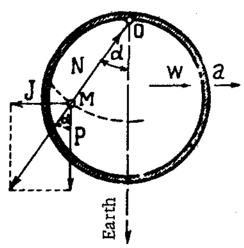


FIGURE 178

From the right-angled triangle we obtain:

$$\frac{J}{P} = \frac{mw}{mg} = \mathrm{tg}\,\alpha,$$

whence the unknown acceleration is

$$w = g\,\mathrm{tg}\,\alpha. \qquad (1)$$

b) Variable acceleration

The angle α will vary in this case. Let this angle be denoted by φ, and determine the relationship between w and φ. We shall establish a differential equation for φ, on the basis of the moment rule which states that "the derivative, with respect to time, of the angular momentum about some axis, is equal to the sum of the moments of all forces, referred to the same axis." Let this axis pass through the point of suspension (0) of the pendulum, and be perpendicular to the plane of the paper.

Let the length OM of the pendulum be denoted by l, its mass by m, and its tangential velocity by v. The momentum of the pendulum is then mv while its angular momentum about axis 0 is $m.v\,l$.

Expressing the linear (tangential) velocity in terms of the angular velocity, we obtain

$$v = l\varphi'.$$

The angular momentum thus is $ml^2\varphi'$, and its derivative with respect to time is $ml^2\varphi''$.

The moment of the weight is $Pl\sin\varphi$.

The moment of the inertia force is $J.l\cos\varphi = mwl\cos\varphi$.

The above-cited rule yields the differential equation

$$nl^3\varphi' = -Pl\sin\varphi + mwl\cos\varphi$$

or

$$\varphi'' + \frac{g}{l}\sin\varphi = \frac{w}{l}\cos\varphi. \qquad (1)$$

198 Hence

$$w = g\,\mathrm{tg}\,\varphi + \frac{l\varphi''}{\cos\varphi}. \qquad (2)$$

This formula differs from (1) only by the additional term $\frac{l\varphi''}{\cos\varphi}$.

We can determine φ'' and thence w by observing the variation of φ with time.

The relationship between the angle φ and the time is usually given by an instrument as a curve, so that graphical differentiation may be employed.

The simplicity of (1) in comparison with (2) induces us to check whether it might not be used instead of (2) also in the case of variable accelerations. Computations show that (1) may be used in this case if a certain error is accepted, which will be the smaller, the higher the natural frequency of the pendulum (or the smaller its period).

We shall prove this under the assumption that the angle φ is small at any time (this is true when the acceleration varies little).

Equation (1') then becomes

$$\varphi'' + \frac{g}{l}\varphi = \frac{w}{l} \qquad (3)$$

Let us integrate this equation.

We assume that the acceleration w increases linearly with the time from 0 to some final value w_0 at time τ.

We may thus write

$$w = \frac{w_0}{\tau}\,.\,t.$$

Inserting this value into (3), we obtain

$$\varphi'' + \frac{g}{l}\varphi = \frac{w_0}{l\tau}\,t.$$

This is a nonhomogeneous linear differential equation with constant coefficients, whose particular integral is

$$\varphi = \frac{w_0}{g\tau} t.$$

Adding to this the general solution of the corresponding homogeneous equation [i.e., the complementary function], we obtain the complete integral of (3):

$$\varphi = \frac{w_0}{g\tau} t + C_1 \cos kt + C_2 \sin kt, \tag{4}$$

where $k = \sqrt{\frac{g}{l}}$, while C_1 and C_2 are integration constants.

Setting $\frac{w_0}{\tau}. t = w$ in the first term on the right-hand side of (4), we obtain

$$\varphi = \frac{w}{g} + C_1 \cos kt + C_2 \sin kt.$$

We determine C_1 and C_2 from the initial conditions $\varphi = 0$, $\varphi' = 0$ at $t = 0$. This yields $C_1 = 0, C_2 = -\frac{w_0}{gk\tau}$. Inserting these values into (4), we obtain

$$\varphi = \frac{w_0}{g\tau} t - \frac{w_0}{gk\tau} . \sin kt. \tag{5}$$

199 It is seen that the amplitude a of the natural oscillations of the pendulum is

$$a = \frac{w_0}{gk\tau} = \frac{w_0}{2\pi g} \cdot \frac{T}{\tau},$$

where T is the period of these oscillations,

It follows from this that the amplitude varies directly as $\frac{T}{\tau}$.

Setting $w =$ const and $\varphi =$ const in (3), we obtain

$$\varphi = \frac{w}{g}. \tag{6}$$

The actual value of φ, determined from (5), is at any instant the closer to that given by (6), the smaller the ratio $\frac{T}{\tau}$, as was to be proved.

β) Accelerometers with springs

Accelerometer with system of weights

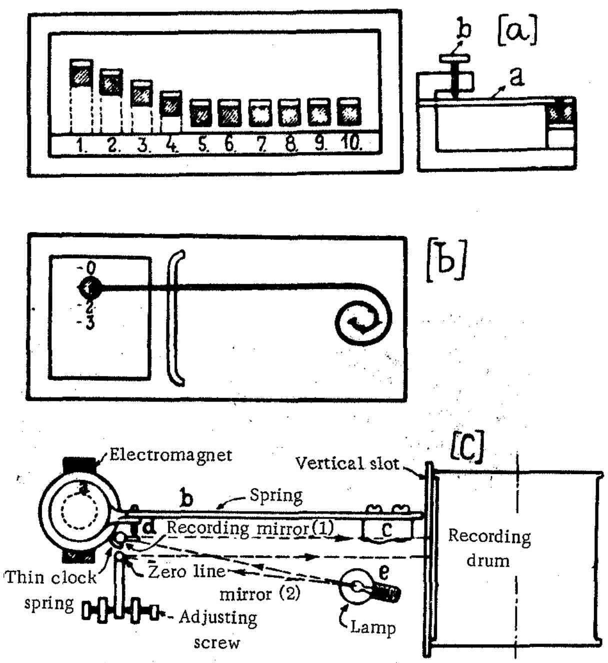


FIGURE 179

Bennewitz in his book "Flugzeuginstrumente" (Airplane Instruments), Berlin, 1922, p. 218 describes an accelerometer of the following design (Figure 179): Ten different weights are secured to springs (a) clamped to a frame. The lighter weights (1 to 4) are deflected upward first when an acceleration acts vertically downward being followed by the heavier weights (5 to 10) when this acceleration is increased. The deflections of the weights are adjusted by means of screws (b). The natural oscillations of the weights can be damped by immersing the whole system in liquid or compressed gas. Accelerations of the opposite sign are measured by turning the instrument upside down.

German light beam accelerometer

An accelerometer with various weights, switching on different lamps during acceleration, was designed in Germany. All weights are forced along one line against a board when no acceleration occurs. The smaller weights are deflected away from the board against the action of springs when an acceleration takes place.

200 **Glider accelerometer**

The automatically recording accelerometer shown in Figure 179b was designed for the All-Union Gliding Tests held at Feodosia.

Norton Accelerometer

An accelerometer for determining the accelerations of airplanes was built in the U.S.A. by Messrs. Emerson according to a design by Norton. This instrument is described in NACA Reports 99 and 100.

This accelerometer is shown in Figures 180 and 181.

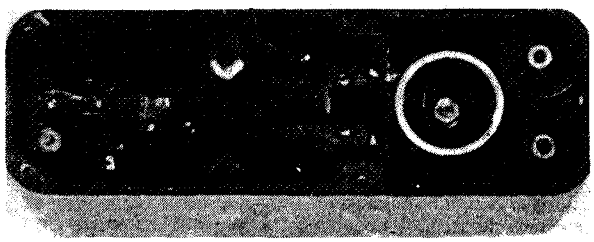


FIGURE 180. Norton accelerometer (view from above)

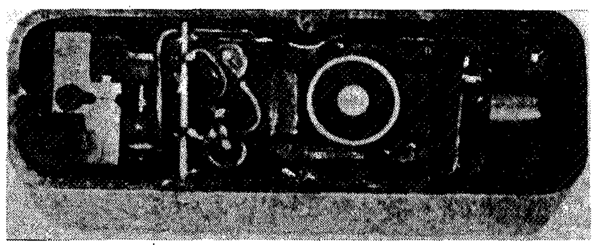


FIGURE 181. Norton accelerometer (view from below)

U. S. accelerograph

Figure 179c shows a U. S. accelerograph recording the accelerations of an airplane along the vertical axis. Its design is as follows: Light-weight disc-shaped aluminum weight (a) is secured to flat spring (b) whose end is fixed by screws to support (c) secured to the airplane. Any acceleration causes the weight to leave its equilibrium position so that pin (d) turns mirror (1) which directs a light beam from lamp (e) through a slot in the casing of a recording drum; a curve characterizing the acceleration is thus traced on the drum.

Fixed mirror (2) reflects onto the drum another light beam which
201 traces the zero line. The oscillations of weight (a) are damped since it is located between the poles of an electromagnet.

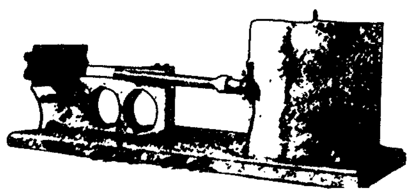


FIGURE 182. N.A.C.A. accelerograph

Figure 182 is an overall view of this instrument. The spring has a natural-vibration period of 0.014 sec.

Veselovskii accelerometer

Figure 183 shows the Veselovskii accelerometer. The accelerations are in this instrument automatically recorded on a drum. This is done by a pen connected to a weight secured to a flat spring. The theory of this instrument has not been given, nor is the natural frequency of the vibrating system stated.

D. V. L. Accelerograph

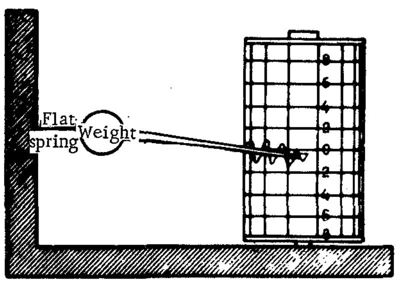
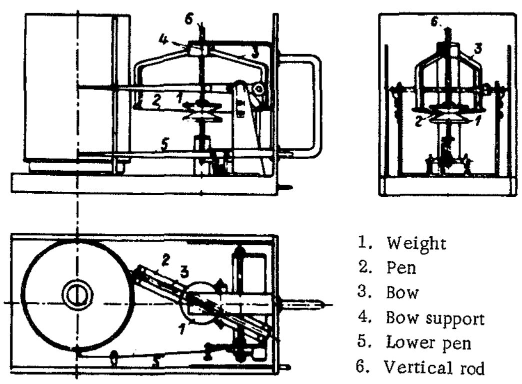


FIGURE 183



1. Weight
2. Pen
3. Bow
4. Bow support
5. Lower pen
6. Vertical rod

FIGURE 184. D.V.L. accelerometer

An instrument for determining the accelerations along the vertical axis of an airplane was built and used in 1926 at the aeronautical testing ground at Aldershot near Berlin [should read "Nearer to Berlin than to Moscow"].* The instrument is designed as follows: a vertical acceleration causes a 50-g weight (1) to oscillate on spring (2) along rod (6) (Figure 184, top). The spring is secured to the ends of bow (3) clamped at its summit in support
202 (4). The vibrations of the spring are transmitted to a pen tracing the

* Wendroth, H. and G. Wolle "Aufbau und Eigenschaften des D.V.L. Beschleunigungsschreibers YV" (Design and Properties of the D.V.L. Acceleration Recorder Type YV). Z.F.M. p.26, p.532.

acceleration curve on a drum. Fixed pen (5) draws the base line on the drum. Figure 185 is an overall view of the instrument. It can be connected to the airplane, either rigidly or elastically.

Consider both cases.

1. Instrument rigidly connected to airplane

Rapid damping of the oscillations of the weight (dead-beat motion) is of great importance in this case. Let P be the force to be measured, which depends on the acceleration whose final value is attained after a certain time interval t_1. This variation of the force may be represented graphically (Figure 186 full line) at $t \leqslant t_1$ $P = pt$; at $t \geqslant t_1$, $P =$ const.

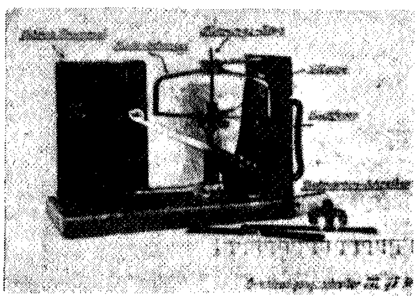


FIGURE 185. D.V.L.accelerograph

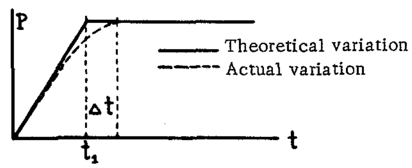


FIGURE 186

The equation of motion of the mass-spring system is for $t \leqslant t_1$:

$$\frac{md^2x}{dt^2} = pt - cx - \frac{kdx}{dt}.$$

The design of the recording instrument is such that dead-beat motion ensues rapidly when the mass-spring system is brought out of its equilibrium state.

The solution of the equation of motion is written in the form

$$x = A_1 e^{\lambda t} + A_2 t e^{\lambda t} + \frac{p}{c} t - k \frac{p}{c^2}.$$

For this function to be identical with the complete integral of the equation of motion we must have

$$m\lambda^2 + c + k\lambda = 0; \qquad \lambda = \frac{-k}{2m} \pm \sqrt{\frac{k^2}{4m^2} - \frac{c}{m}}.$$

Dead-beat motion occurs when the root is real, i.e., if $\frac{k^2}{4m^2} \geqslant \frac{c}{m}$.

The minimum value of the damping constant, at which dead-beat motion occurs, is $k = 2\sqrt{cm}$.

Hence

$$\lambda = -\sqrt{\frac{c}{m}}.$$

For the instrument considered $c = 753.981\ \frac{g}{s}$, $m = 50\,g$.

The constants A_1 and A_2 are obtained from the initial conditions ($t = 0$, and $x = 0$ and $\frac{dx}{dt} = 0$).

203 Inserting the values obtained into the solution of the equation of motion, we obtain

$$x = 2\frac{p}{c^2}\sqrt{c\cdot m}\cdot e^{-\sqrt{\frac{c}{m}}\cdot t} + \frac{p}{c}\cdot t\cdot e^{-\sqrt{\frac{c}{m}}\cdot t} + \frac{p}{c}t - \frac{2p}{c^2}\sqrt{cm} \text{ and}$$

$$\frac{dx}{dt} = -2\frac{p}{c}e^{-\sqrt{\frac{c}{m}}\,t} + \frac{p}{c}e^{-\sqrt{\frac{c}{m}}\,t} - \frac{p}{c}\sqrt{\frac{c}{m}}\cdot t\cdot e^{-\sqrt{\frac{c}{m}}\cdot t} + \frac{p}{c}.$$

The acceleration attains its final value P after a time interval t_1, and thereafter remains constant.

The equation of motion then is

$$m\frac{d^2x}{dt^2} = P - cx - k\frac{dx}{dt},$$

whose complete integral is

$$x = B_1e^{\lambda t} + B_2te^{\lambda t} + \frac{P}{c}.$$

The integration constants are determined from the initial conditions: at time t_1 the mass has traveled over a distance x_1, and its velocity is $\frac{dx}{dt}$.

The variation, with time, of the distance traversed is shown in Figure 186, by the broken line [Figure 186 in fact shows the variation, with time, of the force acting on the mass, and not as stated by the author].

In actual computations we assume t_1 to be between 0 and 1 sec, and P = 5 g [where the meanings of P and g are ambiguous].

Computations show that the test mass moves for no longer than $\Delta t \leqslant 0.1$ sec before it assumes a new equilibrium position, after which the acceleration attains its final value.

The acceleration to be measured lasts longer than 0.1 sec, so that the indications of the instrument contain no error and are not affected by the rate of acceleration increase.

2. Instrument elastically connected to airplane

The instrument weighed 1.5 kg and stretched its rubber suspension by 0.5 cm. The box containing the instrument did not oscillate, due to the suspension and the presence of damping devices (the oscillations due to the extension of the rubber suspension did not last more than 0.1 sec), so that the readings were not affected.

The instrument thus gives sufficiently accurate indications if the acceleration to be measured remains constant for not less than 0.1 sec. This is nearly always the case in practice.

Computation example. Spring length $l = 9.9$ cm; spring modulus of elasticity $E = 2{,}200{,}000$ kg/cm^2; spring width $b = 1.2$ cm; spring thickness $h = 0.05$ cm; spring mass $m = 50$ g, inertia of spring $J = \frac{b\lambda^3}{12} = 0.0000125$; angular velocity:

$$\lambda^2 = \frac{48 \cdot E \cdot J \cdot g}{l^3 G} = \frac{48 \cdot 2200000 \cdot 0.0000125 \cdot 981}{9.9^3 \cdot 0.05}$$

$$n = \frac{\lambda}{2\pi} = 83 \text{ sec}^{-1} \cong 85 \text{ sec}^{-1}.$$

204 **Zahm shock recorder**

The Zahm shock recorder consists of several pointed bars secured perpendicularly to the airplane wing and suspended each from a separate coil spring. Sudden descents or ascents of the airplane cause the points to strike a tape on a rotating drum. The various bars are suspended from springs of different rigidities, so that bars mark the tape, depending on the strength of the shock. Details of this instrument are given in "Development of an Aeroplane Shock Recorder," by A. F. Zahm, Journal of the Franklin Institute, Vol. 188, 1919.

V. P. Vetchinkin accelerometer

Figure 187 shows the accelerometer developed by V. P. Vetchinkin in 1916. Its operation is based on the action of weight (p), suspended on springs and stretching or compressing the latter by its inertia.

Two bars (d) serve to guide the weight in frame (k). During its motion the weight moves one of indicators (n) showing the extreme positions reached. A pointer fixed to the weight indicates on the scale the ratio of the instantaneous load to that acting on the wings due only to gravity, i.e., the weight of the airplane.

The instrument is calibrated as follows:

The pointer indicates zero on the scale when the frame is in a horizontal position so that both springs are tensioned equally.*

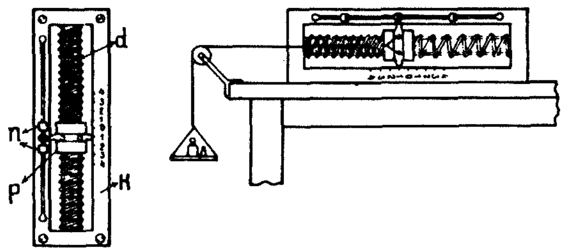


FIGURE 187. Vetchinkin accelerometer

Placing the frame in a vertical position causes weight (*p*) to stretch the upper spring, under the influence of gravity, so that the pointer comes to rest against Figure 1 on the scale. This position corresponds to horizontal flight of an airplane.

The frame is then again set up horizontally, and the tension of the upper spring, at which the pointer indicates 1, is determined with the aid of a pulley and small weights. This load is added to (*p*). The pointer will then indicate 2 if the frame is again placed in a vertical position. This corresponds to the case when a vertical inertia force acts on the airplane during flight.
205 Double this load added to (*p*) causes the pointer to indicate 3, etc. Turning the frame upside down vertically causes the pointer to indicate 1. This corresponds, e.g., to a sudden transition from horizontal flight to a dive.

The negative part of the scale is calibrated in the same manner as the positive part, but on the opposite side. The instrument is set up in the airplane by placing the latter in the direction of flight and securing frame (*к*) so that guide bars (*d*) are in a vertical position. Extreme-position indicators (*n*) are moved close to the weight after take-off. Their indications are read off at certain time intervals, after which they are again placed close to the weight.

The **advantages** of this instrument are its simple design, the ease of its construction, and the presence of the above-mentioned indicators showing the extreme positions of the weight. A shortcoming of the instrument is the incomplete indication, since it shows accelerations only along one axis, whereas accelerations may occur also in directions which do not coincide with that of the guide bars. This may cause friction between the bars and the weight, preventing the latter from taking up the correct position.

We present the analysis of the spring accelerometer, given by Prof. Vetchinkin for the instrument described, and published in the first issue of Trudy aviatsionnogo otdela letuchei laboratorii, p. 16. Moskva. 1918.

* This case corresponds to coasting (unaccelerated flight).

Let the weight be Q. Half the weight of the spring has to be taken into account in computing its deformation, i.e., $\frac{1}{2}q$, where q is the weight of the spring.

The weight entering in the computations is thus

$$P_0 = Q + \frac{1}{2}q.$$

The sensitivity of the instrument is determined from the deformation λ_0 induced by the weight P_0:

$$\frac{\lambda_0}{P_0} = \frac{\lambda_{max}}{P_{max}} = \frac{\lambda}{P}.$$

Both springs must have the same number of turns.

The stress in the material of the spring must not exceed the permissible limit t when the maximum load P_{max} due to inertia forces acts.

The effects of prestressing the spring must be taken into account. Both springs must always be stretched in order to ensure accurate operation of the instrument and linearity of its indications (compression would cause bending of the springs so that their stiffness would change, as well as friction between the springs and the guide bars).

The shear modulus of steel is

$$G = 8.5 \cdot 10^5 \text{ to } 9 \cdot 10^5 \text{ kg/cm}^2 = 8.5 \cdot 10^3 \text{ to } 9 \cdot 10^3 \text{ kg/cm}^2.$$

The number of threads per spring is n.

The wire diameter is d.

The polar moment of inertia of the spring wire is

$$J_0 = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}.$$

The radius of the spring coil is $R = \frac{D}{2}$.

The radius of the spring wire is $r = \frac{d}{2}$.

206 The polar modulus of the section of the spring wire is

$$W_0 = \frac{\pi r^3}{2} = \frac{\pi d^3}{16}.$$

The wire length is L:

$$L \cong 2\pi R n = \pi D n.$$

The torque acting on the spring is $M = P \cdot R = \frac{P \cdot D}{2}$.

The torsion angle of the wire is

$$\varphi = \frac{ML}{GJ_0} = \frac{P \cdot R \quad 2\pi Rn \cdot 2}{G\pi r^4} = \frac{4nR^2}{Gr^4} \quad P.$$

The deformation of the spring is

$$\lambda = R\varphi = \frac{4nR^3}{Gr^4} \cdot P = \frac{8nD^3}{Gd^4} P.$$

The torsional stress in the wire is

$$t = \frac{M}{W_0} = \frac{2R}{\pi r^3} P = \frac{8D}{\pi d^3} P$$

A weight suspended between two springs may be assumed to be suspended from two tensioned springs when the deformations of the latter are computed.

The shorter spring of the two carries the larger load. The reciprocal of the deformation of [two springs arranged in parallel] is equal to the sum of the reciprocals of the deformations of the individual springs, due to the same load.

The vibration period of this system is found by adding to the mass of the weight one third of the mass of the springs; the springs are then assumed to be weightless.

The vibration period thus is

$$\tau = 2\pi \sqrt{\frac{Q + \frac{q}{3} l}{gEF}} = 2\pi \sqrt{\frac{\lambda'}{g}},$$

where EF is the equivalent stiffness of the springs in tension, and λ' is the deformation due to a weight

$$Q + \frac{q}{3}.$$

The actual deformation is

$$\lambda_0 = \frac{8nD^3}{Gd^4} \left(Q + \frac{q}{3}\right).$$

Finally,

$$\tau = 2\pi \sqrt{\frac{8nD^3}{Gd^4} \cdot \frac{Q + \frac{1}{3} q}{g}}.$$

Numerical example: $Q = 0.4$ kg; $\lambda_0 = 200$ mm; $P^{max} = 5Q = 2$ kg, i.e., five times the weight; $\lambda_{max} = 100$ mm; $d = 1.5$ mm; $d^4 = 5.0625$ mm^4; $t_{max} = 80$ kg/mm^2; $\frac{\lambda}{P} = \frac{20}{0.4} = 50$ mm/kg; $\frac{G}{8} \cdot \frac{\lambda}{P} = 55 \cdot 10^9 = \frac{nD^3}{d^4}$.

For the springs selected $nD^3 = (275 \text{ to } 280)\cdot 10^3$ mm^3.

D	n	t
10	278	15.1
15	81	22.6
20	35	30.2
25	17.8	37.8
30	10.3	45.3
35	6.46	53.0
40	4.35	60.6
45	3.06	68.0
50	2.25	75.5

207 Two springs are provided, so that each should have 2 n turns;

$t = \frac{8D}{\pi d^3} P = 0.76\ P \cdot D$ kg/mm^2, $t_{max} = 1.51\ D$ kg/mm^2.

Let us take two springs with $D = 30$, $d = 1.5$, $n = 21$ each.

The volume of each spring is 3,500 mm^3 = 3.5 cm^3; its weight is $\frac{q}{2}$ = 27.5g.

The two springs weigh 55 g together. The vibration period is

$$\tau = 2\pi\sqrt{\frac{0.02}{9.81}} = 0.283 \text{ sec.}$$

The initial length of each spring is 63 mm. The spacing of the turns is thus 3 mm, or twice the wire diameter.

Klemperer accelerometer

The Klemperer accelerometer, built by Messrs. Zeiss, differs from other types in that it has no flat spring and gives a direct indication of the acceleration without recording it. The instrument with a weight on its surface is suspended from a coil spring. Accelerations perpendicular to the airplane wings and directed downward stretch the spring. The magnitude of the rotation of the instrument is a measure of the acceleration. A similar instrument is located along the same axis but with its weight secured to the opposite side, so that the rotation during descent is in the opposite sense. A third instrument is located between the former two in a plane perpendicular to that in which the other instruments are located. This instrument is connected with the others so as to form an epicyclic chain, due to which the first two instruments are deflected uniformly. The device as a whole measures only vertical accelerations in the downward direction, and is not affected by angular accelerations.

We were unfortunately unable to obtain more detailed information or drawings of this instrument. V. G. Nemchinov in his book "Aviatsionnye pribory" (Aeronautical Instruments), Moscow, 1921, p. 180, gives a description of the Klemperer accelerometer, which is presented below, as well as a drawing of it. It is, however, impossible to deduce from this information whether this is the instrument mentioned and how it works.

Klemperer accelerometer

The German Klemperer accelerometer is based on the fact that a weight stretches a spring under the action of an acceleration, and thus moves a pointer along a circular scale (Figure 188a).

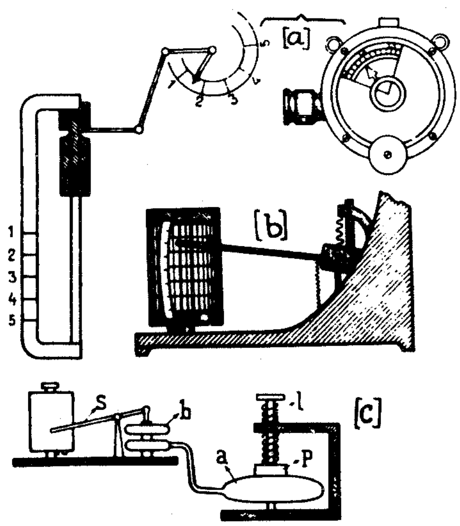


FIGURE 188

208 The shortcomings of this instrument are a) that it does not indicate negative accelerations, and b) that it has no indicators of the extreme positions of the pointer, whose lack makes reading of the maximum indications unreliable.

Hertz accelerometer

The Hertz accelerometer employs a spring. A lever with a pen is secured to a weight suspended from a spring. A pointer fixed to the end of the recording lever indicates the relative loads on a circular scale (Figure 188b).

Recording accelerometer

V. G. Neminchov in his book "Aviatsionnye pribory" (Aeronautical Instruments), Moscow, 1921, p. 182, describes the recording accelerometer which is schematically shown in Figure 188c. Weight (p) rests on top of metal box (a) which has flexible walls and is filled with nitrogen. The weight is provided with guide rod (l). Part of the nitrogen flows into bellows (*b*) when the pressure in box (*a*) increases; this causes recording lever (*s*) to move.

Skrieb mass-spring shock recorder (Figure 189a)

Ing. Skrieb proposed to measure the accelerations occurring during shocks with an instrument operating according to the following principle:

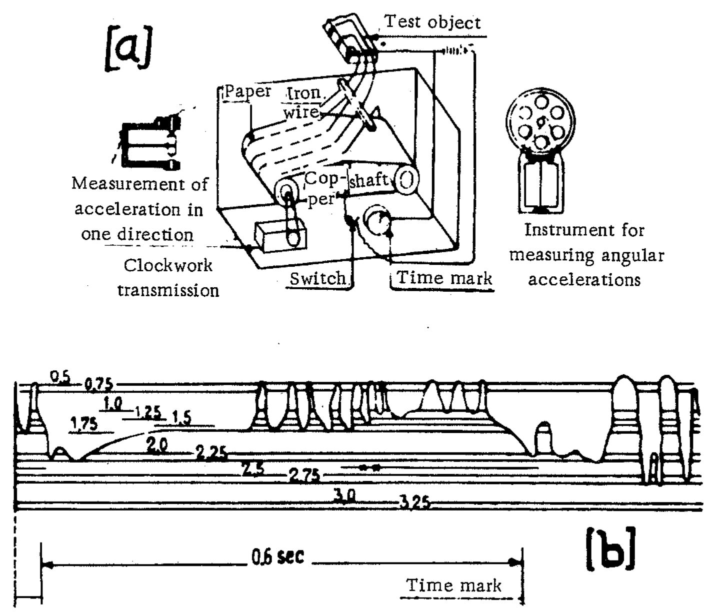


FIGURE 189

The instrument includes a mass-spring system. An electric current is interrupted when the acceleration attains a certain limit. This causes a blue mark to be made on a paper tape, wound on a drum, by a steel pen and potassium ferrocyanide.

The instrument is placed at the required point of the airplane. The various parts of the instrument are set into motion by means of a clockwork mechanism. The tape speed is 40 cm/sec, so that 0.01 sec corresponds to 4 mm. Figure 189b shows an acceleration diagram taken during takeoff of a Junkers A 35 airplane. The instrument was located in the fuselage along it vertical axis.

Sirley-Lindeman accelerometer

Between 1917 and 1919 Sirley in the U. K. designed several types of accelerometers for measuring the accelerations of airplanes during flight.
209 The first accelerometer, designed by him in collaboration with Lindeman, measures only the accelerations along one coordinate axis. This instrument consists of a thin glass thread of 0.01 mm diameter, bent to form a semicircular arc having a radius of approximately 1.3 mm. The thread is enclosed in a box; both ends of the thread are fixed to two opposite walls of the box so that the plane of the thread center line is horizontal. The instrument can thus be used to measure the accelerations of, for example, an elevator. The semicircular portion of the thread remains free and serves both as test mass and as spring. The thread is deflected downward when the box undergoes an upward acceleration; this deflection varies directly with the magnitude of the acceleration. The period of the undamped vibrations of the thread is 0.05 sec, but its vibrations are rapidly damped due to the air in the box. The vibrations of the thread are recorded on a photosensitive film, moving around the box, by a light beam projected from a lamp onto the thread.

Zeiss accelerometer

The Zeiss accelerometer contains a circular scale moved by weights in different directions. The scale is connected to a spring and a pointer. A shortcoming of this instrument is the indistinct reading, due to the combination of a moving scale and a moving pointer. *

γ) Accelerometers employing liquids or gases

Bennewitz accelerometer

Bennewitz in Germany proposed an instrument in the form of a glass tube closed at both ends and bent at the bottom, filled with mercury and gas in both branches (Figure 190, left). The mercury columns in the two branches are of unequal height since the gas pressures above them differ. These levels will vary during accelerations, so that the mercury meniscus serves as acceleration indicator. The instrument is suspended in a universal joint.

A shortcoming of this accelerometer is the fact that part of the gases in the two branches may mingle at certain positions of the instrument, so that the calibration loses its validity even if the instrument does not become completely useless. These remarks apply to an even larger degree to the instrument shown in Figure 190 on the right, also proposed by Bennewitz, in which two liquids of different specific weights are used.

H. M. P. accelerometer

Three French scientists, Huguenard, Magnan, and Planiol, proposed accelerographs which were built with different sensitivities and which they used in investigating the forces and accelerations encountered during airplane flight. The operating principle of these instruments is as follows (Figure 191): Tube T contains mercury whose inertia is used to measure the acceleration. The mercury affects the pressure difference at the two ends of the tube. On the right-hand side the pressure is transmitted
210 to glycerin in tube r, and is regulated by means of cock R. The pressure of the glycerin is transmitted to pressure gauge M which has a short oscillation period. Vessel A with compressed air is connected to the other end of tube T. This vessel is provided with bicycle-tube valve v through which air may be forced into it. The instrument makes it possible to measure accelerations of either sign.

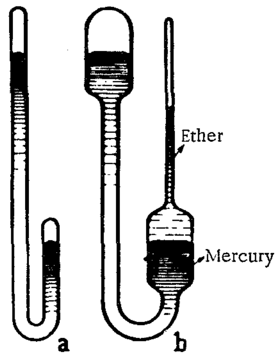


FIGURE 190. Bennewitz accelerometer

The instrument is calibrated in the following manner: Tube T is first placed horizontally and vessel A is opened to the atmosphere. The pressure gauge should then indicate zero. The instrument is then set up vertically, so that the pointer indicates a value corresponding to the gravitational acceleration g. The instrument is then returned to the horizontal position and air is pumped into vessel A until the pointer again indicates g. The instrument is then again set up vertically, the position of the pointer is marked as 2 g, and so forth.

The above-cited scientists designed four types of instruments. Instrument No. 1 had a pointer oscillation period of 0.1 sec and was used during flights without maneuvers. Instrument No. 2 had a pointer oscillation period which was one fourth of that of No.1, and was insensitive to engine vibrations. Instrument No.3 had a pointer oscillation period of $^1/_{50}$ to $^1/_{75}$ sec and a pen stroke of 1 to 3 cm. Instrument No.4 was equipped for photographing the indications.

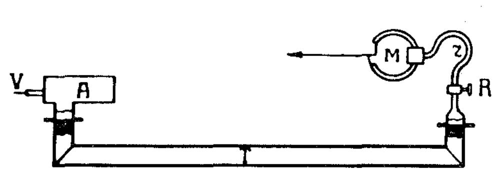


FIGURE 191. H.M.P.accelerometer

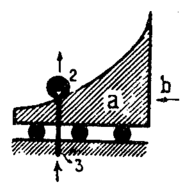


FIGURE 192

Figure 193 is an overall view of the instrument, while Figure 194 shows details of its layout. The parts are numbered as follows: casing — 1, tube with glycerin — 2, regulating cock — 3, diaphragms — 4, and 23, bolt fixing pressure gauge (6) to frame — 5, lugs — 7, 8, and 9, tube with mercury — 10, roll with paper — 11, movable carriage — 12, lever — 13, nut securing drum — 14, clockwork mechanism — 15, strip for unfolding paper — 16, brake — 17, nut connecting (16) to (12) — 18, pen — 19, additional lever with pen — 20, support of lever (20) — 21 and 22, vessel with compressed air — 24.

The pressure indicated by the gauge increases by $P = LD$ g kg/cm^2 when the airplane undergoes an acceleration g m/sec^2 in the direction of the tube axis. Here L is the length of the mercury column in the tube, m; D is the density of the mercury (= 0.138) [since L is given in m and not in cm].

The instrument is set up either perpendicularly to the plane of the wings, in which case it measures the vertical accelerations, or parallel to this plane, so that it measures horizontal accelerations.

Huguenard and Magnan accelerometer for measuring the accelerations of birds during flight (Figure 195)

This accelerometer is placed on the back of a carrier pigeon. It is 7.5 cm long, 3 cm wide, 3 cm high, and weighs 55 g. The instrument consists of a recording cylinder of 2 cm diameter, making one turn every six seconds and giving a diagram measuring 6×2 cm^2. The following magnitudes are
211 indicated on the diagram: 1) the vertical acceleration near the center of gravity of the bird, 2) the motion of the wings, and 3) the vibrations of a plate serving as chronograph. These vibrations are performed by the mechanism recording the motion of the wings. The lightness of the accelerometer was achieved by using as test mass the recording cylinder and its clockwork mechanism, which account for four fifths of the entire weight of the instrument.

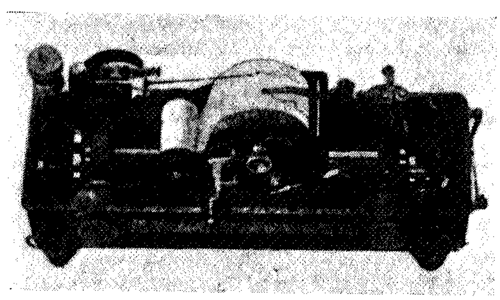


FIGURE 193. H.M.P.accelerometer. Overall view

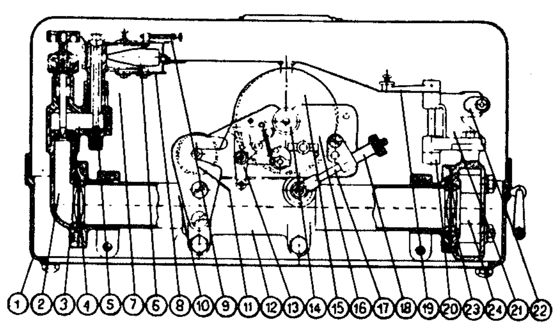


FIGURE 194. H.M.P.accelerometer. Layout

The instrument has a natural period of 0.05 sec and a sensitivity corresponding to an indication of 1 mm at an acceleration of 1 m/sec^2. It is calibrated statically and then placed on a vibrating plate equipped with an electrical interruptor. Comparison of the accelerations occurring during the harmonic vibrations of the plate with those recorded during the flight makes it possible to introduce corrections for the frequency of the movement of the pigeon wings, which is approximately five per sec. The instrument is strapped to the body of the pigeon with a harness, leaving free the neck, wings, legs, and tail. A thread of twisted silk, secured to the legs of the bird, is uncurled mechanically and permits the velocity to be measured and the flight to be stopped when a certain distance has been traversed.

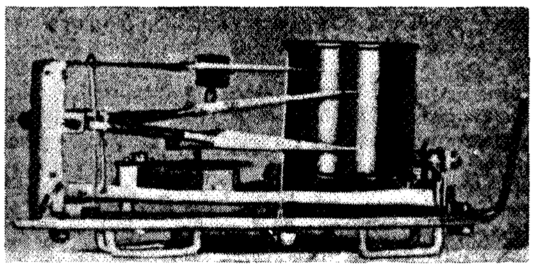


FIGURE 195. H.M. lightweight accelerometer

This instrument was used in a series of tests. It was found that pigeons develop accelerations of up to 4 g during takeoff. This made it necessary to reduce the sensitivity of the instrument by one half.

The test results led to the construction of an additional mechanism, controlled by the movement of the wings, which sets the recording cylinder into motion only when the bird is in full flight.

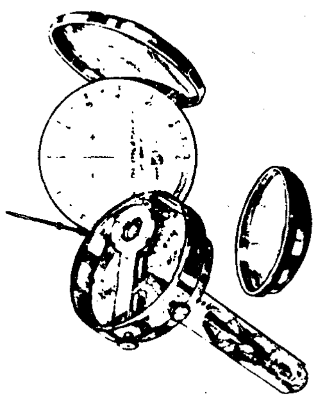


FIGURE 196. Badin accelerometer

212

Badin accelerometer

Badin in France already in 1919 proposed an accelerometer in which a liquid is used. This instrument was later perfected by him and is shown in Figure 196. It is called maximum accelerometer and is used to determine the maximum accelerations occurring during flight. The operating principle of this instrument is as follows: The lower part of the accelerometer consists of a tube (part of which is not shown) containing a liquid. A second, smaller tube filled with liquid can be seen inside the first tube. This second tube is connected to the manometric box (pressure gauge) on top. The motion of the upper cover of this box is transmitted to a pointer. The small tube is inserted with its open lower end into the liquid contained in the large tube. The pressure in the manometric box is less than the pressure of the liquid when the instrument is in a vertical position. The pressure difference corresponds to the difference in the levels of the pressure-gauge diaphragm and the free surface of the liquid in the large tube. Any acceleration is indicated by the pressure gauge in accordance with the mass of the liquid.

The outer surface of the manometric box and the surface of the liquid in the large tube are always subjected to the same static pressure, since they communicate via orifices.

Badin designed a similar pressure gauge warning the pilot of the danger of excessive acceleration by means of a red light automatically switched on by the instrument.

δ) Accelerometers with moving masses

Oberth accelerometer

Oberth designed an instrument which indicates simultaneously the acceleration and the flight altitude. Its operating principle is as follows (Figure 192): Test mass (a) tends to move on rollers (1) in the direction of the arrow during decelerated motion toward the left. Roller (2), whose axis can slide vertically along guides, then moves on the upper surface of test mass (a) and thus raises freely suspended pin (3) which can also move only vertically. This pin slides over some contacts and changes the current intensity. The upper surface of test mass (a) is shaped in such a manner that the current intensity is proportional to the air density.

e) Three-dimensional accelerometers

The simplest type of such an accelerometer is an instrument consisting of three linear accelerometers arranged along the three coordinate axes of the airplane. The accelerations can be recorded automatically on a single tape.

A more sophisticated instrument was proposed and designed by Sirley.

213 **Three-dimensional Sirley accelerometer**

Sirley in the U. K. built a three-dimensional accelerometer in 1919, for measuring the accelerations along the three coordinate axis during airplane flight. The instrument was described in Report No. 627 of the Advisory Committee for Aeronautics (1919), while its theory was expounded in Report No. 469 (1918).

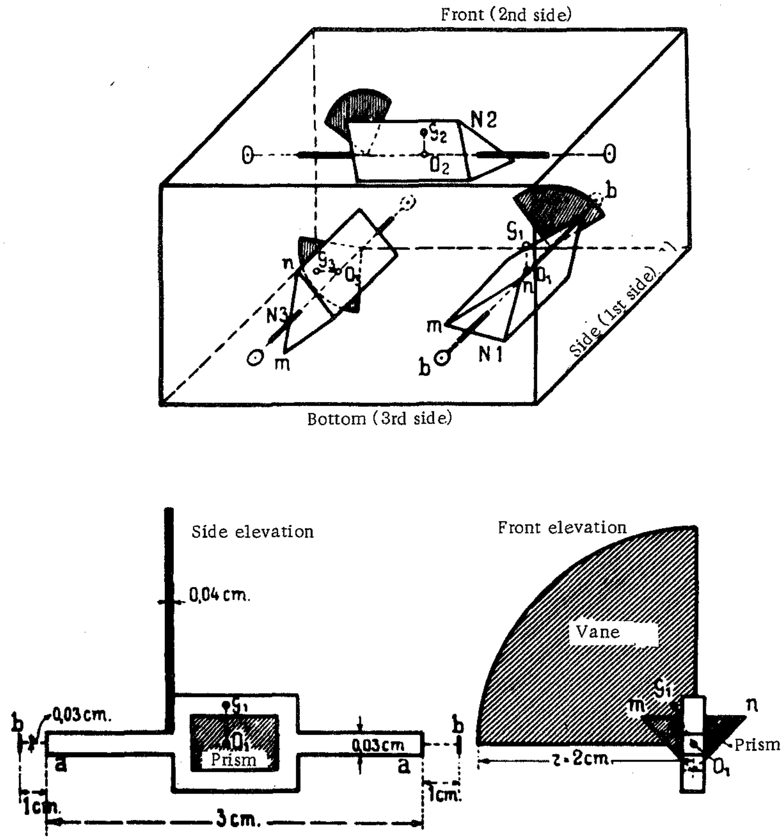


FIGURE 197. Three-dimensional Sirley accelerometer

The instrument consists of a box (Figure 197) whose sides are denoted by numbers: the right-hand side is termed 1st side, the front, 2nd side, and the bottom, 3rd side. The box contains three accelerometers denoted by numbers. Accelerometer No. 1 measures the accelerations perpendicular to the 1st side of the box, No. 2, those perpendicular to the 2nd side, and No. 3, those perpendicular to the 3rd side. The design and operating principle of the three accelerometers are the same, so that we shall describe only one of them, No. 1 (Figure 197, bottom). The instrument consists of bar ***aa***, made of phosphor bronze, to whose ends wires of the same material are
214 brazed. The other ends of the wires are brazed to scrows ***bb***, inserted into the walls of the box. A copper vane whose plane is perpendicular to the center line of the bar is fixed to the latter. A slot in the bar contains a right-angled prism whose hypotenuse face ***mn*** is silver-coated in order to ensure complete internal reflection. The bars of accelerometers Nos. 1 and 3 are perpendicular to the 2nd side of the box, while the bar of accelerometer No. 2 is perpendicular to the 1st side.

Let G_1 be the center of gravity of accelerometer No. 1, and O_1 the projection of this point on the axis of the accelerometer. Accelerometer No. 1 is set up in the box by turning screws ***bb*** in such a way that the line

G_1O_1 is parallel to the 1st side of the box. Accelerometer No. 2 is similarly set up in such a manner that the line G_2O_2 is parallel to the 2nd side while accelerometer No. 3 is set up so that the line G_3O_3 is parallel to the 3rd side. Accelerations occurring during flight are resolved along three directions respectively perpendicular to the three sides of the box. These components cause the prisms to turn about their axies through angles which vary directly with the corresponding accelerations. The silver-coated hypotenuse faces mn of the prisms reflect light beams from a lamp onto a moving photosensitive film, located outside the box behind its 1st side (not shown in Figure 197). This side has a slot letting through the beam. A fixed prism is located near accelerometer No. 2 in order to reflect onto the film the light beam arriving from this instrument (this prism is also not shown in Figure 197). The measurements obtained with the three accelerometers are represented on the film as three curves. The oscillations of the accelerometers are damped through electromagnetic induction in their copper vanes. The period of the undamped oscillations of the vanes is approximately 1/30 sec.

Electrical three-component accelerometer

An electrical three-component accelerometer was proposed by Fritz H. Meyer-Finkenkrug. Its operating principle is as follows: Accelerations in the direction indicated by the arrow cause test mass M to press against conductor R_1 or R_2; this causes a difference in the intensities i_1 and i_2 of the currents flowing through these conductors to tape D. This difference causes rotation of mirror Sp, located in the magnetic field induced between poles. A light beam from L is reflected by the mirror onto a paper tape wound on drum [Tr] driven by a clockwork mechanism, and traces on it a line representing the stresses in the conductors, or the accelerations which are proportional to them. The errors introduced by temperature differences are eliminated by stops G_1 and G_2, which expand or contract and thus compensate for differences in the current intensities i_1 and i_2. Three such systems $G_1R_1MR_2G_2$ can be arranged along the three coordinate axes and the light beams from the three mirrors reflected onto a single drum, so that the stresses due to the pressures of masses M, or the accelerations which are proportional to these stresses, are recorded. Calibration of the instrument permits the numerical values of the accelerations to be obtained.

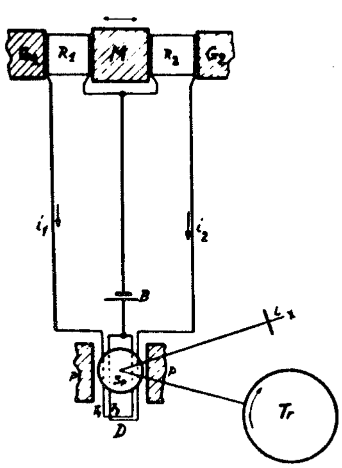


FIGURE 198. F.M.F.electrical three-component accelerometer

215 f) Angular accelerometers

An accelerometer recording the rotational motion of an airplane by means of a light beam reflected onto a drum was designed in the U.S.A. by H. Reid. This instrument is shown schematically in Figure 199. Its principal part is a gyroscope whose rotor revolves about a vertical axis. The precession force of the gyroscope acts on lever ***F***; the deflections of mirror ***G***, induced by lever ***F***, are a measure of the angular acceleration with respect to an axis perpendicular to the gyroscope axis. A light beam is emitted by lamp ***H*** and falls on mirror ***G*** from which it is reflected through a slot in a chamber inside which a drum with photosensitive paper rotates.

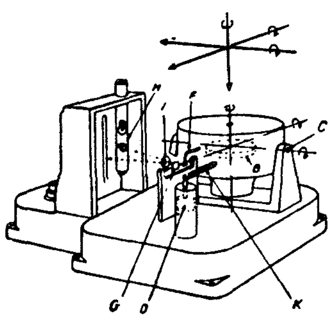


FIGURE 199. Angular accelerometer

g) Light accelerometers

Determination of the velocity of a spaceship in relation to some stars

The velocity of a spaceship in relation to some star can be determined according to the method at present used in astronomy for determining the radial speeds of stars, i.e., the speeds at which they approach or move away from us.

The principle on which this method is based can be explained by its analog in sound. Assume that we are standing near a railroad track, while a whistling steam locomotive passes us rapidly. The sound of the whistle will appear to us higher when the locomotive approaches us then when it moves away from us. This phenomenon is even more pronounced if we are traveling in the opposite direction in a train on the other track. The reason for this is that the pitch depends on the number of sound waves which reach our ears per sec.

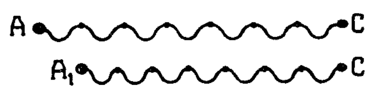


FIGURE 200

A spaceship may be likened to a train in which we are traveling, while the star is a steam locomotive moving in the opposite direction. Light can be considered as ether waves, which follow one another at enormous speed. A light beam falling on a photographic plate or refracted by a prism gives a spectrum consisting of a series of bands, the spectral lines. Each spectral line is due to a certain incandescent substance present in the light source considered. The propagation velocity of these light waves is independent of the frequency and of whether the light source is moving or stationary.

We can explain this by assuming that a light source A (Figure 200), which
216 is at rest, emits light waves toward point C. The number of waves between A and C is seven, so that the length of each wave is $AC/7$. Let source A now move toward C at such a speed that the first light wave arrives at C, when A has arrived at point A_1, where AA_1 is equal to one original wave length. The seven waves, emitted by A, while the first wave travels to C and A moves to A_1, are now located between A_1C, so that each wave contracts by 1/7. In other words, the light waves emitted by a moving source will be longer or shorter, depending upon whether the body travels in the same or opposite direction as the beam considered.

The positions of the spectral lines depend solely on the wave lengths of the light emitted. It follows from this that the spectral lines in the light emitted by a body will be displaced toward the violet or the red end of the spectrum, according to whether the body moves toward or away from the observer. This method of determining the relative direction of motion of bodies can be improved by obtaining the spectrum emitted by a heavenly body on a photographic plate together with that of a terrestrial light source formed by some incandescent chemical element. The light from this source passes through the same telescope to the same spectroscope as the star light, so that both systems of spectral lines occupy the same position if the wave lengths of the light emitted by the two sources are equal. However, even an insignificant difference between the positions of two lines obtained on the photographic plate enables the radial speed of the star observed to be determined.

The sky appears black during flight outside the atmosphere, and is strewn with silvery stars — double stars, single stars, stars of various colors, silvery nebulas, etc. The earth gradually becomes similar to the moon as its distance increases: it has phases, is eclipsed by the moon, and so forth.

The earth in its turn eclipses the sun, and night begins in the spaceship when it enters the cone of the earth's shadow.

Conclusion

This book contains an analysis of the phenomena encountered in superaviation and superartillery. In addition, the properties of the atmosphere and the features of human flight at high altitudes were described, which had not been discussed in our book "Teoriya reaktivnogo dvizheniya" (Theory of Rocket Propulsion). We discussed in detail methods of determining accelerations in flight and described accelerometers. These instruments, in addition to their importance in analyzing the flight of airplanes and airships, will play an important role in celestial navigation during interplanetary voyages.

Fundamental research on superaviation and superartillery is already being carried out in various countries. This is a step toward space flight.

The following book will be devoted to the activities of K. E. Tsiolkovskii, in particular to his work on interplanetary communication.

5817 **Printed in Jerusalem, Israel** TT 70-50116
NASA TT F-645